

# SUPERSYMMETRIC HIGGS YUKAWA COUPLINGS TO BOTTOM QUARKS AT NNLO

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- I Introduction
- II  $\phi^0 b \overline{b}$  Couplings
- III Conclusions

in collaboration with D. Noth



### <u>MSSM</u>

- 2 Higgs doublets  $\xrightarrow{\text{ESB}}$  5 Higgs bosons:  $h, H, A, H^{\pm}$
- LO: 2 input parameters:  $M_A$ ,  $tg\beta = \frac{v_2}{v_1}$ • radiative corrections  $\propto m_t^4 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \rightarrow M_h \lesssim 135 \text{ GeV}$ Haber, Hempfling Carena,... Heinemeyer,... Martin Harlander,...  $\dots$
- Yukawa couplings:  $tg\beta\uparrow \Rightarrow g_u^{\phi}\downarrow g_d^{\phi}\uparrow g_V^{\phi}\downarrow$

• LHC: 
$$gg \rightarrow \phi$$
 dominant for  $tg\beta \lesssim 10$   
 $gg \rightarrow \phi b\overline{b}$  dominant for  $tg\beta \gtrsim 10$ 





- QCD corrections to  $\phi^0 \rightarrow b\bar{b}$  known to NNNLO
- large SUSY–QCD corrections to  $\phi^0 \rightarrow b\bar{b}$

Braaten, Leveille Drees, Hikasa Kataev,... Chetyrkin,... etc.

 $(\Delta\Gamma/\Gamma\sim 10\%)$ 

Hall,... Carena,... Nierste,... Guasch,... etc.

dominated by scale dependence of  $\alpha_s$ 

Guasch, Häfliger, S.





 $\begin{aligned} \underline{\text{SUSY-QCD Corrections to } b\bar{b}\phi^{0}} & [\Delta \lesssim 1\%] \\ \mathcal{L}_{eff} &= -\lambda_{b}\overline{b_{R}} \left[ \phi_{1}^{0} + \frac{\Delta_{b}}{\text{tg}\beta} \phi_{2}^{0*} \right] b_{L} + h.c. \quad \text{valid to all orders in } \Delta_{b} \\ &= -m_{b}\overline{b} \left[ 1 + i\gamma_{5} \frac{G^{0}}{v} \right] b - \frac{m_{b}/v}{1 + \Delta_{b}} \overline{b} \left[ g_{b}^{h} \left( 1 - \frac{\Delta_{b}}{\text{tg}\alpha} \text{tg}\beta} \right) h \\ &+ g_{b}^{H} \left( 1 + \Delta_{b} \frac{\text{tg}\alpha}{\text{tg}\beta} \right) H - g_{b}^{A} \left( 1 - \frac{\Delta_{b}}{\text{tg}^{2}\beta} \right) i\gamma_{5}A \right] b \end{aligned}$ 

$$\Delta_{b} = \Delta_{b}^{QCD(1)} + \Delta_{b}^{elw(1)}$$

$$\Delta_{b}^{QCD(1)} = \frac{2}{3} \frac{\alpha_{s}(\mu_{R})}{\pi} M_{\tilde{g}} \mu \, \mathrm{tg}\beta \, I(m_{\tilde{b}_{1}}^{2}, m_{\tilde{b}_{2}}^{2}, M_{\tilde{g}}^{2})$$

$$\Delta_{b}^{elw(1)} = \frac{\lambda_{t}^{2}(\mu_{R})}{(4\pi)^{2}} \mu \, A_{t} \, \mathrm{tg}\beta \, I(m_{\tilde{t}_{1}}^{2}, m_{\tilde{t}_{2}}^{2}, \mu^{2})$$

$$I(a, b, c) = \frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a - b)(b - c)(a - c)}$$
Carena, Garcia, N

 $\Rightarrow$  resummed Yukawa couplings

Carena, Garcia, Nierste, Wagner Guasch, Häfliger, S.







• LET:  $v_2 \rightarrow \sqrt{2}\phi_2^{0*}$ 

Ellis,... Shifman,...





Bednyakov,... Martin Bauer,...



- 2-loop self-energies @ vanishing momentum
- dimensional regularization in  $n = 4 2\epsilon$  dimensions
- integration by parts: reduction to 1-point functions

$$A_0(m) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2}$$

and 2-loop master integrals

$$T_{134}(m_1, m_3, m_4) = \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k - q)^2 - m_3^2](q^2 - m_4^2)}$$

- $\alpha_s, \lambda_t$ : MS scheme [5 flavours] masses,  $A_t$ : on-shell
- dim. reg. violates SUSY: anomalous counter terms

$$\hat{g}_{s} = g_{s} \left[ 1 + \left( \frac{C_{A}}{6} - \frac{C_{F}}{8} \right) \frac{\alpha_{s}}{\pi} \right]$$

$$\lambda_{Hbb} = \lambda_{H\tilde{b}\tilde{b}} \left[ 1 + \frac{C_{F}}{4} \frac{\alpha_{s}}{\pi} \right] = \lambda_{\tilde{H}\tilde{b}b} \left[ 1 + \frac{3}{8} C_{F} \frac{\alpha_{s}}{\pi} \right]$$
Martin, Vaughn



• 
$$m_t^2 \ll m_{\tilde{g}}^2 = m_{\tilde{b}_i}^2 = \mu^2 = m_{\tilde{t}_i}^2 \equiv M^2$$
:  

$$\Delta_b^{QCD(1)} = \frac{C_F \alpha_s(\mu_R)}{4 \pi} \text{tg}\beta$$

$$\Delta_b^{QCD(2)} = \frac{\alpha_s}{\pi} \left\{ \frac{C_A}{3} + C_F + \frac{N_F + 1}{4} + \frac{1}{6} \log \frac{M^2}{m_t^2} + \beta_0^L \log \frac{\mu_R^2}{M^2} \right\} \Delta_b^{QCD(1)}$$

$$\Delta_b^{elw(1)} = \frac{1}{2} \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} \frac{A_t \text{tg}\beta}{M}$$

$$\Delta_b^{elw(2)} = C_F \frac{\alpha_s}{\pi} \left\{ \frac{7}{4} + \frac{3}{2} \log \frac{\mu_R^2}{m_t M} \right\} \Delta_b^{elw(1)} \text{ Noth, S.}$$

$$\lambda_t(\mu_R) = \lambda_{t,MO}^{\tilde{h}^{\pm}}(\mu_R) \left\{ 1 - C_F \frac{\alpha_s}{\pi} \left[ \frac{3}{8} \log \frac{\mu_R^2}{M^2} - \frac{1}{8} \right] \right\} \qquad [\mu_R \ll M]$$
  
$$\beta_0^L = \frac{11C_A - 2N_F}{12}$$



$$\begin{split} \Delta_{b}^{QCD\,(1)} &= \frac{C_{F}\alpha_{s}(\mu_{R})\mu\mathrm{tg}\beta}{2} \left\{ -\log\frac{m_{\tilde{g}}^{2}}{m_{\tilde{b}_{2}}^{2}} + \frac{m_{\tilde{b}_{1}}^{2}}{m_{\tilde{b}_{2}}^{2} - m_{\tilde{b}_{1}}^{2}}\log\frac{m_{\tilde{b}_{2}}^{2}}{m_{\tilde{b}_{1}}^{2}} \right\} \\ \Delta_{b}^{QCD\,(2)} &= \frac{\alpha_{s}}{\pi} \left\{ \frac{4}{3}C_{A} + C_{F} \left[ \log\frac{m_{\tilde{g}}^{2}}{m_{\tilde{b}_{2}}^{2}} - \frac{m_{\tilde{b}_{1}}^{2}}{m_{\tilde{b}_{2}}^{2} - m_{\tilde{b}_{1}}^{2}}\log\frac{m_{\tilde{b}_{2}}^{2}}{m_{\tilde{b}_{1}}^{2}} + \frac{5}{2} \right] - \frac{N_{F} + 1}{2} \\ &+ \frac{1}{6}\log\frac{m_{\tilde{g}}^{2}}{m_{t}^{2}} + \frac{1}{24}\sum_{\tilde{q}}\log\frac{m_{\tilde{g}}^{2}}{m_{\tilde{q}}^{2}} + \beta_{0}^{L}\log\frac{\mu_{R}^{2}}{m_{\tilde{g}}^{2}} \right\} \Delta_{b}^{QCD\,(1)} \\ & \Delta_{b}^{elw\,(2)} = C_{F}\frac{\alpha_{s}}{\pi} \left\{ \frac{23}{8} + \frac{3}{2}\log\frac{\mu_{R}^{2}}{m_{t}m_{\tilde{g}}} \right\} \Delta_{b}^{elw\,(1)} \\ & \lambda_{l}(\mu_{R}) \to \lambda_{l,MO}^{\tilde{h}^{\pm}}(m_{l}) \end{split}$$
Noth, S.

$$\lambda_t(\mu_R) = \lambda_{t,MO}^{\tilde{h}^{\pm}}(\mu_R) \left\{ 1 - C_F \frac{\alpha_s}{\pi} \left[ \frac{3}{8} \log \frac{\mu_R^2}{m_{\tilde{g}}^2} + \frac{9}{16} \right] \right\} \qquad [\mu_R \ll m_{\tilde{g}}]$$



small  $\alpha_{eff}$  scenario [modified]

Carena, Heinemeyer, Wagner, Weiglein

$$\begin{array}{rcl} \mathrm{tg}\beta &=& 30\\ M_{\tilde{Q}} &=& 800 \ \mathrm{GeV}\\ M_{\tilde{g}} &=& 1000 \ \mathrm{GeV} &\longleftarrow\\ M_2 &=& 500 \ \mathrm{GeV}\\ A_b = A_t &=& -1.133 \ \mathrm{TeV}\\ \mu &=& 2 \ \mathrm{TeV} \end{array}$$

$$\begin{array}{rcl} m_{\tilde{t}_1} &=& 679 \,\, {\rm GeV} & m_{\tilde{t}_2} = 935 \,\, {\rm GeV} \\ m_{\tilde{b}_1} &=& 601 \,\, {\rm GeV} & m_{\tilde{b}_2} = 961 \,\, {\rm GeV} \end{array}$$





















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## $\vee \underline{SUMMARY}$

- Higgs boson searches @ Tevatron, LHC, ILC major endeavours
- MSSM: bottom Yukawa couplings dominate Higgs boson production & decay processes for large  $tg\beta$
- large NLO SUSY corrections to bottom Yukawa couplings [ $\leftarrow \Delta_b$ ]
- NNLO corrections to  $\Delta_b$ :  $\mathcal{O}(10\%)$ ,  $\Delta_{scale} \sim \mathcal{O}(10\%) \rightarrow \mathcal{O}(1\%)$ [mutual agreement with Mihaila, Reisser]

### $\rightarrow$ HDECAY

Djouadi, Kalinowski, Mühlleitner, S.

• applicable for  $e^+e^- \rightarrow b\overline{b}\phi^0, t\overline{b}H^-$  and  $pp \rightarrow b\overline{b}\phi^0, t\overline{b}H^-$ 



Comparison with A. Bauer *et al.*, JHEP **0902** (2009) 037 • MSSM decoupling relations:  $\overline{m}_b^{(5)}(\mu_R) = \zeta_{m_b} m_b^{(SQCD)}(\mu_R)$  (bottom mass)

(i) <u>Equal masses</u>:  $m_t^2 \ll m_{\tilde{g}}^2 = m_{\tilde{b}_i}^2 = \mu^2 = m_{\tilde{t}_i}^2 \equiv M^2$ • decoupling heavy particles from  $\alpha_s$  and  $A_b$ :  $(L_i = \log \frac{\mu_R^2}{m_i^2})$ 

$$\alpha_s^{SUSY}(\mu_R) = \alpha_s^{\overline{MS}}(\mu_R) \left\{ 1 + \frac{\alpha_s}{\pi} \left[ C_A \frac{1 + 2L_M}{12} + \frac{L_t}{6} + \frac{N_F + 1}{12} L_M \right] \right\}$$
$$A_b^{SUSY}(\mu_R) = A_b^{\overline{MS}}(\mu_R) \left\{ 1 - \frac{3}{4} C_F \frac{\alpha_s}{\pi} \frac{A_b - \mu \text{tg}\beta}{A_b} L_M \right\}$$

$$\begin{split} \bar{\zeta}_{m_b}^{(2)} &= C_F \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\mu \text{tg}\beta}{M} \left\{ C_A \frac{1+3L_M}{16} + C_F \frac{4-3L_M}{16} + \frac{N_F+1}{12} \frac{3-3L_M}{4} \right\} + \Delta_{\alpha_s} + \Delta_{A_b} \\ &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ C_A \frac{1+3L_M}{4} + C_F \frac{4-3L_M}{4} + \frac{N_F+1}{12} (3-3L_M) \right\} + \Delta_{\alpha_s} + \Delta_{A_b} \\ \Delta_{\alpha_s} &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ C_A \frac{1+2L_M}{12} + \frac{L_t}{6} + \frac{N_F+1}{12} L_M \right\} \\ \Delta_{A_b} &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \frac{3}{4} C_F \frac{\alpha_s}{\pi} L_M \qquad \qquad \beta_0^L = \frac{11C_A - 2N_F}{12} \\ \overline{\zeta}_{m_b}^{(2)} &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ \frac{C_A}{3} + C_F + \frac{N_F+1}{4} + \frac{1}{6} \log \frac{M^2}{m_f^2} + \beta_0^L \log \frac{\mu_R^2}{M^2} \right\} \end{split}$$

(ii) Large gluino mass:  $m_t^2, m_{\tilde{b}_i}^2, \mu^2, m_{\tilde{t}_i}^2 \ll m_{\tilde{g}}^2$   $[m_{\tilde{b}}^2 = m_{\tilde{b}_1}^2 = m_{\tilde{b}_2}^2]$ • decoupling heavy particles from  $\alpha_s$  and  $A_b$ :  $(L_i = \log \frac{\mu_R^2}{m_i^2})$ 

$$\alpha_s^{SUSY}(\mu_R) = \alpha_s^{\overline{MS}}(\mu_R) \left\{ 1 + \frac{\alpha_s}{\pi} \left[ C_A \frac{1 + 2L_{\tilde{g}}}{12} + \frac{L_t}{6} + \frac{1}{24} \sum_{\tilde{q}} L_{\tilde{q}} \right] \right\}$$
$$A_b^{SUSY}(\mu_R) = A_b^{\overline{MS}}(\mu_R) \left\{ 1 - C_F \frac{\alpha_s}{\pi} \frac{A_b - \mu \mathrm{tg}\beta}{A_b} \left[ \log \frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2} + \frac{9}{8} + \frac{3}{4} L_{\tilde{g}} \right] \right\}$$

$$\begin{split} \overline{\zeta}_{m_b}^{(2)} &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ C_A \left[ \frac{5}{4} + \frac{3}{4} L_{\tilde{g}} \right] + C_F \left[ \frac{3}{8} - \frac{3}{4} L_{\tilde{g}} \right] - \frac{N_F + 1}{4} (2 + L_{\tilde{g}}) \right\} + \Delta_{\alpha_s} + \Delta_{A_b} \\ \Delta_{\alpha_s} &= \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ C_A \frac{1 + 2L_{\tilde{g}}}{12} + \frac{L_t}{6} + \frac{1}{24} \sum_{\tilde{q}} L_{\tilde{q}} \right\} \\ \Delta_{A_b} &= \Delta_{QCD}^{(1)} C_F \frac{\alpha_s}{\pi} \left\{ \log \frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2} + \frac{9}{8} + \frac{3}{4} L_{\tilde{g}} \right\} \qquad \beta_0^L = \frac{11C_A - 2N_F}{12} \end{split}$$

$$\overline{\zeta}_{m_b}^{(2)} = \Delta_{QCD}^{(1)} \frac{\alpha_s}{\pi} \left\{ \frac{4}{3} C_A + C_F \left[ \log \frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2} + \frac{3}{2} \right] - \frac{N_F + 1}{2} + \frac{1}{6} \log \frac{m_{\tilde{g}}^2}{m_t^2} + \frac{1}{24} \sum_{\tilde{q}} \log \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} + \beta_0^L \log \frac{\mu_R^2}{m_{\tilde{g}}^2} \right\}$$

#### Decoupling of heavy particles



(i) <u>Equal masses</u>:  $m_t^2 \ll m_{\tilde{g}}^2 = m_{\tilde{b}_i}^2 = \mu^2 = m_{\tilde{t}_i}^2 \equiv M^2$ 

$$\lambda_{t,MO}(\mu_R) = \lambda_{t,MO}^{\tilde{h}}(\mu_R) \left\{ 1 - C_F \frac{\alpha_s}{\pi} \left[ \frac{3}{8} \log \frac{\mu_R^2}{M^2} - \frac{1}{8} \right] \right\}$$

(ii) <u>Large gluino mass</u>:  $m_t^2, m_{\tilde{b}_i}^2, \mu^2, m_{\tilde{t}_i}^2 \ll m_{\tilde{g}}^2$  $\lambda_{t,MO}(\mu_R) = \lambda_{t,MO}^{\tilde{h}}(\mu_R) \left\{ 1 - C_F \frac{\alpha_s}{\pi} \left[ \frac{3}{8} \log \frac{\mu_R^2}{m_{\tilde{g}}^2} + \frac{9}{16} \right] \right\}$ 

$$\begin{split} m_{\tilde{q}}^{0\,2} &= m_{\tilde{q}}^{2} + \delta m_{\tilde{q}}^{2} & m_{\tilde{g}}^{0} = m_{\tilde{g}} + \delta m_{\tilde{g}} & \beta_{0} = \frac{3C_{A} - N_{F} - 1}{4} \\ \lambda_{t}^{0} &= \lambda_{t}(\mu_{R}) + \delta\lambda_{t} & \alpha_{s}^{0} = \alpha_{s}(\mu_{R}) + \delta\alpha_{s} & A_{t}^{0} = A_{t} + \delta A_{t} \\ \delta m_{\tilde{t}_{t}}^{2} &= -iC_{F}g_{s}^{2} \left\{ 2A_{0}(m_{\tilde{t}_{t}}) - 2A_{0}(m_{\tilde{g}}) - 2A_{0}(m_{t}) - 4m_{\tilde{t}_{t}}^{2}B_{0}(m_{\tilde{t}_{t}}^{2}; 0, m_{\tilde{t}_{t}}) \\ + 2(m_{\tilde{t}_{s}}^{2} - m_{\tilde{g}}^{2} - m_{t}^{2})B_{0}(m_{\tilde{t}_{t}}^{2}; m_{\tilde{g}}; m_{t}) \right\} \\ \delta m_{\tilde{b}_{t}}^{2} &= -iC_{F}g_{s}^{2} \left\{ 2A_{0}(m_{\tilde{b}_{t}}) - 2A_{0}(m_{\tilde{g}}) - 4m_{\tilde{b}_{t}}^{2}B_{0}(m_{\tilde{b}_{t}}^{2}; 0, m_{\tilde{b}_{t}}) + 2(m_{\tilde{b}_{t}}^{2} - m_{\tilde{g}}^{2})B_{0}(m_{\tilde{b}_{t}}^{2}; m_{\tilde{g}}, 0) \right\} \\ \frac{\delta m_{\tilde{g}}}{m_{\tilde{g}}} &= -ig_{s}^{2} \left\{ C_{A} \left[ 2B_{0}(m_{\tilde{g}}^{2}; 0, m_{\tilde{g}}) - 2B_{1}(m_{\tilde{g}}^{2}; 0, m_{\tilde{g}}) - \frac{i}{(4\pi)^{2}} \right] \\ -\sum_{q} \left[ B_{0}(m_{\tilde{g}}^{2}; m_{\tilde{q}_{t}}, m_{q}) - B_{0}(m_{\tilde{g}}^{2}; m_{\tilde{q}_{t}}, m_{q}) - B_{1}(m_{\tilde{g}}^{2}; m_{\tilde{q}_{t}}, m_{q}) - B_{1}(m_{\tilde{g}}^{2}; m_{\tilde{g}_{t}}, m_{q}) \right] \right\} \\ \frac{\delta \lambda_{t}}{\lambda_{t}} &= C_{F}\frac{\alpha_{s}}{\pi} \Gamma(1 + \epsilon)(4\pi)^{\epsilon} \frac{3}{4} \left\{ -\frac{1}{\epsilon} + \log \frac{\mu_{R}^{2}}{\mu^{2}} \right\} + iC_{F}g_{s}^{2} \left\{ B_{1}(m_{t}^{2}; m_{\tilde{g}}, m_{\tilde{t}_{t}}) + B_{1}(m_{t}^{2}; m_{\tilde{g}}, m_{\tilde{t}_{s}}) \right\} \\ \frac{\delta \alpha_{s}}{\alpha_{s}} &= \frac{\alpha_{s}}{\pi} \Gamma(1 + \epsilon)(4\pi)^{\epsilon} \left\{ \left( -\frac{1}{\epsilon} + \log \frac{\mu_{R}^{2}}{\mu^{2}} \right) \beta_{0} + \frac{C_{A}}{6} \log \frac{\mu_{R}^{2}}{m_{\tilde{g}}^{2}} + \frac{1}{6} \log \frac{\mu_{R}^{2}}{m_{t}^{2}} + \sum_{\tilde{q}} \frac{1}{24} \log \frac{\mu_{R}^{2}}{m_{\tilde{q}}^{2}} \right\} \\ \frac{\delta A_{t}}{A_{t}} &= -iC_{F}g_{s}^{2} \left\{ \frac{A_{0}(m_{t})}{m_{t}^{2}} + 2B_{0}(m_{t}^{2}; 0, m_{t}) + B_{1}(m_{t}^{2}; m_{\tilde{g}}, m_{\tilde{t}_{t}}) + B_{1}(m_{t}^{2}; m_{\tilde{g}}, m_{\tilde{t}_{s}}) \right\} \\ -\frac{4}{m_{\tilde{t}_{1}}^{2}B_{0}(m_{\tilde{t}_{1}}^{2}; 0, m_{\tilde{t}_{1}}) - m_{\tilde{t}_{2}}^{2}B_{0}(m_{\tilde{t}_{2}}^{2}; 0, m_{\tilde{t}_{s}}) \\ +2\frac{(m_{\tilde{t}_{1}}^{2} - m_{\tilde{t}_{2}}^{2} - m_{\tilde{t}_{2}}^{2})B_{0}(m_{\tilde{t}_{1}}^{2}; m_{\tilde{t}}, m_{t}) - (m_{\tilde{t}_{2}}^{2} - m_{\tilde{t}_{2}}^{2})B_{0}(m_{\tilde{t}_{2}}^{2}; m_{\tilde{g}}, m_{t}) \right\} \end{cases}$$















200 300 500

M<sub>H</sub> [GeV]

10

100

1000

10

100



-gg

 $ZZ \rightarrow \leftarrow c\overline{c}$ 

tt

200 300

M<sub>H</sub> [GeV]

500

1000

BR(H)

tgβ=3



