

Simulation of Intrabeam Scattering

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Thanks to : Y. Papaphilippou and F. Antoniou

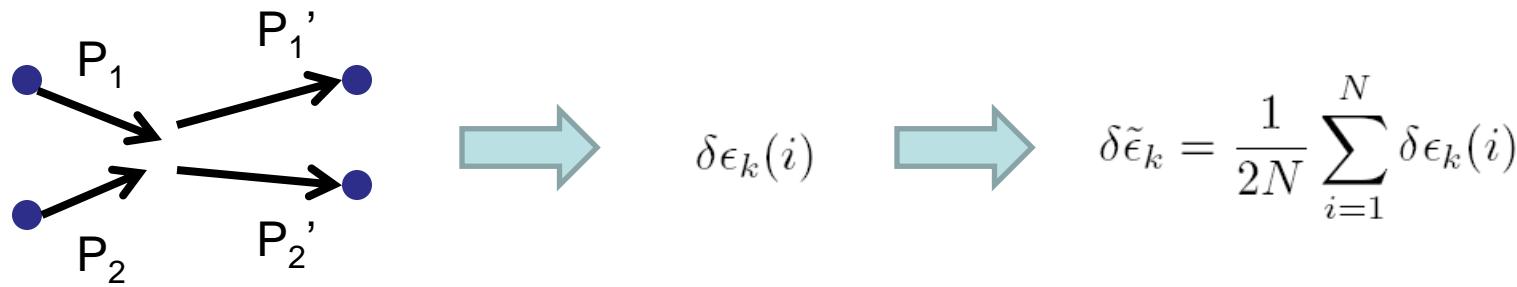
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- Conventional Calculation of IBS
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Introduction: Intra-Beam Scattering in DR

IBS is the effect due to multiple Coulomb scattering between charged particles in the beam:



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Evolution of the emittance:

$$\frac{d\tilde{\epsilon}_k}{dt} = -\frac{1}{\tau_k} (\tilde{\epsilon}_k - \epsilon_k^0) + \frac{\tilde{\epsilon}_k}{T_k}$$

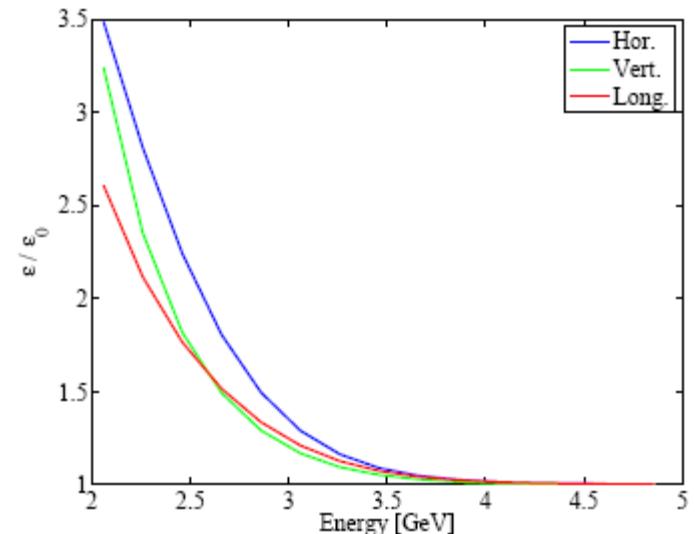
T_k IBS Growth Times

Radiation Damping

Quantum Excitation

IBS

T_k contain the effect of all the scattering processes in the beam at a certain time.



Introduction: Conventional theories of IBS

Conventional IBS theories in Accelerator Physics ([Bjorken-Mtingwa](#), Piwinski–Martini) derive T_k by the formula:

$$\frac{1}{T_k} = \frac{1}{\epsilon_k} \frac{1}{2N} \int d^3x d^3p_1 d^3p_2 d^3p'_1 d^3p'_2 \rho(x, p_1) \rho(x, p_2) |M|^2 [\epsilon_k(x, p'_1) - \epsilon_k(x, p_1)] \frac{\delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)}{(2\pi)^2}$$

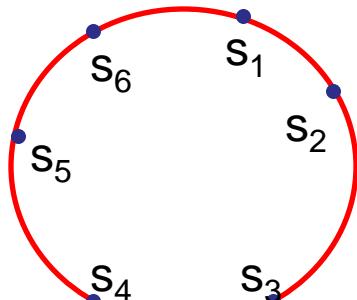
1. The particle distribution is inserted from outside the theory.
2. The integral is too complicate.

In practise, the integral has been solved only for Gaussian particles distribution.

In this case the formulas for the growth times reduce to:

$$\frac{1}{T_k} = \frac{r_0^2 c N(\log)}{8\pi\gamma^4\beta^3\epsilon_x\epsilon_z\epsilon_s} \int_0^\infty d\lambda \frac{\lambda^{1/2}}{|L + \lambda I|^{1/2}} \left\{ Tr L^{(k)} T r (L + \lambda I)^{-1} - 3 Tr L^{(k)} (L + \lambda I)^{-1} \right\}$$

Growth rates are calculated at different points of the lattice and then averaged over the ring:



$$\frac{1}{T_k} = \sum_{i=1}^M \frac{S_{i+1} - S_i}{C} \frac{1}{T_k^i} \quad S_{M+1} = C \quad S_1 = 0$$

10/20/2010

IBS studies for the CLIC Damping Rings

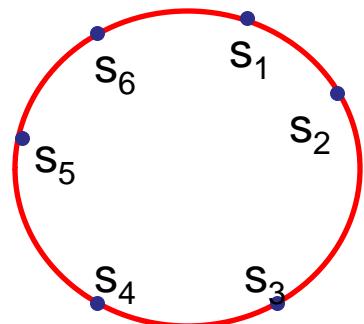
Goals:

1. Follow the evolution of the particle distribution in the DR (we are not sure it remains Gaussian).
2. Calculate IBS effect for any particle distribution (in case it doesn't remain Gaussian).

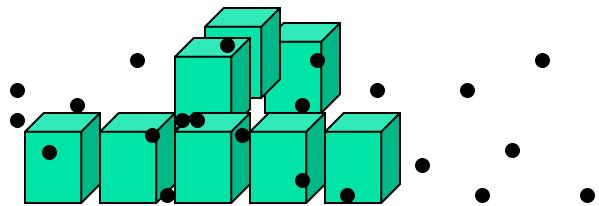


Development development of a tracking code computing the combined effect of radiation damping, quantum excitation and IBS during the cooling time in the CLIC DR.
(Software for IBS and Radiation Effects)

Software for IBS and Radiation Effects



- The lattice is read from a MADX file containing the Twiss functions.
- Particles are tracked from point to point in the lattice by their invariants (no phase tracking up to now).
- At each point of the lattice the scattering routine is called.



- 6-dim Coordinates of particles are calculated.
- Particles of the beam are grouped in cells.
- Momentum of particles is changed because of scattering.
- Invariants of particles are recalculated.

- Radiation damping and excitation effects are evaluated at the end of every loop.
- A routine has also been implemented in order to speed up the calculation of IBS effect.

SIRE: Motion in the DR

The motion of the particles in the CLIC damping rings can be expressed through 3 invariants (and 3 phases).

Transversal invariants:

$$\epsilon_x(i) = \beta_x \left(x'_i - D' \frac{\Delta p_i}{p} \right)^2 + 2\alpha_x \left(x'_i - D' \frac{\Delta p_i}{p} \right) \left(x_i - D \frac{\Delta p_i}{p} \right) + \gamma_x \left(x_i - D \frac{\Delta p_i}{p} \right)^2$$

$$\epsilon_z(i) = \beta_z {z'}_i^2 + 2\alpha_z z_i z'_i + \gamma_z {z_i}^2$$

Longitudinal invariant:

$$\epsilon_s(i) = \left(\frac{\Delta p_i}{p} \right)^2 + \frac{(2\pi)^2 \nu_s^2}{(\alpha - \frac{1}{\gamma^2})^2 C^2} {\Delta s}_i^2 \quad i = 1, \dots, \text{Num.Part.}$$

Emittance: $\tilde{\epsilon}_k = \frac{1}{2N} \sum_{i=1}^N \epsilon_k(i) \quad k = x, z, s$

Zenkevich-Bolshakov algorithm (from MOCAC)

Laboratory Frame:

$$p_1^\mu = \left(\frac{E_1}{c}, \vec{p}_1 \right)$$



$$p_2^\mu = \left(\frac{E_2}{c}, \vec{p}_2 \right)$$

$$\vec{P}_1 = P_0 \{x'_1 \vec{e}_x + z'_1 \vec{e}_z + (1 + \delta_1) \vec{e}_s\}$$

$$\vec{P}_2 = P_0 \{x'_2 \vec{e}_x + z'_2 \vec{e}_z + (1 + \delta_2) \vec{e}_s\}$$



Relativistic Center of Mass Frame:

$$\tilde{\vec{P}}_1 = \frac{\tilde{\Delta P}}{2} \vec{e}_{s'}$$



$$\tilde{\vec{P}}_2 = -\frac{\tilde{\Delta P}}{2} \vec{e}_{s'}$$



$$\delta \tilde{\vec{P}}_1 (\tilde{\vec{P}}_2) = \tilde{\vec{P}}_1' - \tilde{\vec{P}}_1 = \frac{\tilde{\Delta P}}{2} \left\{ \sqrt{\frac{4\pi c \rho(\tilde{\vec{P}}_2) r_{cl}^2}{\gamma_0^2 \beta_{CM}^3}} L_c \Delta t (\cos \Xi \vec{e}_x' + \sin \Xi \vec{e}_z') - \left(\frac{2\pi c \rho(\tilde{\vec{P}}_2) r_{cl}^2}{\gamma_0^2 \beta_{CM}^3} L_c \Delta t \right) \vec{e}_{s'}' \right\}$$

$$\tilde{\vec{P}}_2' = -\tilde{\vec{P}}_1'$$

(P.R. Zenkevich, O. Boine-Frenkenheim, A. E. Bolshakov, *A new algorithm for the kinetic analysis of inta-beam scattering in storage rings*, NIM A, 2005)

Radiation damping and quantum excitation are calculated with the formula:

$$\Delta \epsilon_k(i) = \left(e^{-\frac{\Delta T}{\tau_k}} - 1 \right) \epsilon_k(i) \quad \Delta \xi(i) = N \left(0, \sigma_\xi \sqrt{1 - e^{-\frac{\Delta T}{\tau_\xi}}} \right)$$

$$k = x, z, s$$

$$i = 1, \dots, N_{part}$$

$$\xi = x_\beta, x'_\beta, z_\beta, z'_\beta, \Delta s, \frac{\Delta p}{p}$$

Lattice Recurrences

Elements of the lattice with twiss functions differing of less than 10% are considered equal.

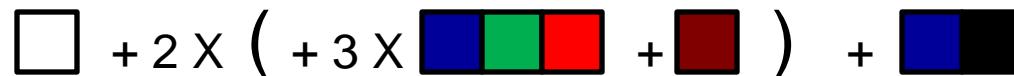
Lattice:



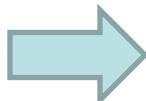
First reduction:



Second reduction:



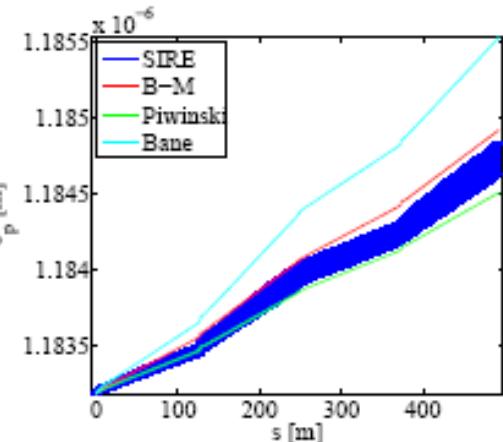
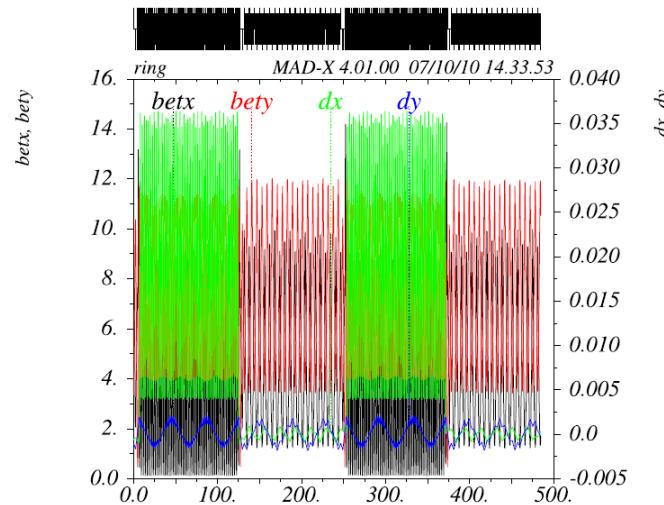
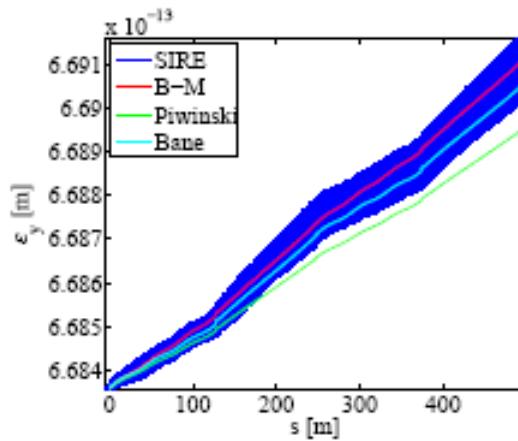
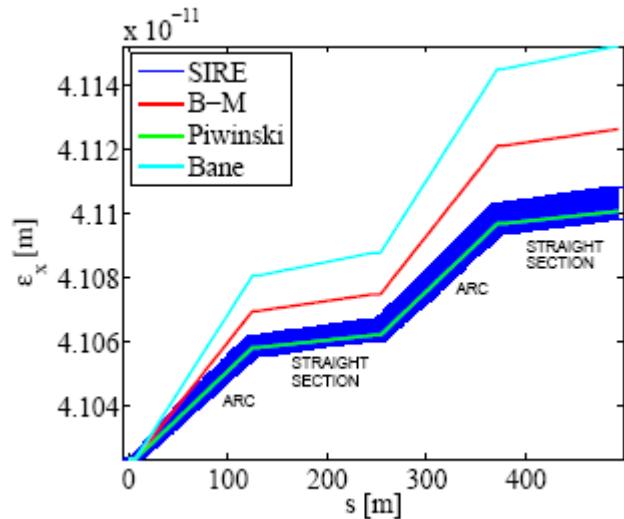
$$\frac{\Delta\epsilon}{\epsilon} = \frac{\Delta t_w}{T_w} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_g}{T_g} + \frac{\Delta t_r}{T_r} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_g}{T_g} + \dots = \frac{\Delta t_w}{T_w} + \frac{6\Delta t_b}{T_b} + \frac{6\Delta t_g}{T_g} + \frac{6\Delta t_r}{T_r} + \frac{2\Delta t_{br}}{T_{br}} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_{bl}}{T_{bl}}$$

CLIC DR LATTICE: 14400 elements  420 elements

SIRE: Benchmarking (Gaussian Distribution) CLIC DR

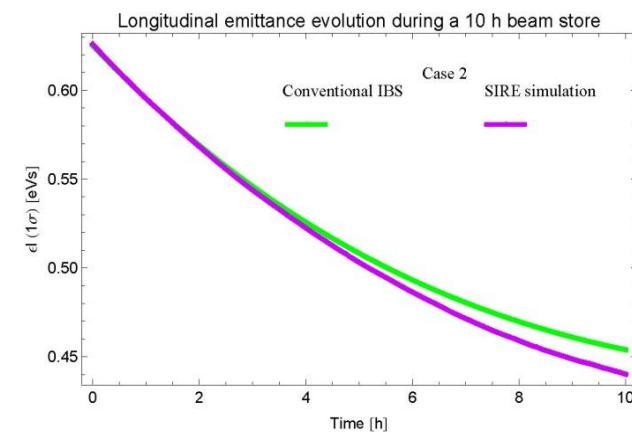
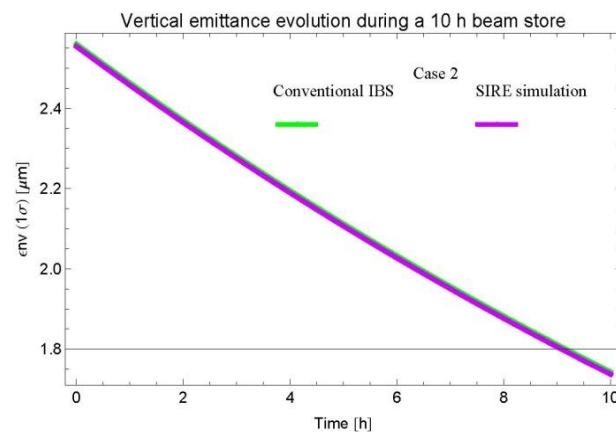
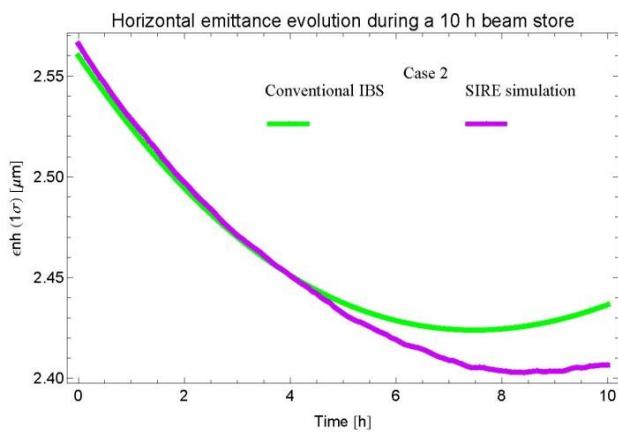
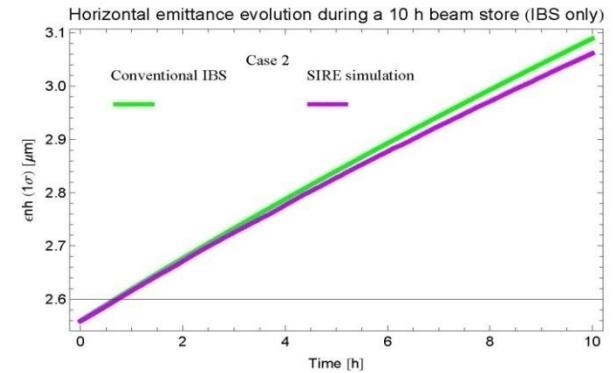
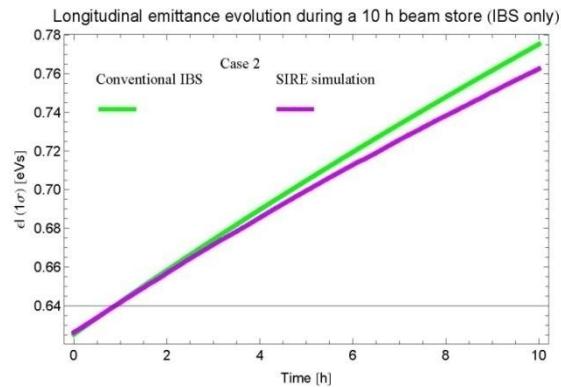
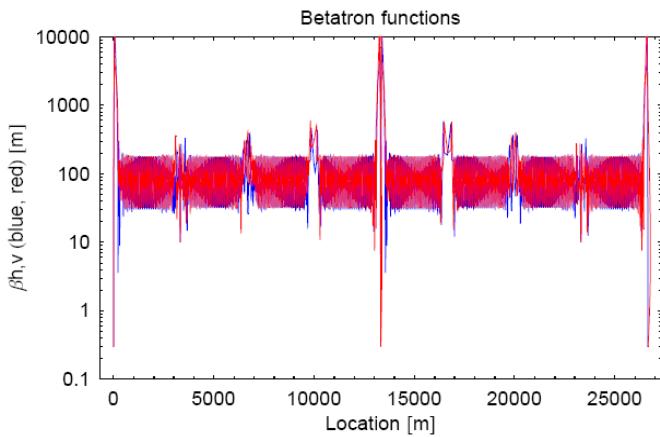
Table 1: Parameters for validation

Parameter	Unit	Value
Energy	GeV	2.86
Bunch Population	10^9	4.07
Circumference	m	493.16
Norm. Emittance H,V	nm	229.6 , 3.74
Momentum Spread	%	0.109
Bunch Length	mm	0.922
N. macro-particles	10^3	200
N. cells	10^3	200
ΔT	μs	1.645



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SIRE: Benchmarking (Gaussian Distribution) on LHC

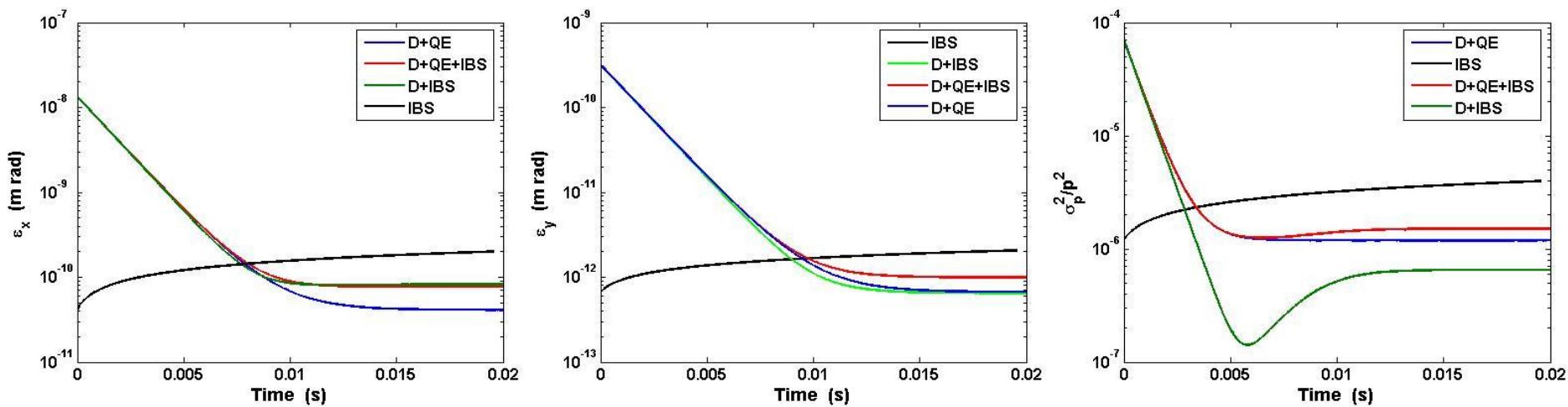


SIRE: Distribution Study

Case studies:

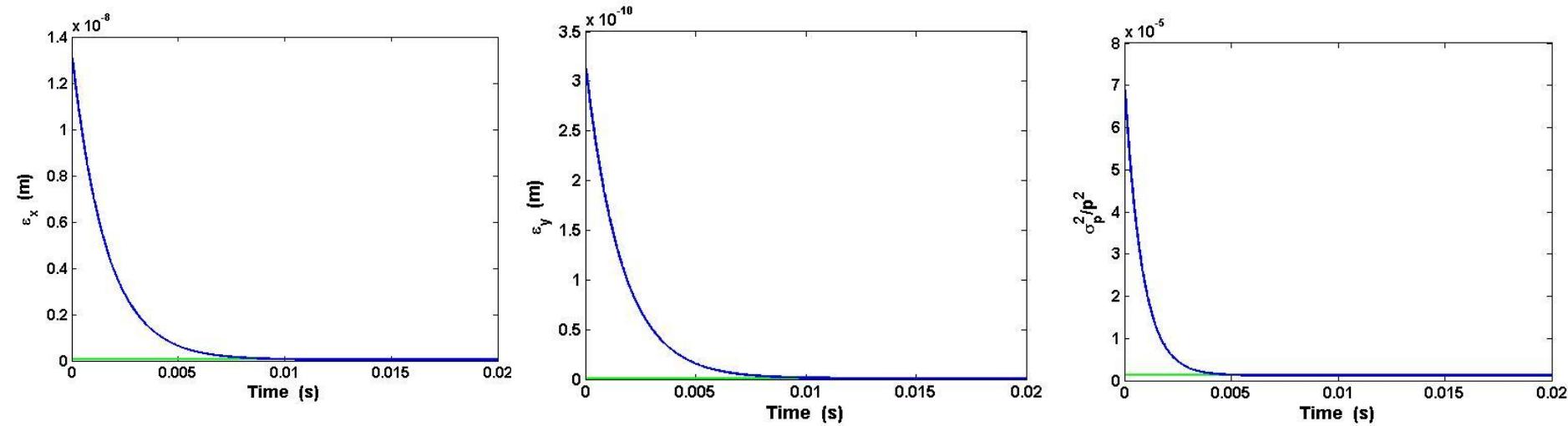
- A – Damping + QE
- B – Damping + IBS + QE
- C – Damping + IBS
- D – IBS

Parameter	A	B	C	D
INITIAL $\gamma\varepsilon_x, \gamma\varepsilon_y, \sigma_z\sigma_p$ (m,m,eV m)	74.3e-6, 1.8e-6, 1.71e+5	74.3e-6, 1.8e-6, 1.70e+5	74.3e-6, 1.8e-6, 1.71e+5	229.7e-9, 3.7e-9, 2.87e+3
FINAL $\gamma\varepsilon_x, \gamma\varepsilon_y, \sigma_z\sigma_p$ (m,m,eV m)	229.7e-9, 3.76e-9, 2.88e+3	435.6e-9, 5.54e-9, 3.65e+3	458.5e-9, 3.61e-9, 1.58e+3	1.12e-6, 1.16e-8, 9.61e+3



CLIC DR A: Damping + QE

Simulation of the CLIC Damping Rings case A:



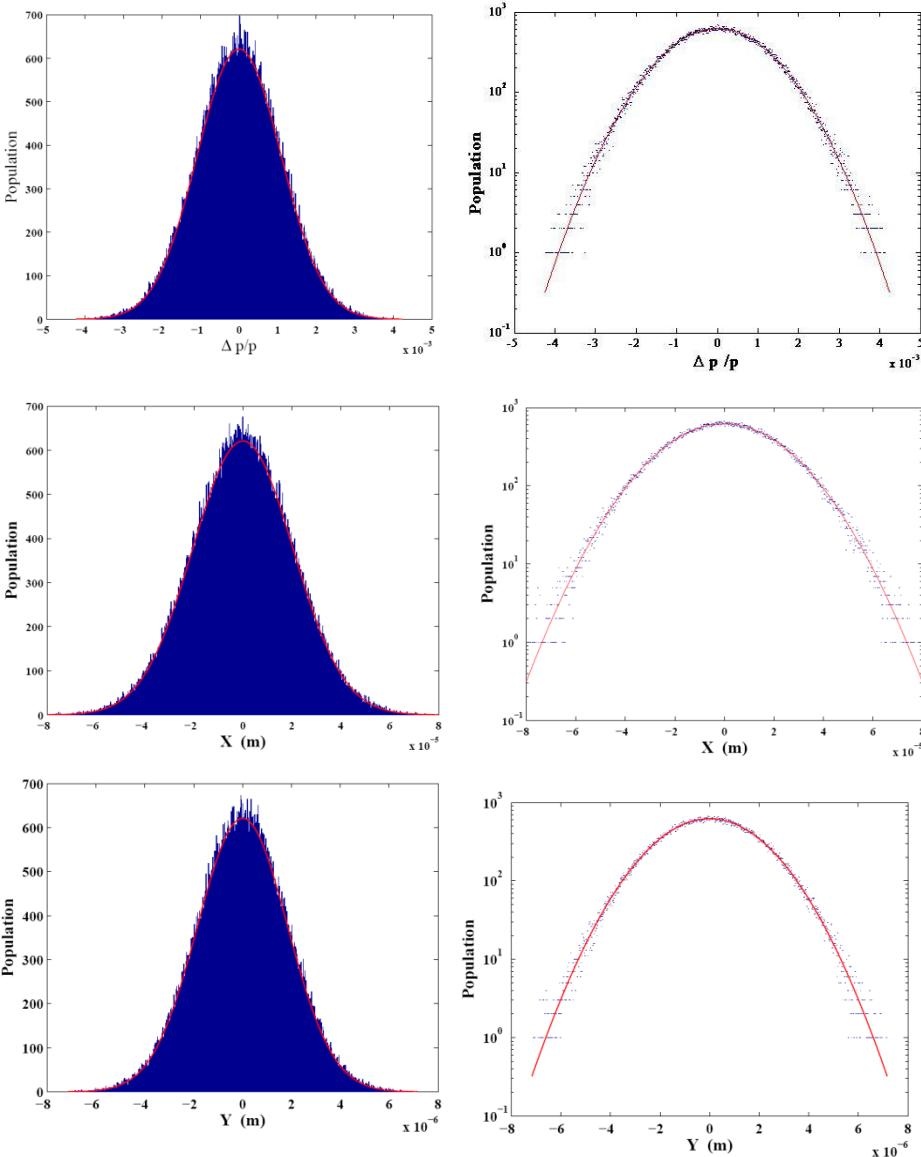
Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	13.27e-9	321.6e-12	1.71e+5
Extraction (SIRE)	4.104e-11	6.72e-13	2.88e+3
Extraction (MAD-X)	4.102e-11	6.69e-13	2.87e+3

Table 2: Comparison SIRE – conventional formalisms

Formalism	$1/T_H$ (s $^{-1}$)	$1/T_V$ (s $^{-1}$)	$1/T_L$ (s $^{-1}$)
Bjorken-Mtingwa	1579	739	969
SIRE	1186	665	800
SIRE (compressed)	1239	687	786
Modified Piwinski	1300	626	775

SIRE: IBS Distribution study A: Damping + QE



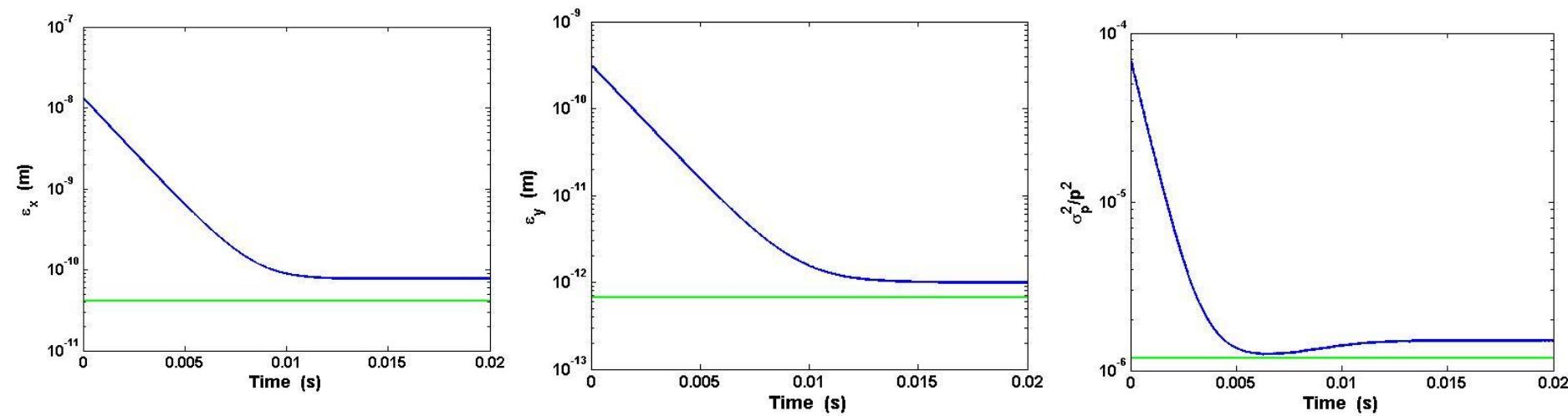
$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

Parameter	χ^2_{999}	Confidence
$\Delta P/P$	964.2251	0.7876
X	976.2195	0.6988
Y	957.4559	0.8290

Parameter	Value
Eq. ε_x (m rad)	4.1039e-011
Eq. ε_y (m rad)	6.7113e-013
Eq. σ_δ	1.0901e-3
Eq. σ_z (m)	9.229e-4

CLIC DR B: Damping + IBS + QE

Simulation of the CLIC Damping Rings case B:

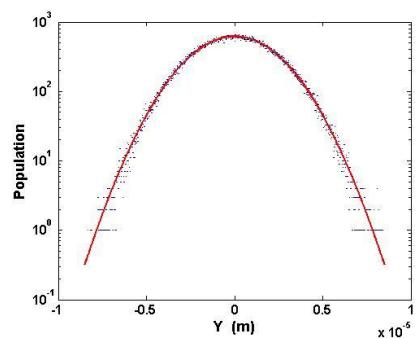
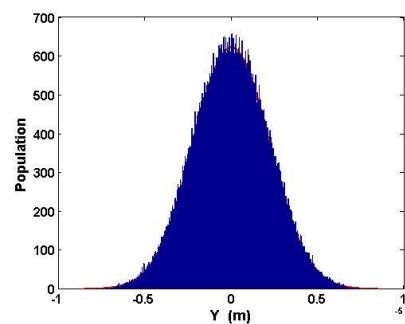
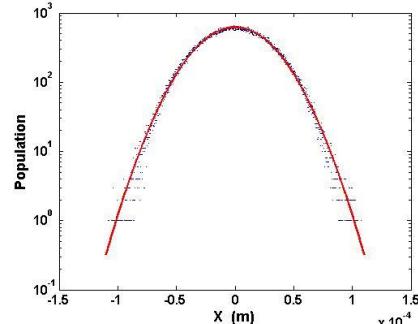
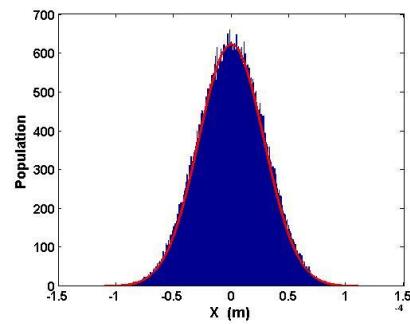
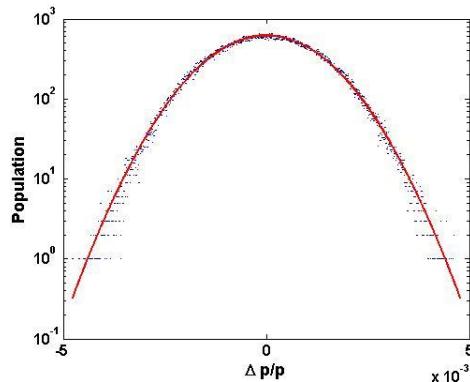
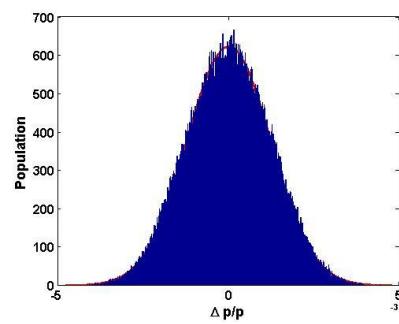


Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	1.327e-8	3.177e-10	1.704e+5
Extraction (SIRE)	7.783e-11	9.901e-13	3.651e+3
Extraction (B-M)	-	-	-

	$1/T_x$ (s $^{-1}$)	$1/T_y$ (s $^{-1}$)	$1/T_z$ (s $^{-1}$)
Bjorken-Mtingwa	448	244	372
SIRE compressed (Gauss)	309	209	283
SIRE not compressed (Gauss)	310	213	289
SIRE compressed	294	200	262
SIRE not compressed	283	191	258

SIRE: IBS Distribution study B: Damping + IBS + QE

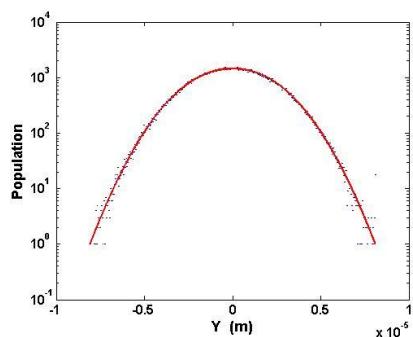
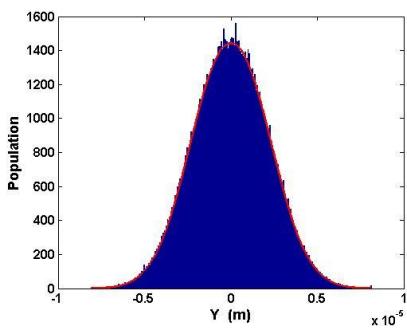
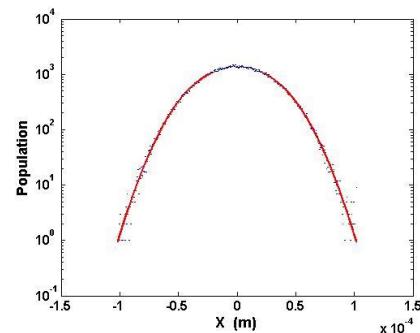
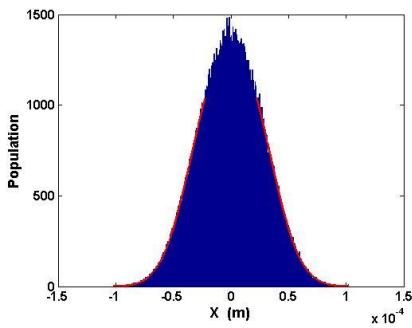
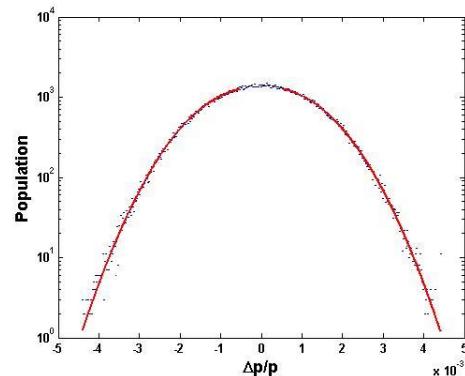
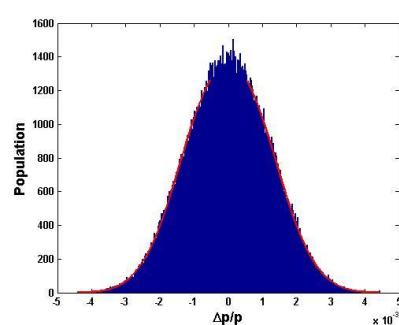


$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

Parameter	χ^2_{999}	Confidence
$\Delta p/p$	1166	2.0e-4
X	1212	4.1e-6
Y	1121	4.3e-3

Parameter	Value
Eq. ε_x (m rad)	7.783e-11
Eq. ε_y (m rad)	9.901e-13
Eq. σ_δ	1.228e-3
Eq. σ_z (m)	1.04e-3

SIRE: IBS Distribution study B: Damping + IBS + QE



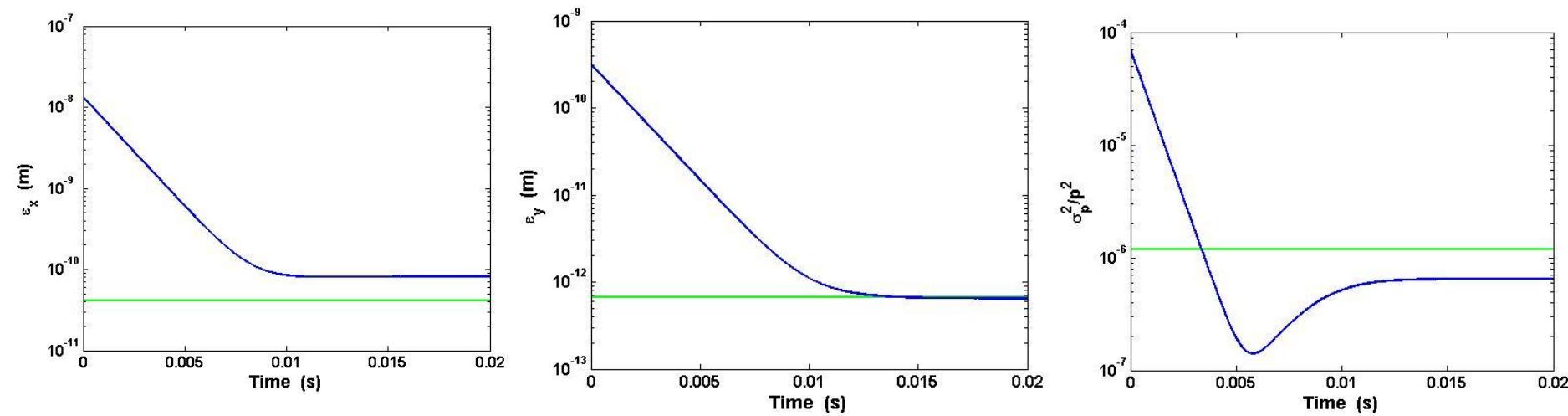
$$p_k(\xi_k) = N_k e^{-\alpha_k (\xi_k^2)^{\beta_k}}$$

Parameter	χ^2_{37}	Confidence	Sample %
$\Delta P/P$	37.86	0.430	67
X	37.49	0.450	44
Y	44.84	0.176	100

Parameter	Value
α_p	1.166e+6
β_p	1.108252
α_x	1.325e+10
β_x	1.160051
α_y	4.3711e+11
β_y	1.058449

CLIC DR C: Damping + IBS

Simulation of the CLIC Damping Rings case C:

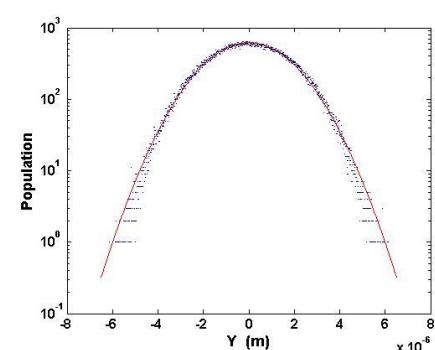
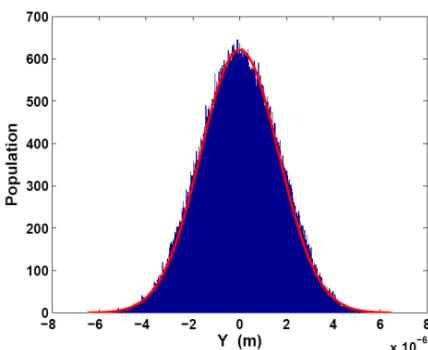
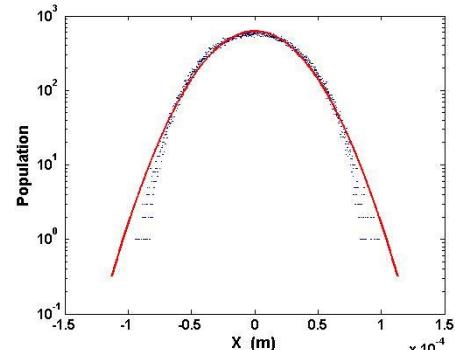
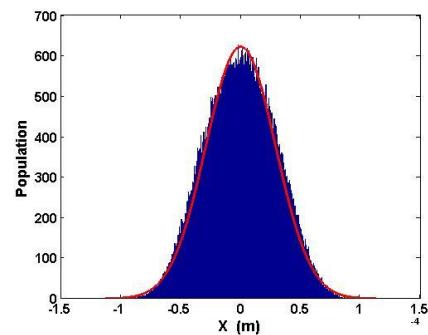
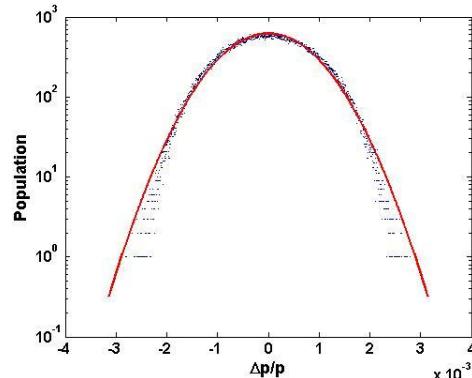
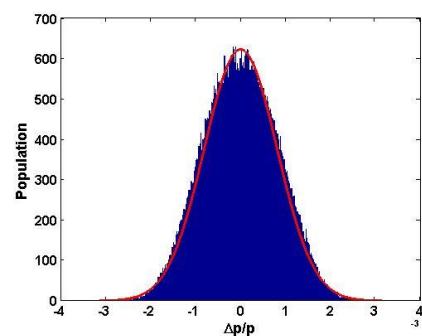


Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	1.328e-8	3.153e-10	1.71e+5
Extraction (SIRE)	8.19e-11	6.46e-13	1577
Extraction (B-M)	-	-	-

	$1/T_x$ (s $^{-1}$)	$1/T_y$ (s $^{-1}$)	$1/T_z$ (s $^{-1}$)
Bjorken-Mtingwa	1021	846	1928
SIRE compressed (Gauss)	725	734	1466
SIRE not compressed (Gauss)	705	706	1441
SIRE compressed	619	616	1231
SIRE not compressed	611	603	1244

SIRE: IBS Distribution study C: Damping + IBS

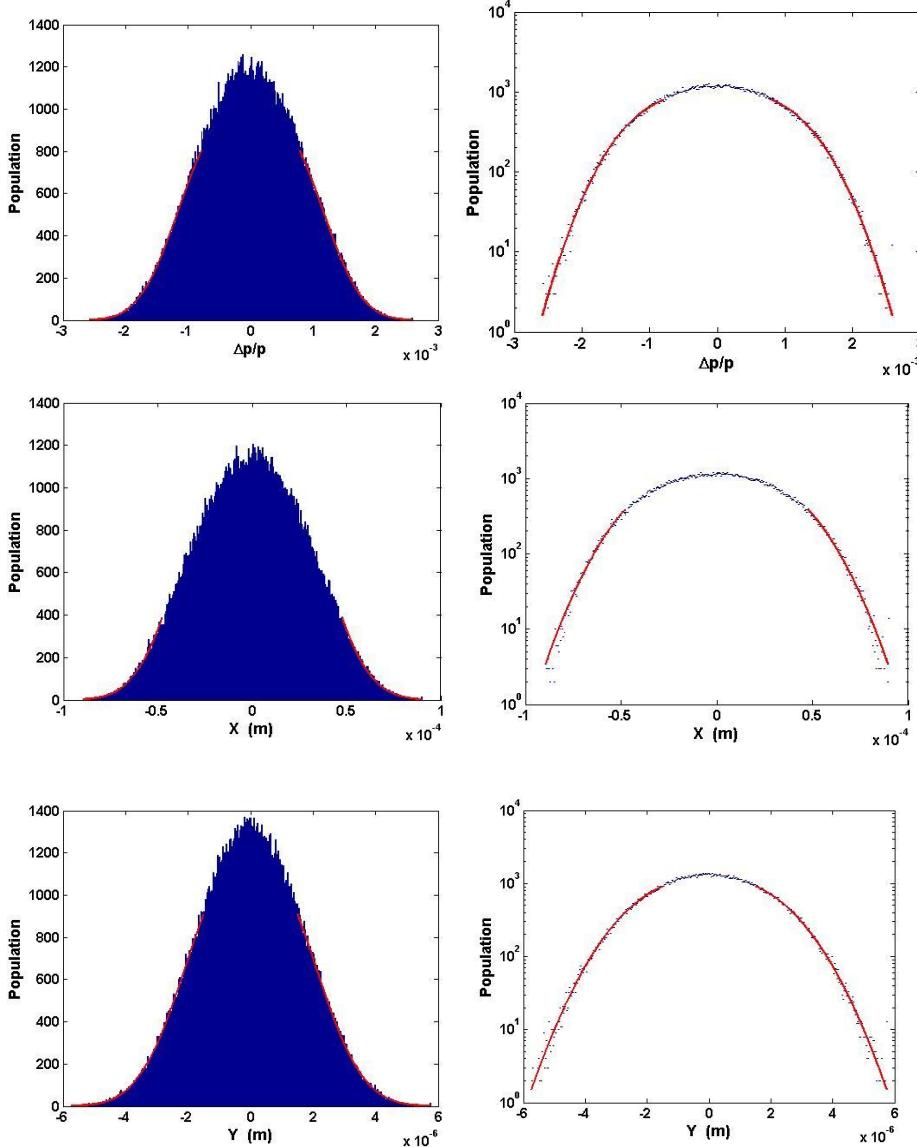


$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

Parameter	χ^2_{999}	Confidence
$\Delta p/p$	2497	<1e-15
X	2770	<1e-15
Y	1346	1.313e-12

Parameter	Value
Eq. ε_x (m rad)	8.192e-11
Eq. ε_y (m rad)	6.460e-13
Eq. σ_δ	8.072e-4
Eq. σ_z (m)	6.833e-4

SIRE: IBS Distribution study C: Damping + IBS



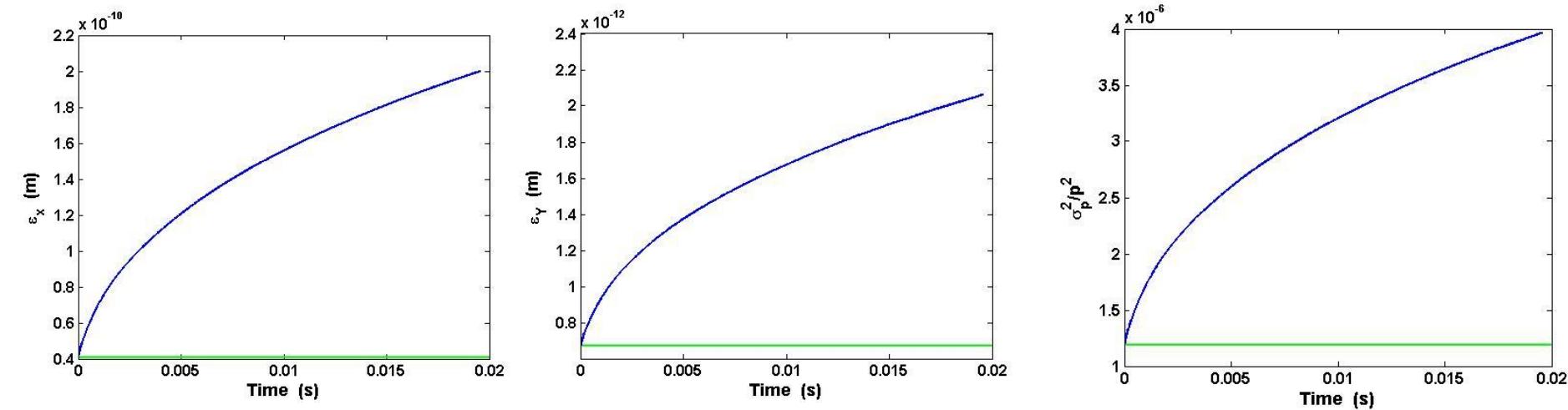
$$p_k(\xi_k) = N_k e^{-\alpha_k (\xi_k^2)^{\beta_k}}$$

Parameter	χ^2_{37}	Confidence	Sample %
$\Delta P/P$	39.04	0.38	35
X	73.19	3.60e-4	44
Y	38.71	0.39	38

Parameter	Value
α_p	1.722e+8
β_p	1.4350
α_x	9.0614e+10
β_x	1.2576
α_y	2.7499e+13
β_y	1.2036

CLIC DR D: IBS

Simulation of the CLIC Damping Rings case D:

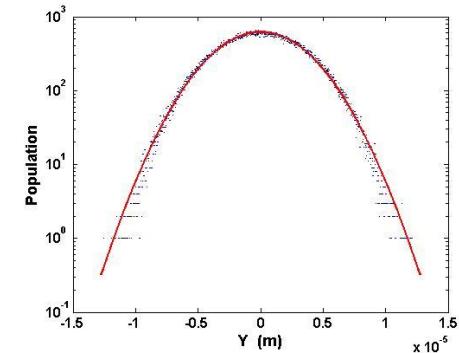
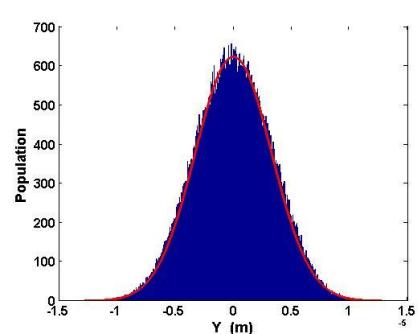
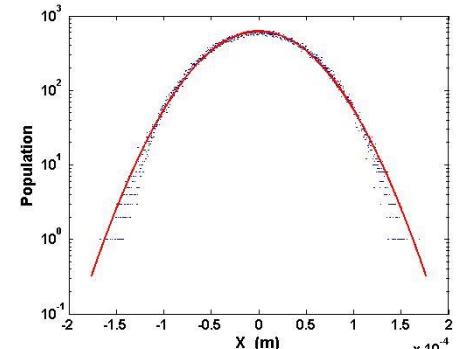
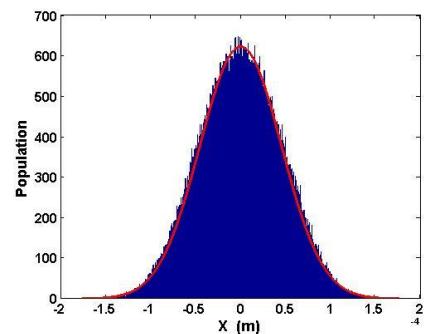
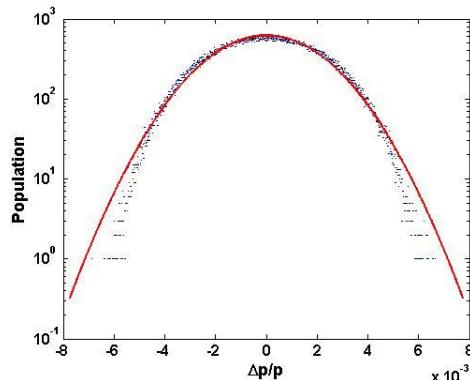
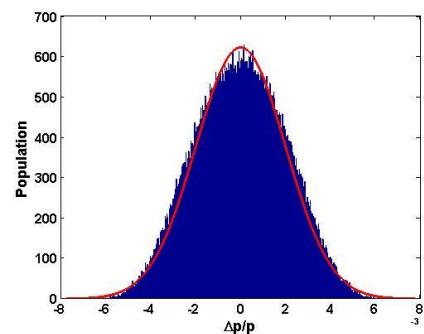


Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	4.104e-11	6.663e-13	2871
Extraction (SIRE)	2.001e-010	2.064e-12	9609
Extraction (B-M)	-	-	-

	1/Tx (s ⁻¹)	1/Ty (s ⁻¹)	1/Tz (s ⁻¹)
Bjorken-Mtingwa	29.6	21.0	28.9
SIRE compressed (Gauss)	21.6	17.8	20.6
SIRE not compressed (Gauss)	18.1	18.0	19.3
SIRE compressed	17.0	14.6	17.2
SIRE not compressed	18.3	15.3	16.5

SIRE: IBS Distribution study D: IBS

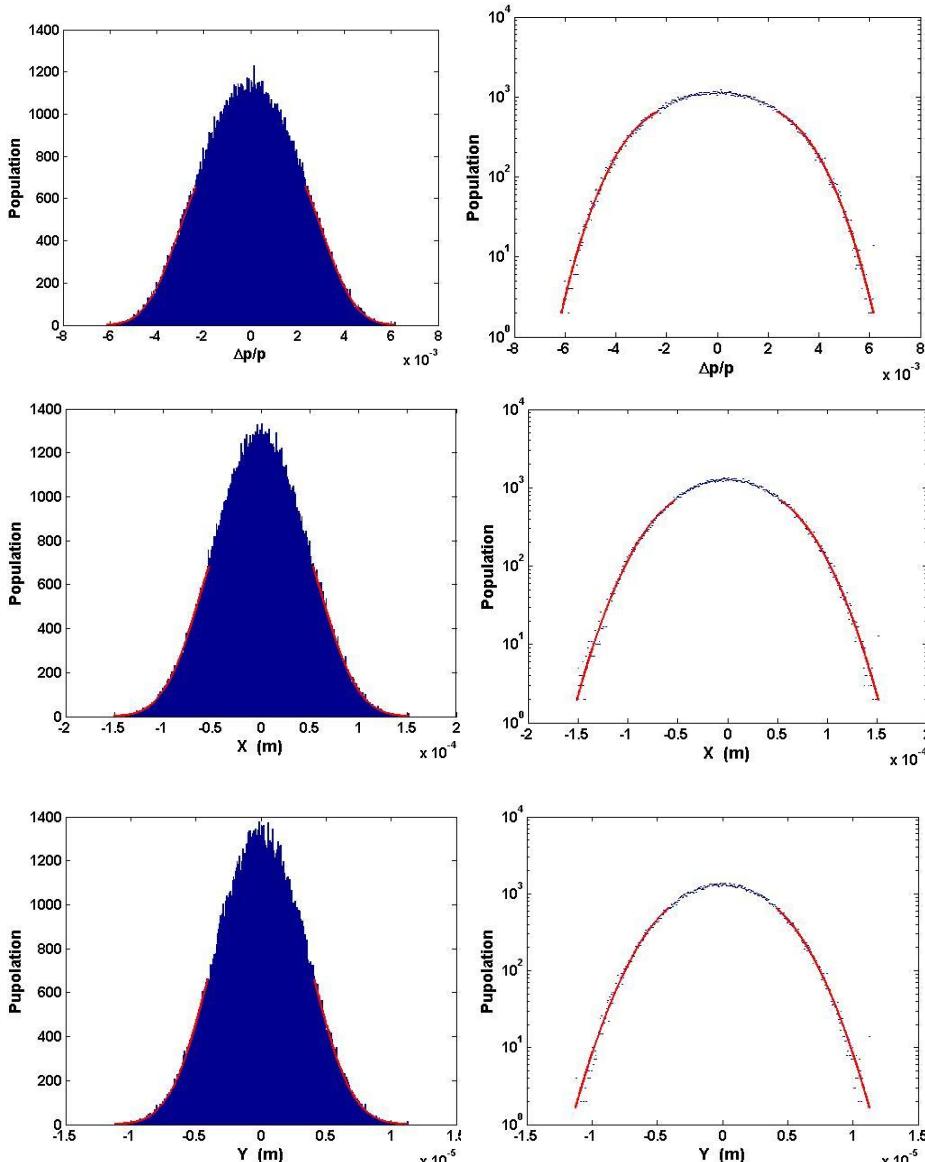


$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

Parameter	χ^2_{999}	Confidence
$\Delta p/p$	3048.7	<1e-15
X	1441.7	<1e-15
Y	1466.9	<1e-15

Parameter	Value
Eq. ε_x (m rad)	2.001e-10
Eq. ε_y (m rad)	2.064e-12
Eq. σ_δ	1.992e-3
Eq. σ_z (m)	1.687e-3

SIRE: IBS Distribution study D: IBS



$$p_k(\xi_k) = N_k e^{-\alpha_k (\xi_k^2)^{\beta_k}}$$

Parameter	χ^2_{37}	Confidence	Sample %
$\Delta P/P$	38.81	0.39	26
X	36.73	0.48	25
Y	46.83	0.13	22

Parameter	Value
α_p	5.281e+7
β_p	1.568
α_x	3.840e+10
β_x	1.280
α_y	4.557e+12
β_y	1.196

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Conclusions

- Investigation of IBS effect in the CLIC DR has been carried out with the simulation code SIRE:
 - Benchmarking with conventional IBS theories gave good agreement.
 - Evolution of the particle distribution shows deviations from Gaussian behaviour due to IBS effect.
 - IBS tends to develop tails of the particle distribution with a faster decay than for a Gaussian corresponding to the same beam parameters.
 - IBS effect on the actual distribution is reduced compared to the effect evaluated on a Gaussian corresponding to the same beam parameters.
 - In the CLIC DR this deviation is small (also due to the short cooling time) and discrepancy with Bjorken-Mtingwa calculation remains within the usual discrepancy of the theories.

THANKS.

The End