

# Spin Analysis of Supersymmetric Particles

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New Theory  $\Leftrightarrow$  SUSY?

Smuons  
General Analysis  
Selectrons  
Charginos/Neutralinos

Summary

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## New Theory

- Particles: mass, **spin**, parity, internal quantum numbers
- Scenarios: potentially **isomorphic** patterns

$$\begin{aligned} \text{SUSY} : \tilde{q}_L &\rightarrow q\tilde{\chi}_2^0 \rightarrow q\ell^+\tilde{\ell}^- \rightarrow q\ell^+\ell^-\tilde{\chi}_1^0 \rightarrow q\ell^+\ell^- E_{miss} \\ \text{UED} : q_1 &\rightarrow qZ_1 \rightarrow q\ell^+\ell_1^- \rightarrow q\ell^+\ell^-\gamma_1 \rightarrow q\ell^+\ell^- E_{miss} \end{aligned}$$

LHC  $[q, \ell^+, \ell^-]$  invariant masses affected by intermediate spins

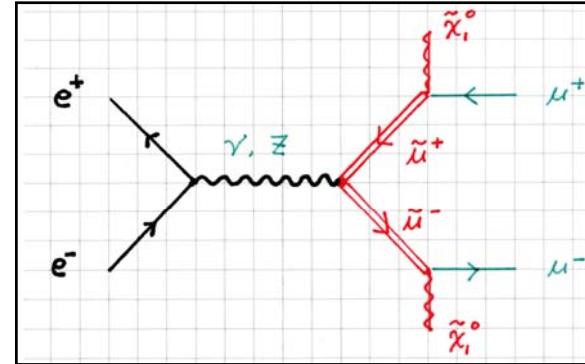
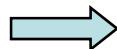
ILC model-independent spin analysis for

$$\begin{aligned} e^+e^- &\rightarrow \tilde{\mu}_R^+\tilde{\mu}_R^- \text{ and } \tilde{e}_R^+\tilde{e}_R^- \\ e^+e^- &\rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \text{ and } \tilde{\chi}_2^0\tilde{\chi}_2^0 \end{aligned}$$

- (i) threshold excitation
- (ii) production angular distribution
- (iii) decay angular distributions

## Smuons

$$e^+ e^- \rightarrow \tilde{\mu}_R^+ \tilde{\mu}_R^- \\ \rightarrow \mu^+ \mu^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$



SUSY

threshold :  $\sigma [\tilde{\mu}_R^+ \tilde{\mu}_R^-] = \frac{\pi \alpha^2}{6s} \beta^3 [Q_L^2 + Q_R^2] \sim \beta^3$

ang distrib :  $\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta} [\tilde{\mu}_R^+ \tilde{\mu}_R^-] = \frac{3}{4} \sin^2 \theta \sim \sin^2 \theta$

UED

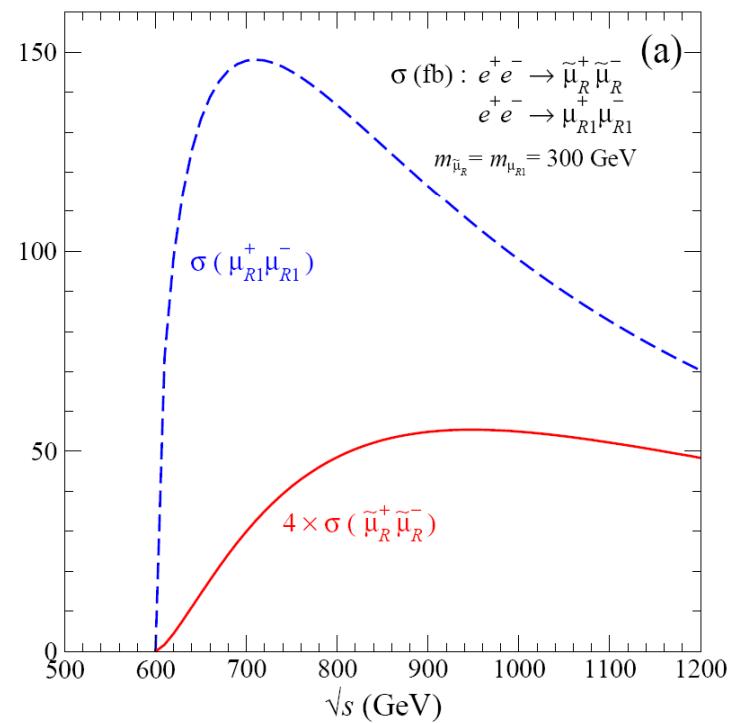
threshold :  $\sigma [\mu_{R1}^+ \mu_{R1}^-] = \frac{2\pi \alpha^2}{3s} \beta \frac{(3-\beta^2)}{2} [Q_L^2 + Q_R^2] \sim \beta$

ang distrib :  $\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta} [\mu_{R1}^+ \mu_{R1}^-] = \frac{3}{8} \frac{2}{(3-\beta^2)} [1 + \cos^2 \theta + (1 - \beta^2) \sin^2 \theta]$

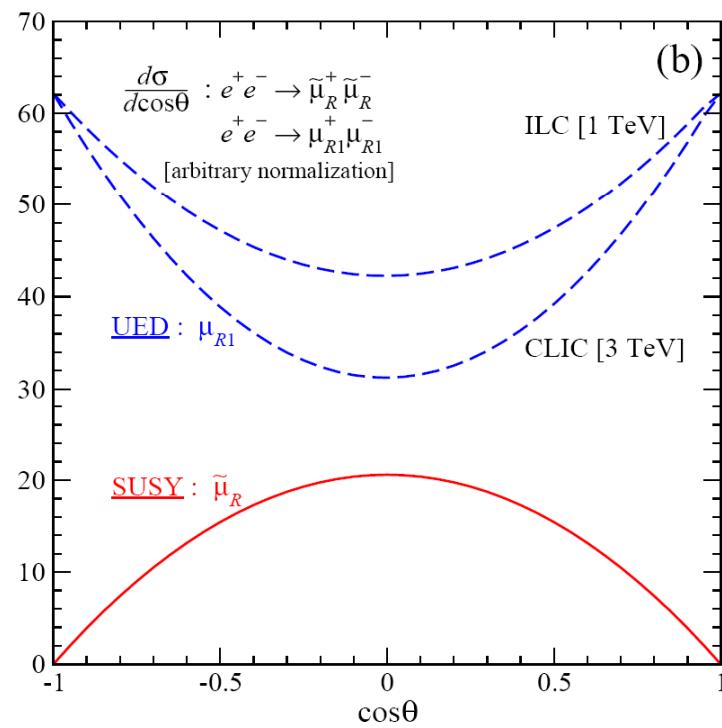
$\sim 1 \rightarrow [1 + \cos^2 \theta]$

## SUSY $\leftrightarrow$ UED

Threshold excitation



Polar-angle distribution



## General Analysis

$$e^+ e^- \rightarrow F_{\lambda_1}^J \bar{F}_{\lambda_2}^J \text{ via } \gamma, Z \text{ exchange}$$

$$\sigma = \frac{\pi \alpha^2}{3s} \beta (Q_L^2 + Q_R^2) [\sum_{\lambda} (|\mathcal{T}_{\lambda,\lambda-1}|^2 + |\mathcal{T}_{\lambda,\lambda+1}|^2) + \sum_{\lambda} |\mathcal{T}_{\lambda\lambda}|^2]$$

$$\frac{d\sigma^s}{d\cos\theta} = \frac{\pi \alpha^2}{4s} \beta (Q_L^2 + Q_R^2) \left[ \frac{1 + \cos^2\theta}{2} \sum_{\lambda} (|\mathcal{T}_{\lambda,\lambda-1}|^2 + |\mathcal{T}_{\lambda,\lambda+1}|^2) + \sin^2\theta \sum_{\lambda} |\mathcal{T}_{\lambda\lambda}|^2 \right]$$

Fermions

$$j_{\mu} = \bar{\psi}_{\alpha_1 \dots \alpha_n} \gamma_{\mu} \psi^{\alpha_1 \dots \alpha_n} \quad [\text{spin } J = n + 1/2]$$

$$= j_{\mu}^e + j_{\mu}^m : \quad j_{\mu}^e = \frac{1}{2m} \bar{\psi}_{\alpha_1, \dots, \alpha_n} i \overleftrightarrow{\partial}_{\mu} \psi^{\alpha_1, \dots, \alpha_n} \quad \text{Fierz-Pauli}$$

$$j_{\mu}^m = \frac{1}{2m} \partial^{\nu} (\bar{\psi}_{\alpha_1, \dots, \alpha_n} \sigma_{\mu\nu} \psi^{\alpha_1, \dots, \alpha_n})$$

$$\mathcal{T}_{\lambda\lambda}^e = \beta^2 \mathcal{Q}_{\lambda}^J$$

$$\mathcal{T}_{\lambda\lambda}^m = -\mathcal{Q}_{\lambda}^J$$

$$\mathcal{Q}_{\lambda}^J = \frac{\gamma}{\sqrt{2}} \left[ \frac{(J+\lambda)}{2J} Q_{\lambda-1/2}^{J-1/2}(\gamma) - \frac{(J-\lambda)}{2J} Q_{\lambda+1/2}^{J-1/2}(\gamma) \right]$$

$$Q_n^N(\gamma) = \frac{2^N (N+n)! (N-n)!}{(2N)!} \sum' \prod_{i=1}^N \frac{(2\gamma^2 \delta_{\lambda_i 0} - 1)}{(1+\lambda_i)! (1-\lambda_i)!}$$

$$\begin{aligned} \mathcal{T}_{\lambda, \lambda \pm 1}^m &= \mathcal{Q}_{\lambda \pm}^J \Rightarrow g = 1/J \\ &\Rightarrow g = 2 \quad [\text{non-min cplg :: asy unitarity}] \end{aligned}$$

Hagen et al  
Ferrara et al

J>0: non-zero magnetic contribution in forward-backward directions

## General Analysis

$$e^+ e^- \rightarrow F_{\lambda_1}^J \bar{F}_{\lambda_2}^J \text{ via } \gamma, Z \text{ exchange}$$

$$\sigma = \frac{\pi \alpha^2}{3s} \beta (Q_L^2 + Q_R^2) [\sum_{\lambda} (|\mathcal{T}_{\lambda,\lambda-1}|^2 + |\mathcal{T}_{\lambda,\lambda+1}|^2) + \sum_{\lambda} |\mathcal{T}_{\lambda\lambda}|^2]$$

$$\frac{d\sigma^s}{d\cos\theta} = \frac{\pi \alpha^2}{4s} \beta (Q_L^2 + Q_R^2) \left[ \frac{1 + \cos^2\theta}{2} \sum_{\lambda} (|\mathcal{T}_{\lambda,\lambda-1}|^2 + |\mathcal{T}_{\lambda,\lambda+1}|^2) + \sin^2\theta \sum_{\lambda} |\mathcal{T}_{\lambda\lambda}|^2 \right]$$

### Bosons

$$j_{\mu} = j_{\mu}^e + j_{\mu}^m : \quad j_{\mu}^e = i \varphi_{\alpha_1 \dots \alpha_J}^* \overleftrightarrow{\partial}_{\mu} \varphi^{\alpha_1 \dots \alpha_J} \quad \text{for } J > 0$$

$$j_{\mu}^m = -i \partial^{\nu} (\varphi_{[\mu}^{*\alpha_2 \dots \alpha_J} \varphi_{\nu]\alpha_2 \dots \alpha_J})$$

Hagen et al

$$\mathcal{T}_{\lambda\lambda}^e = \beta \mathcal{Q}_{\lambda}^{\prime J}$$

$$\mathcal{T}_{\lambda,\lambda\pm 1}^m = \beta \mathcal{Q}_{\lambda\pm}^{\prime J} : \text{non-zero contribution} : g = 1/J \Rightarrow 2$$

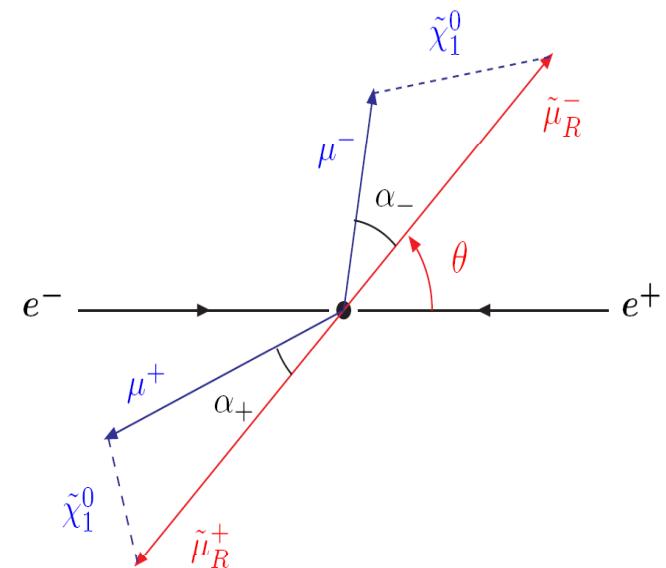
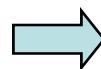
Ferrara et al

Any  $J > 0$ : non-zero magnetic contribution in FB directions  
different from scalar  $J = 0$  particles

## Experimental Analysis

Event axis reconstruction

$$\tilde{\mu}_R^+ \tilde{\mu}_R^- \rightarrow [\mu^+ \tilde{\chi}_1^0] + [\mu^- \tilde{\chi}_1^0]$$



$$m_{\mu_R^\pm}^2 - m_{\tilde{\chi}_1^0}^2 = \sqrt{s} E_{\mu^\pm} (1 - \beta_{\tilde{\mu}_R^\pm} \cos \alpha_\pm)$$



2 solutions: true  $\sim \sin^2 \Theta$  while false  $\sim$  a little flattened

## Experimental Simulation

# threshold excitation

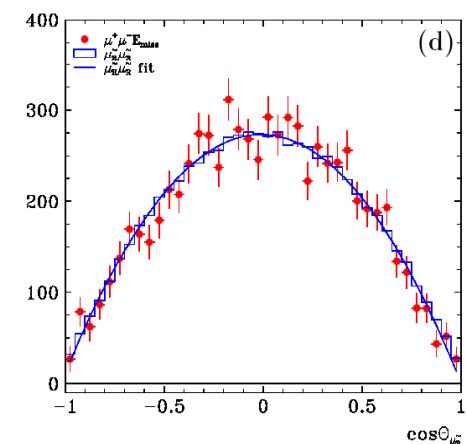
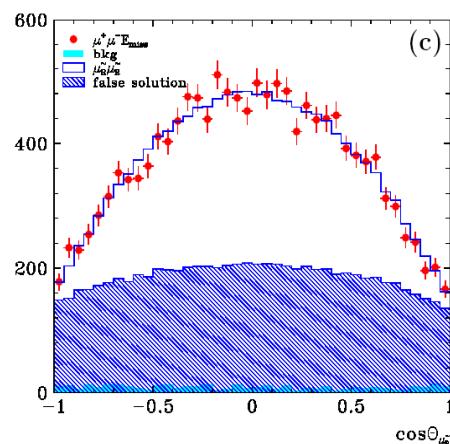
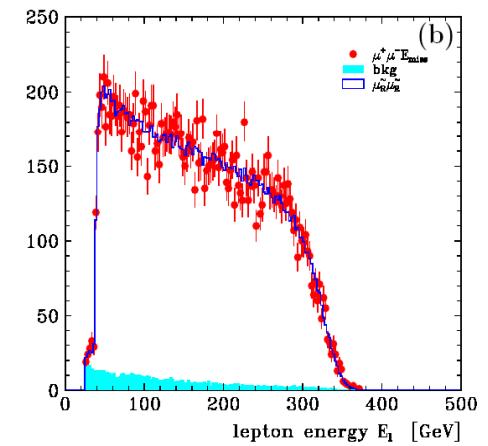
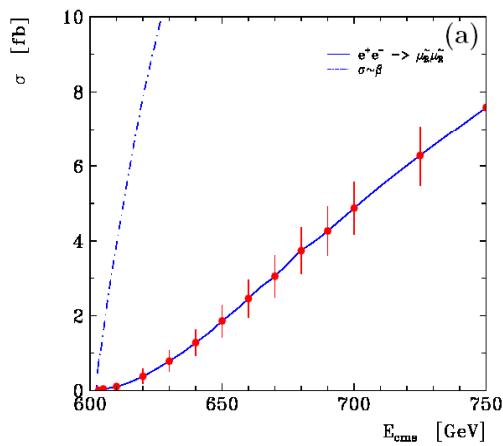
# decay distribution

# polar angle distribution

$$\frac{d\sigma^{\text{exp}}}{d \cos \theta} \sim 1 + a \cos \theta + b \cos^2 \theta$$

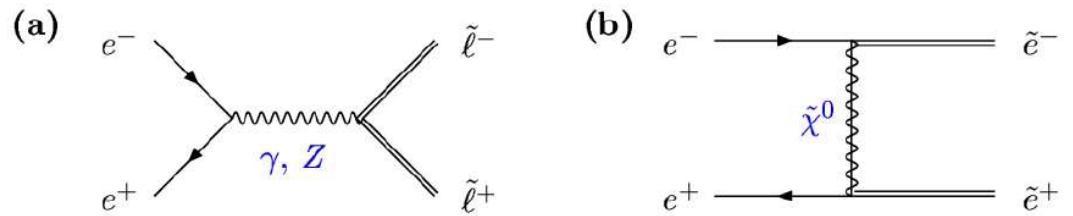
$a = -0.020 \pm 0.016$

$b = -0.979 \pm 0.022$



## Selectrons

$$e^+ e^- \rightarrow \tilde{e}_R^+ \tilde{e}_R^- \\ \rightarrow e^+ e^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$



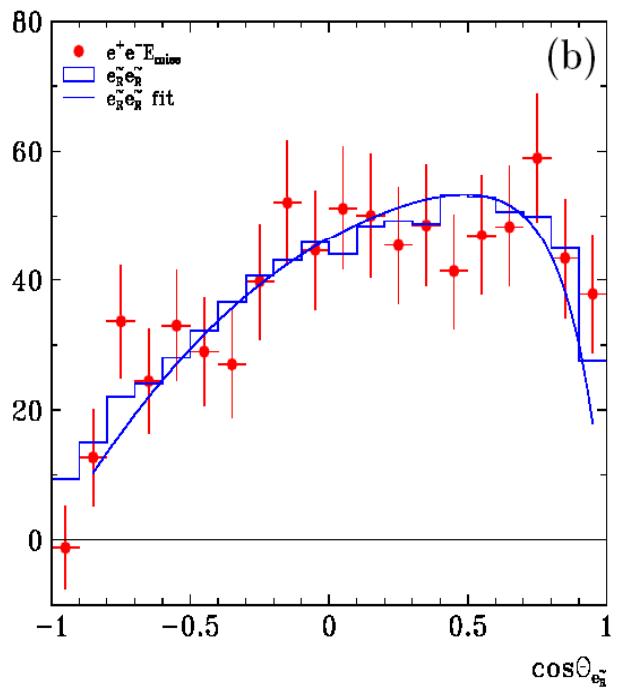
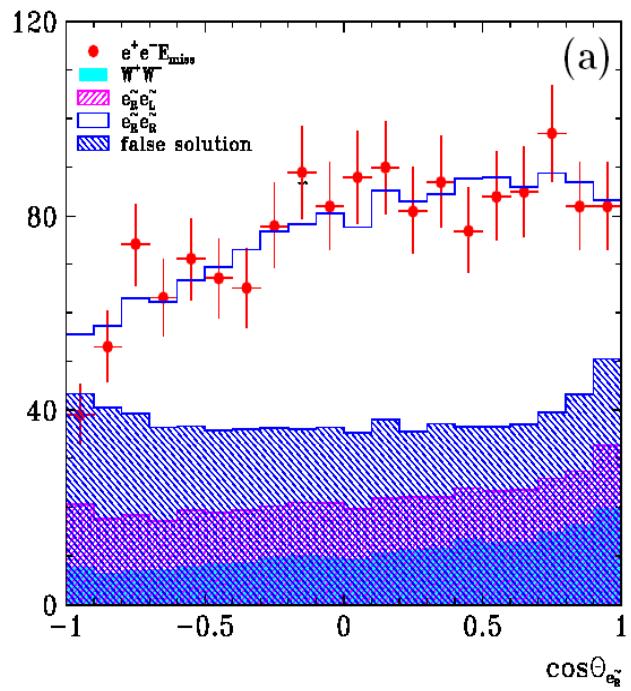
threshold :  $\sigma [\tilde{e}_R^+ \tilde{e}_R^-] \sim \beta^3$

ang distrib :  $\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta} [\tilde{e}_R^+ \tilde{e}_R^-] \sim \sin^2 \theta \mathcal{G}(\cos^2 \theta) \rightarrow \sin^2 \theta$

t-channel: contribution rising steeply above threshold  
 $\Leftrightarrow$  modifying  $\sin^2 \Theta$  law  
 $\Leftrightarrow$  switched off partly with polarized beams

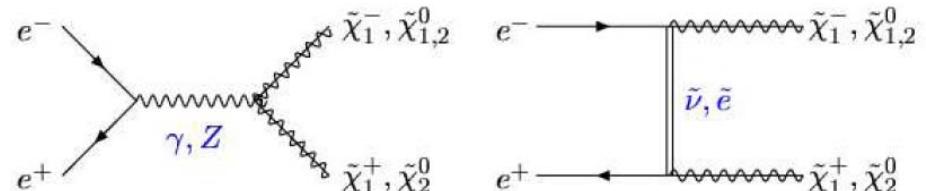
$e_L^-$ not cpld to $\tilde{e}_R^-$	$\mathcal{P}[e^-] = -80\%$
$e_R^+$ not cpld to $\tilde{e}_R^+$	$\mathcal{P}[e^+] = +60\%$

## Experimental Simulation



## Charginos and Neutralinos

$$e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \\ \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$$



### Charginos

$$\sigma = \frac{\pi \alpha^2 f_s}{2s} \beta \left\{ \langle Q_1 \rangle + \beta^2 \langle \cos^2 \theta Q_1 \rangle + 4\mu^2 \langle Q_2 \rangle + 2\beta \langle \cos \theta Q_3 \rangle \right\} \quad \text{threshold : } \sim \beta / s$$

$$\frac{d\sigma}{d \cos \theta} = \frac{\pi \alpha^2}{2s} \beta \left\{ [1 + \beta^2 \cos^2 \theta] Q_1 + 4\mu^2 Q_2 + 2\beta \cos \theta Q_3 \right\} \quad \text{threshold : } \sim 1$$

### Neutralinos

threshold excitation [Majorana /  $\mathcal{P}$ ] :  $\sim \beta^3$

angular distribution :  $\sim [1 + \cos^2 \theta] \rightarrow \cos^2 / \sin^2 \theta$  mix

## Comparison

		thr excitation	thr ang distrib
SUSY	$\tilde{\chi}^+ \tilde{\chi}^-$	$\beta$	flat
UED	$W_1^+ W_1^-$	$\beta$	flat
GENERAL	<i>Dirac pair</i>	$\beta$	flat
SUSY	$\tilde{\chi}^0 \tilde{\chi}^0$ [ <i>Majorana</i> ]	$\beta^3$	$1 + \kappa \cos^2 \theta$
UED	$Z_1 Z_1$ [ <i>Dirac</i> ]	$\beta$	flat
GENERAL	<i>Majorana pair</i>	$\beta^3$	$1 + \kappa \cos^2 \theta$

Threshold/angular distributions are unique  
neither **for charginos** nor **for neutralinos!!**



Final state analysis for polarized spin-1/2 charginos

$$\tilde{\chi}_1^\pm \rightarrow W^\pm \tilde{\chi}_1^0 \quad \Rightarrow \quad \frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_W^*} = \frac{1}{2}(1 + \kappa_W \cos \theta_W^*)$$

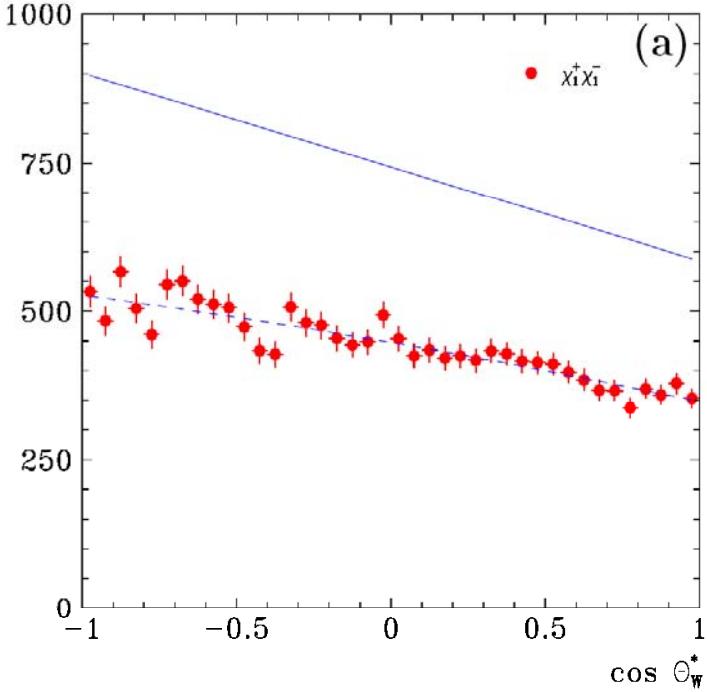
$$E_W = \gamma(E_W^* + \beta p_W^* \cos \theta_W^*)$$

## Experimental Simulation

$$\frac{d\sigma}{d \cos \theta_W^*} = 1 + \sum_{n=1}^{2J} a_n \cos n \theta_W^*$$

exp :  $a_1 = -0.203 \pm 0.020$   
 $a_2 = -0.001 \pm 0.020$

Neutralinos



$$\tilde{\chi}_2^0 \rightarrow Z \tilde{\chi}^0$$

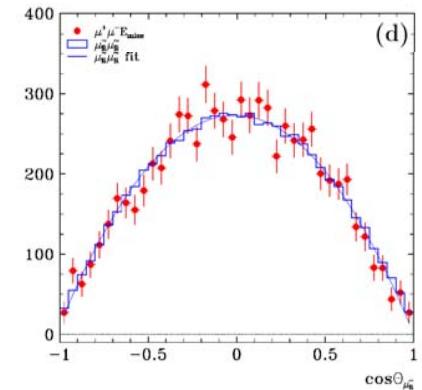
Majorana  $\Leftrightarrow$  flat angular distribution  
 $\Leftrightarrow$  Z-polarization analysis

$$\tilde{\chi}_2^0 \rightarrow \tilde{\ell}_R^+ \ell^-$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{\ell^-}^*} = \frac{1}{2} (1 + \mathcal{P}_{\tilde{\chi}_2^0} \cos \theta_{\ell^-}^*)$$

## Summary

[Sleptons]  
Angular distribution  $\sim \sin^2 \theta$   
in pair production in  $e^+e^-$  collisions:  
unique signal for spin =0



[Charginos]  
Threshold excitation  $\sim \beta$   
Flat angular distribution near threshold  
Decay angular distribution: no  $\cos n \theta$  for  $n > 1$

[Neutralinos]  
Threshold excitation  $\sim \beta^3$   
Angular distribution  $\sim$  mixed cos/sin  
Decays: Z-polarization analysis or lepton angles

ILC  $\Leftrightarrow$  powerful spin determination of SUSY particles