

Status of the Muon g-2

Michel Davier (LAL – Orsay)

- the muon magnetic anomaly
- revisited τ spectral functions: Belle + updated corrections
- ee spectral functions after KLOE and BABAR
- new evaluation using BABAR multihadronic data
- discussion and perspectives
- new precise value for $\alpha(M_Z)$



Lepton Magnetic Anomaly: from Dirac to QED

$$\vec{\mu} = g \frac{e}{2m} \vec{s},$$

$$a = (g - 2)/2$$

Dirac (1928) $g_e = 2$ $a_e = 0$

anomaly discovered:

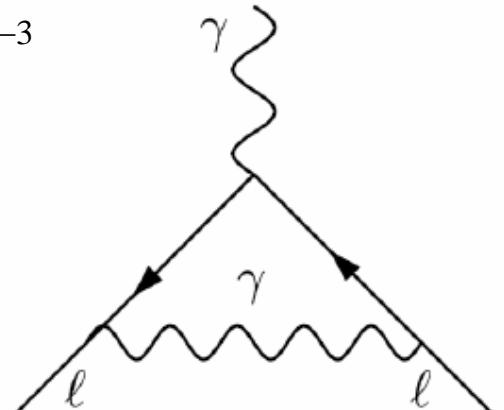
Kusch-Foley (1948) $a_e = (1.19 \pm 0.05) 10^{-3}$

and explained by $O(\alpha)$ QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16 10^{-3}$

first triumph of QED

$\Rightarrow a_e$ sensitive to quantum fluctuations of fields



More Quantum Fluctuations

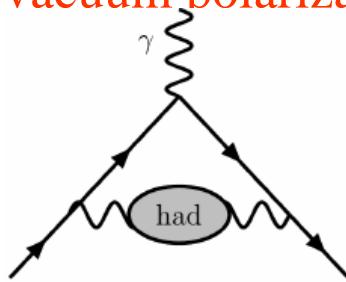
$$a = a^{\text{QED}} + a^{\text{had}} + a^{\text{weak}} + ? \text{ a new physics ?}$$

QED up to $\mathcal{O}(\alpha^4)$, α^5 in progress (Kinoshita et al.)

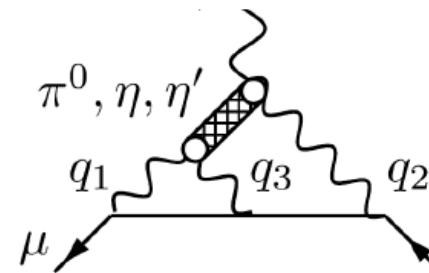


Hadrons

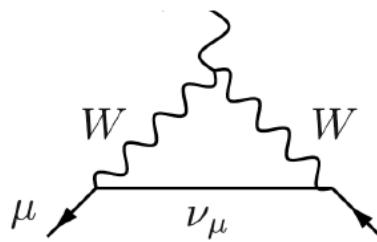
vacuum polarization



light-bv-light (models)

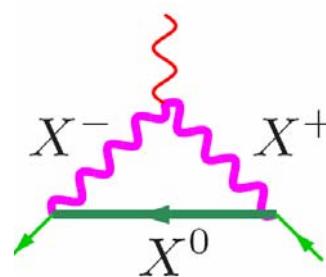


Electroweak



$\Rightarrow a_\mu$ much more sensitive to high scales

new physics at high mass scale



$$\delta a_\ell \propto \frac{m_\ell^2}{M^2}$$

Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

Dominant uncertainty from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \rightarrow \text{hadrons}$ cross section (Bouchiat-Michel 1961)

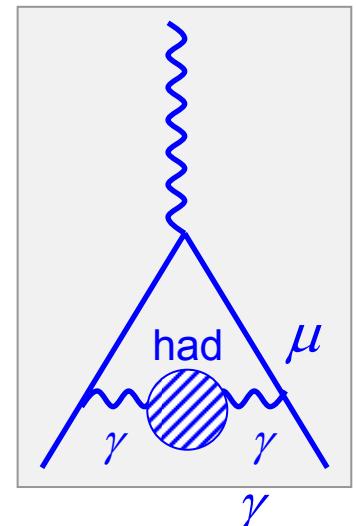
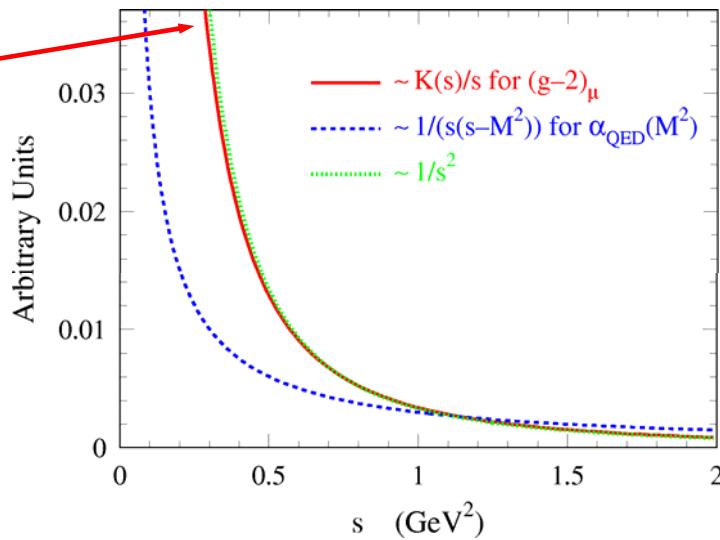
$$\text{Born: } \sigma^{(0)}(s) = \sigma(s) (\alpha / \alpha(s))^2$$

$$12\pi \text{Im} \Pi_\gamma(s) = \frac{\sigma^0 [e^+e^- \rightarrow \text{hadrons}(\gamma)]}{\sigma_{pt}} \equiv R(s)$$

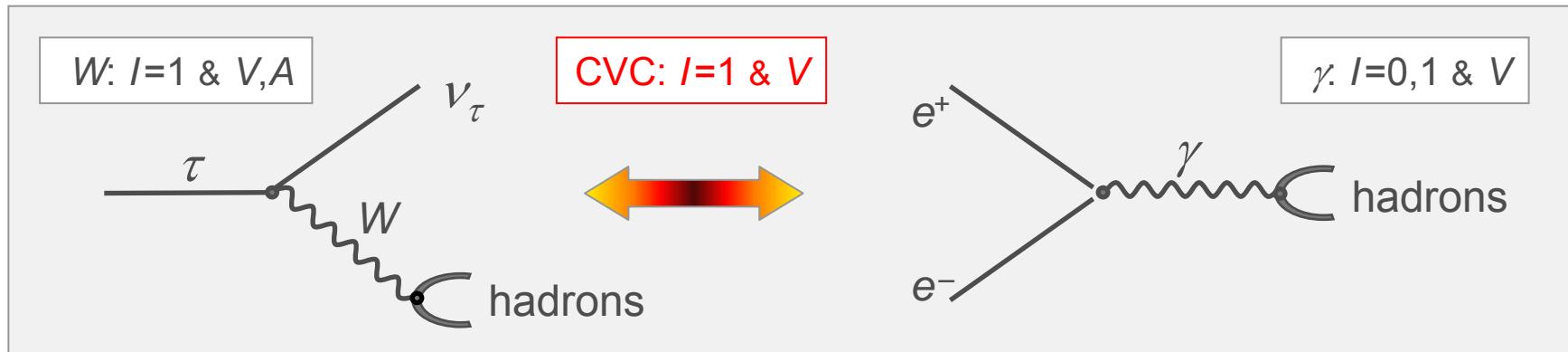
$\text{Im} [\text{hadrons}] \propto |\text{hadrons}|^2$

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty ds \left(\frac{K(s)}{s} \right) R(s)$$

Dispersion relation



The Role of τ Data through CVC – SU(2)



Hadronic physics factorizes (spectral Functions)

$$\sigma^{(I=1)}[e^+e^- \rightarrow \pi^+\pi^-] = \frac{4\pi\alpha^2}{s} v[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]$$

$$v[\tau^- \rightarrow \pi^-\pi^0\nu_\tau] \propto \frac{\text{BR}[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]}{\text{BR}[\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau]} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2(1+s/m_\tau^2)}$$

branching fractions mass spectrum kinematic factor (PS)

R. Alemany, MD, A. Höcker, EPJC 1998

SU(2) breaking corrections

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contrib.:

■ Electroweak radiative corrections:

- ▶ dominant contribution from short distance correction S_{EW}
- ▶ subleading corrections (small)
- ▶ long distance radiative correction $G_{\text{EM}}(s)$

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

■ Charged/neutral mass splitting:

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

- ▶ $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
- ▶ ρ - ω mixing (EM $\omega \rightarrow \pi^-\pi^+$ decay) corrected using FF model
- ▶ $m_{\rho^-} \neq m_{\rho^0}$ *** and $\Gamma_{\rho^-} \neq \Gamma_{\rho^0}$ ***

Flores-Baez-Lopez Castro' 08
Davier et al.'09

- Electromagnetic decays: $\rho \rightarrow \pi\pi\gamma$ ***, $\rho \rightarrow \pi\gamma$, $\rho \rightarrow \eta\gamma$, $\rho \rightarrow l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Situation 2009-10

- including BaBar 2 π results in the e+e- combination + estimate of hadronic LBL contribution (Prades-de Rafael-Vainhstein, 2009) yields

$$a_{\mu}^{\text{SM}}[\text{e+e-}] = (11\ 659\ 183.4 \pm 4.1 \pm 2.6 \pm 0.2) \ 10^{-10}$$

HVP LBL EW (± 4.9)

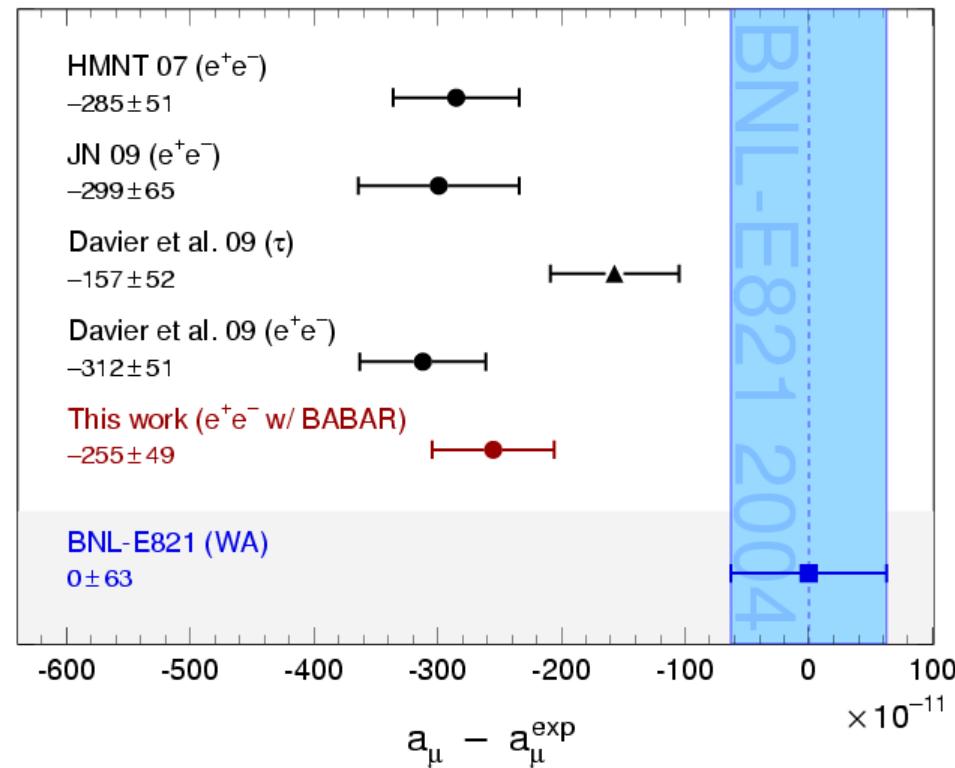
- E-821 updated result

$$11\ 659\ 208.9 \pm 6.3$$

- deviation (ee) 25.5 ± 8.0
(3.2σ)

- updated τ analysis
+Belle +revisited IB corrections

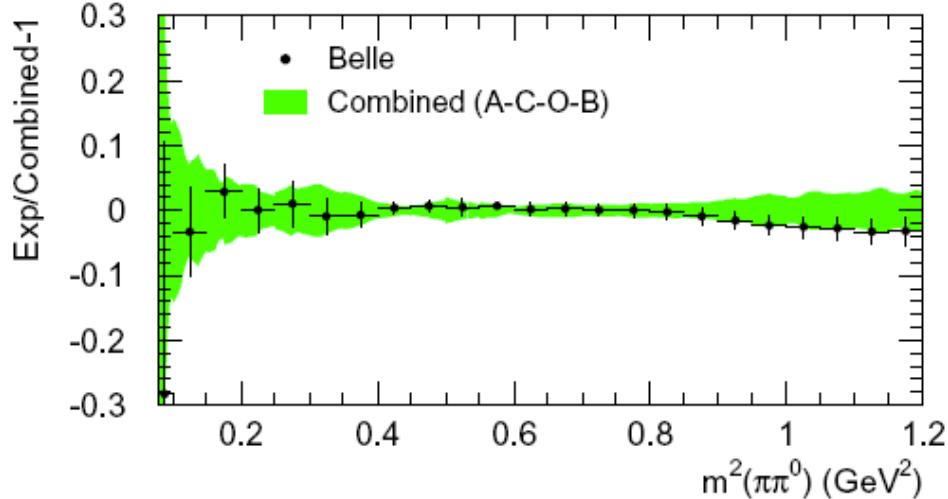
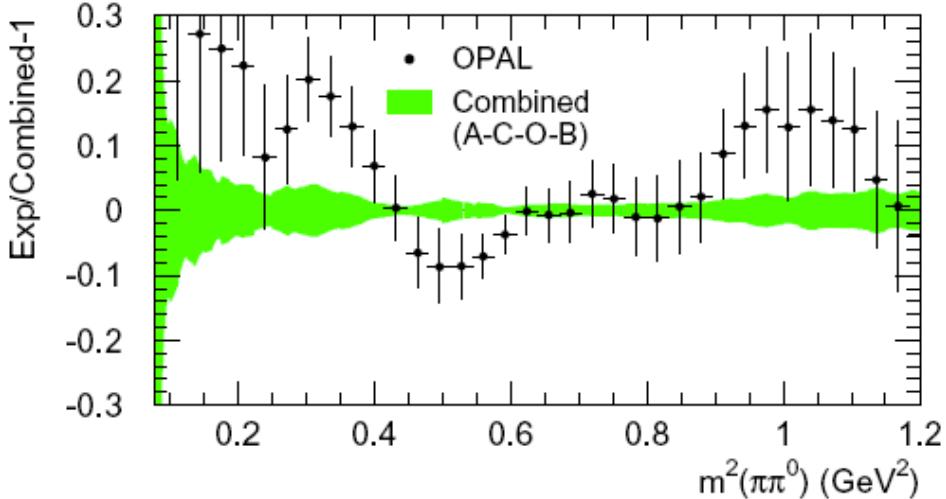
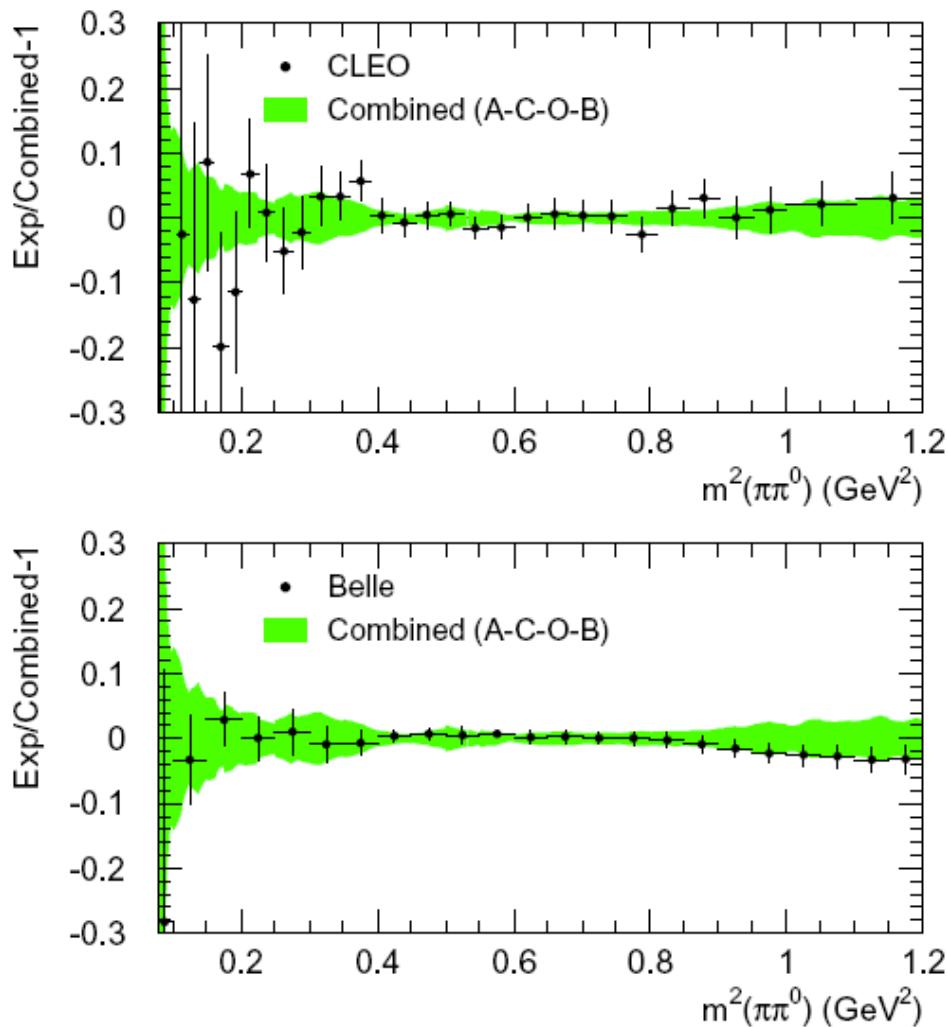
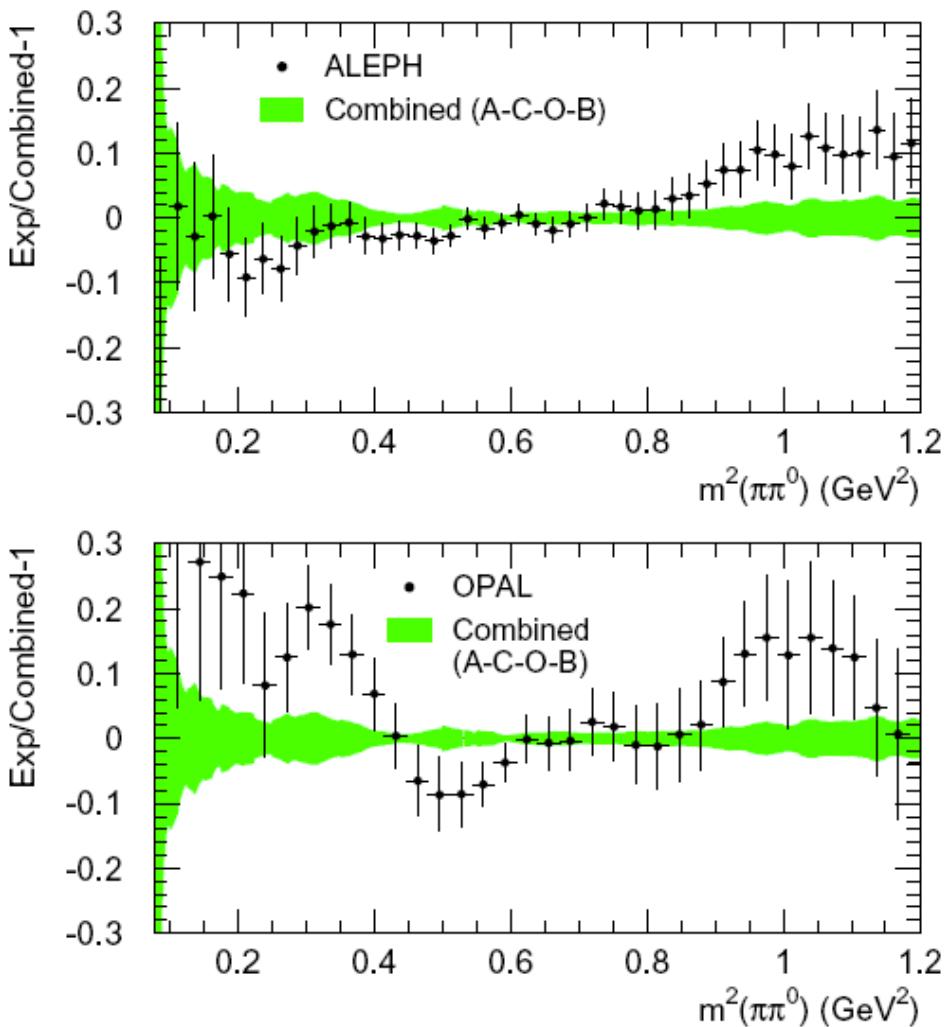
- deviation (τ) 15.7 ± 8.2
(1.9σ)



Most recent data and analyses

- $\tau \rightarrow \pi \pi^0 \nu_\tau$ data from Belle PRD 78 (2008) 072006
- $e^+ e^- \rightarrow \pi^+ \pi^-$ data
 - KLOE PLB 670 (2009) 285
arXiv:1006.5313
 - BaBar arXiv:0908.3589v1 (PRL)
- updated τ -based analysis arXiv:0906.5443v3 (EPJC)
MD, A. Hoecker, G. Lopez Castro, B. Malaescu, X.H.Mo, G. Toledo Sanchez, P. Wang, C.Z. Yuan, Z. Zhang
- updated ee-based analysis arXiv:0908.4300v2 (EPJC)
MD, A. Hoecker, B. Malaescu, C.Z. Yuan, Z. Zhang
- complete re-evaluation arXiv: today
MD, A. Hoecker, B. Malaescu, Z. Zhang

Revisited Analysis using τ Data: including Belle

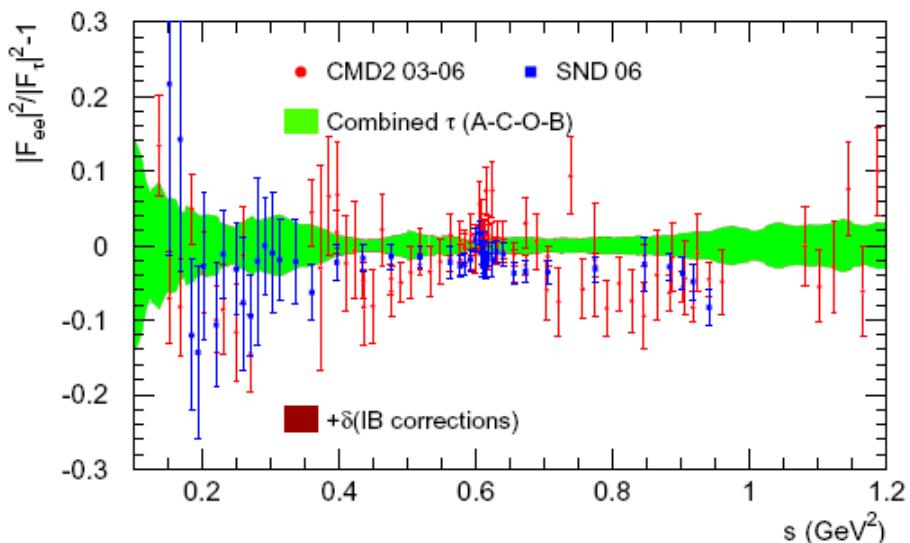


Test of the spectral function shapes from different experiments: WA BR used

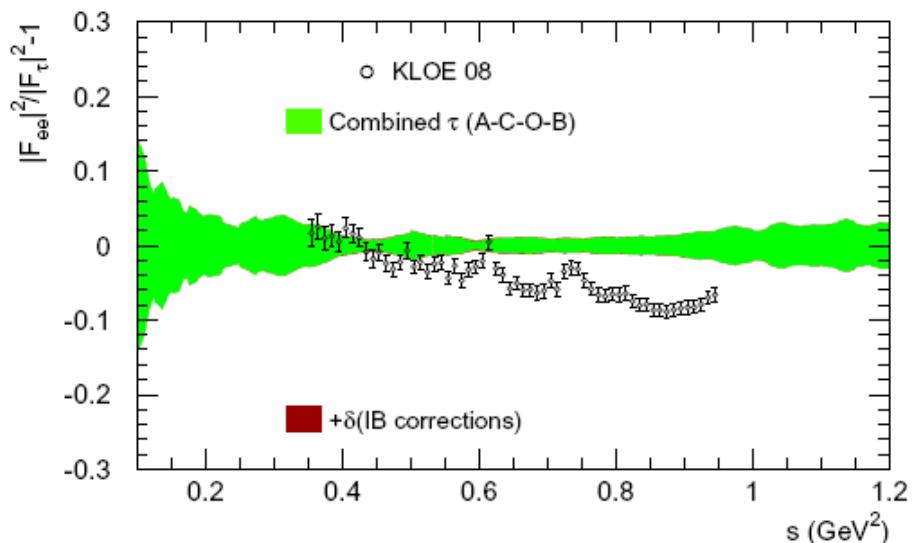
Comparison of ee and τ Data Revisited (1)

Relative comparison of IB-corrected τ and ee spectral functions (τ green band)

CMD-2, SND



KLOE



⇒ better agreement than before with CMD2-SND
⇒ strong disagreement with KLOE : slope...

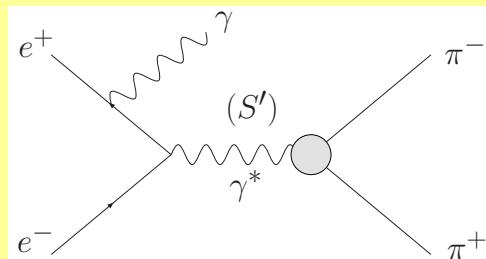
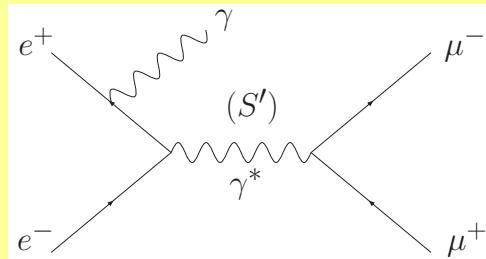
The BaBar Analysis

$e^+ e^- \rightarrow \mu^+ \mu^- \gamma (\gamma) \text{ and } \pi^+ \pi^- \gamma (\gamma)$ measured simultaneously

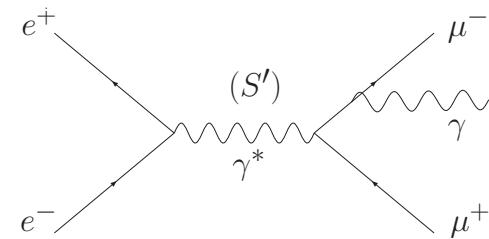
$$x = 2E_\gamma^*/\sqrt{s}$$

$$s' = s(1 - x)$$

ISR

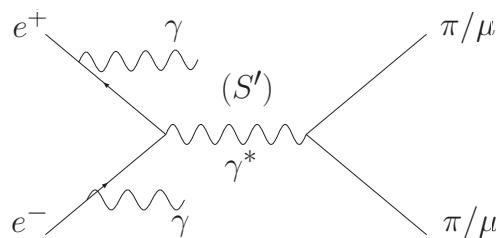


FSR

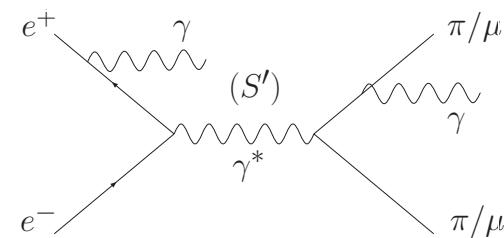


LO FSR negligible for $\pi\pi$
at $s \sim (10.6 \text{ GeV})^2$

ISR + add. ISR

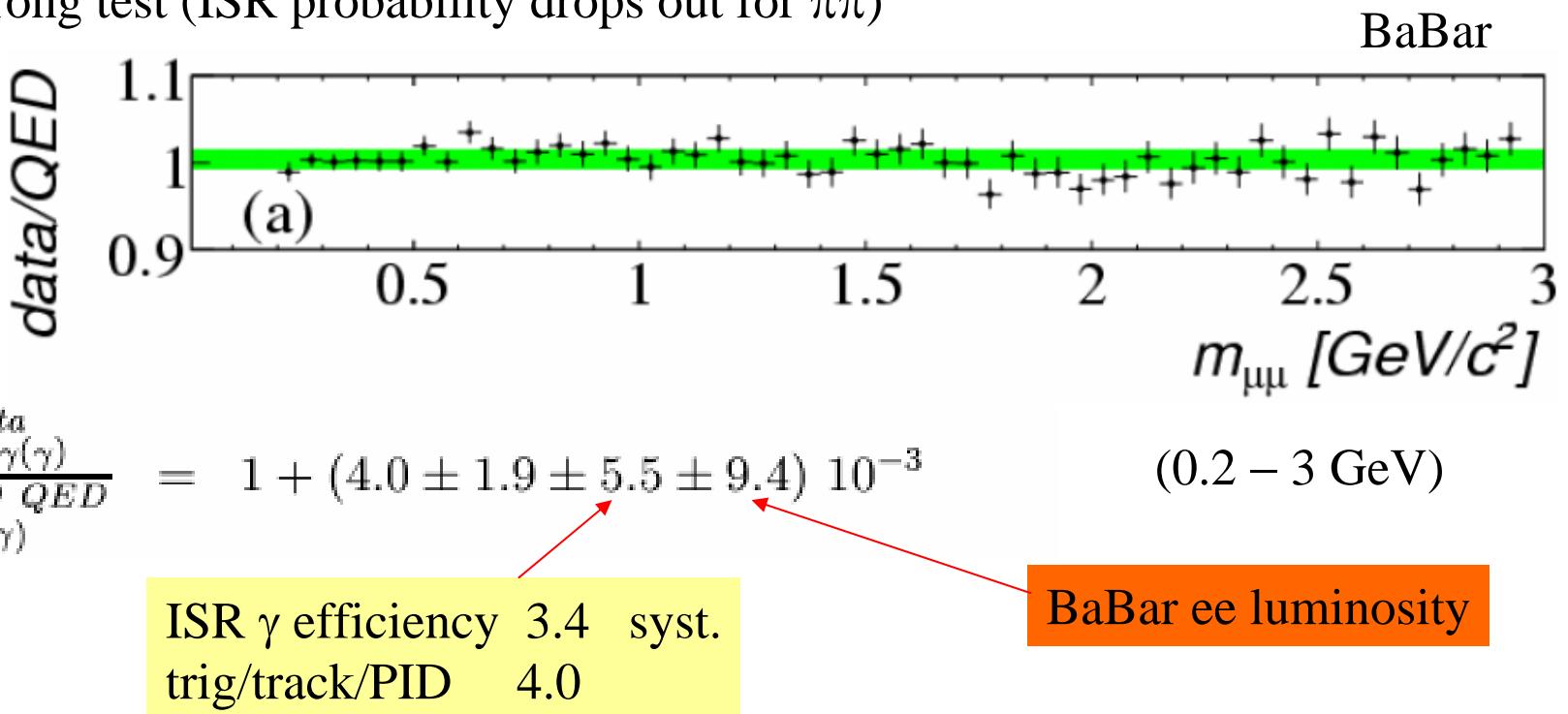


ISR + add. FSR



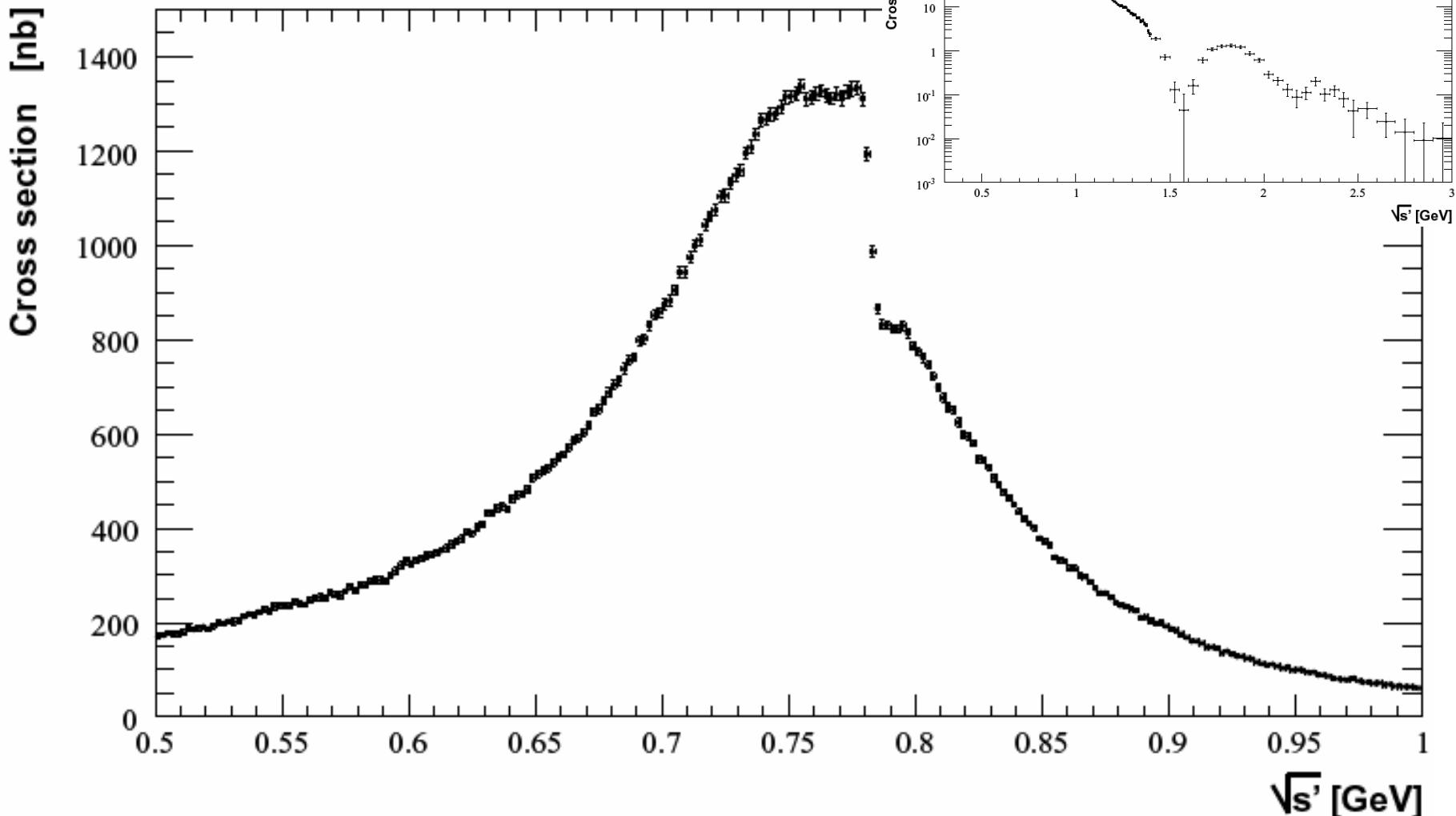
QED Test with $\mu\mu\gamma$ sample

- absolute comparison of $\mu\mu$ mass spectra in data and in simulation
- simulation corrected for data/MC efficiencies
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for $\pi\pi$)



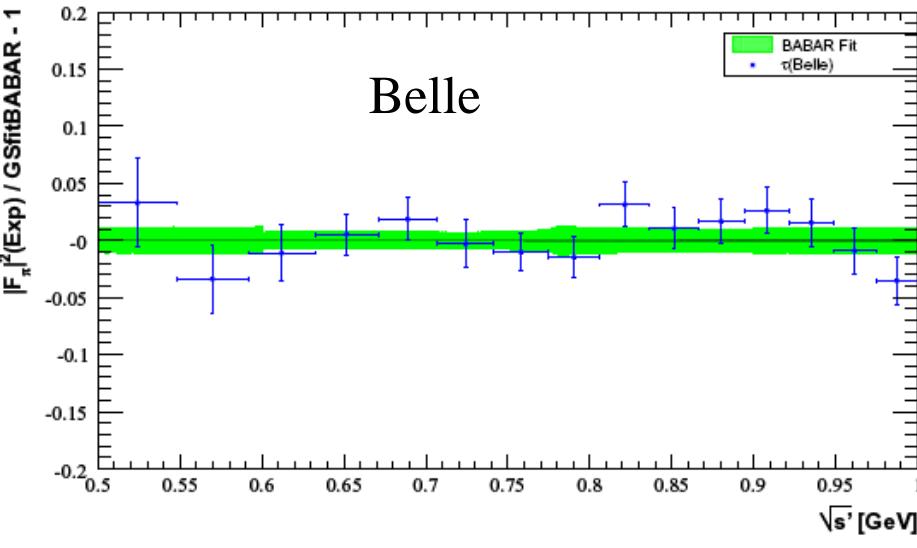
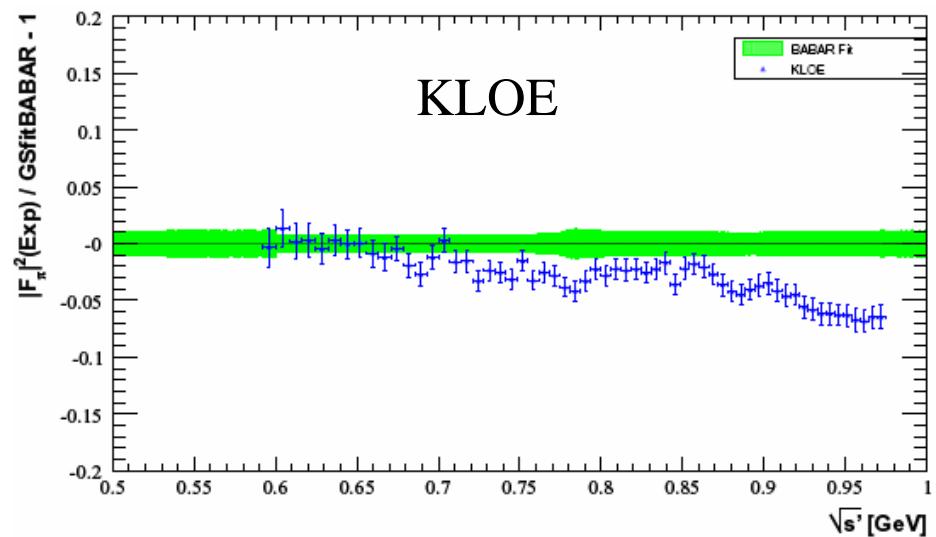
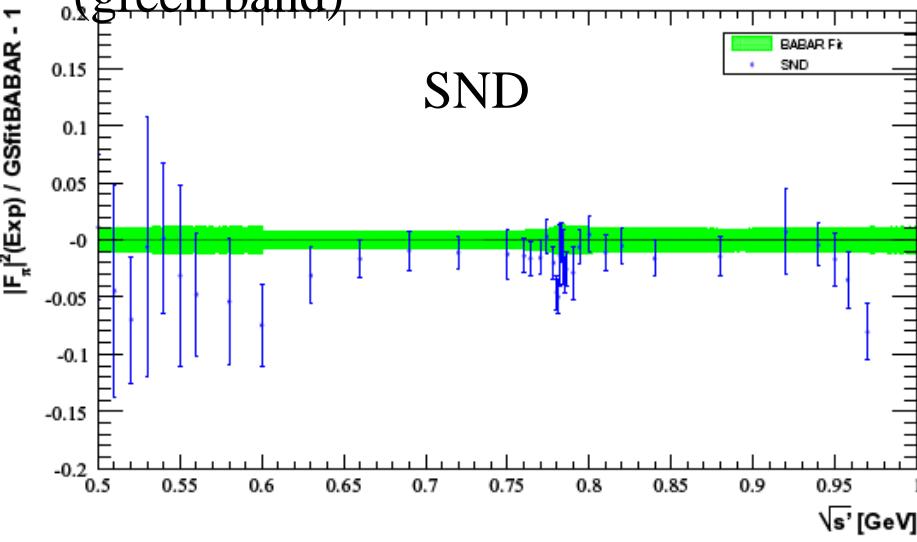
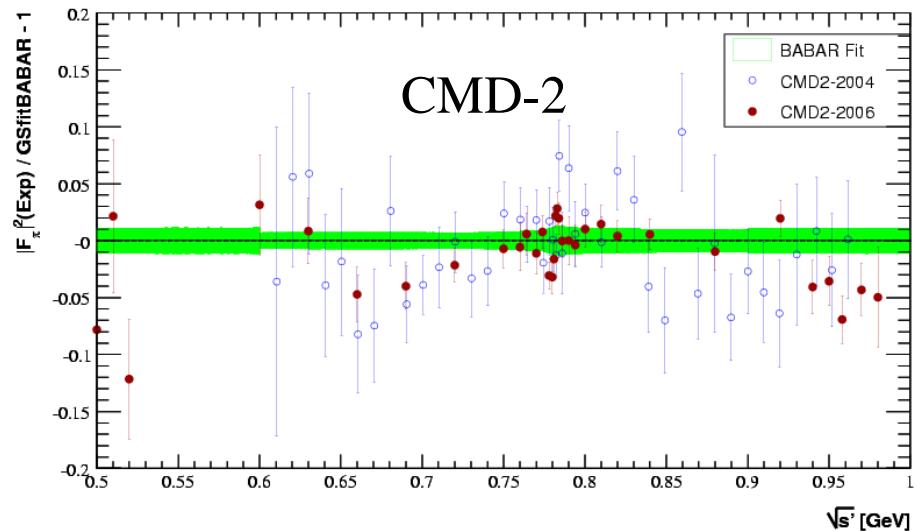
Data on $e^+e^- \rightarrow \pi^+ \pi^-$

BaBar (PRL Dec 2009)



BaBar vs.other ee data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included)
 (green band)



Combining and integrating data

New analysis: MD, A. Hoecker, B. Malaescu, Z. Zhang (DHMZ Oct.2010)

The integration of data points belonging to different experiments, with different within-experiment and inter-experiment correlated systematic errors, and with different data densities requires a **careful treatment**

It is mandatory to test the accurateness of the integration procedure in terms of central value and error using representative models with known truth.

DHMZ approach (HVPTools):

- Quadratic interpolation of the data points/bins for each experiment
- Local weighted average between interpolations performed in infinitesimal bins
- Full covariance matrices: correlations between data points of an experiment (systematic errors), between experiments and channels (VP, luminosity, ...)
- Consistent error propagation using pseudo experiments
- Possible bias tested in 2π channel using a GS model: negligible for quadratic interpolation, but not for linear model (trapezoidal rule)

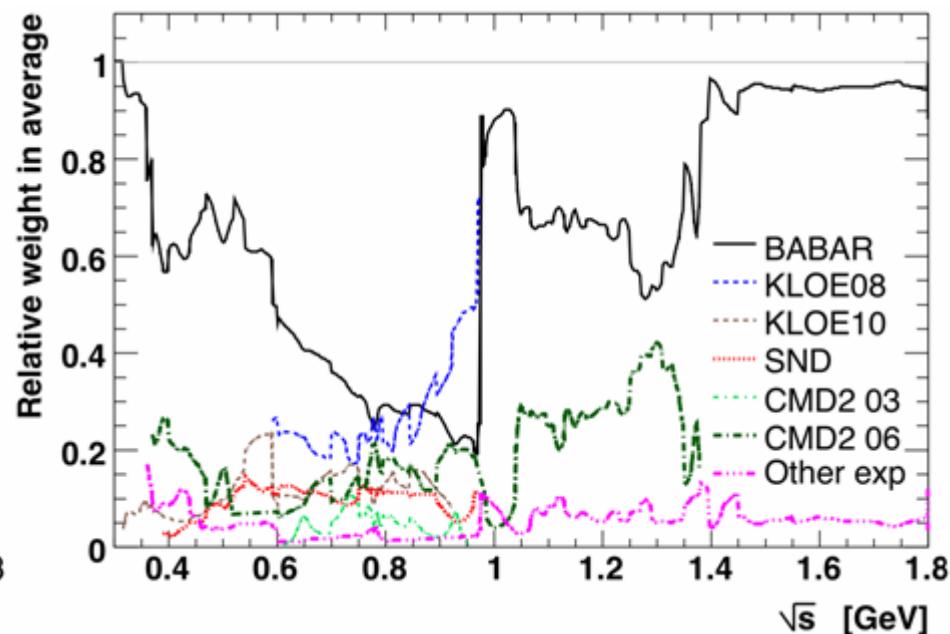
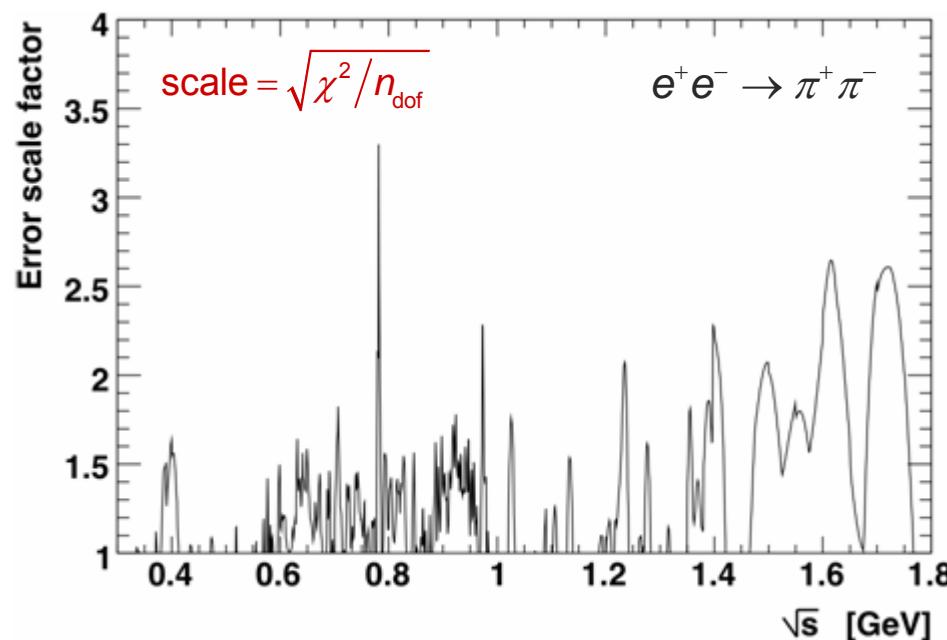
Combining and integrating data

Incompatibilities between data points lead to error rescaling

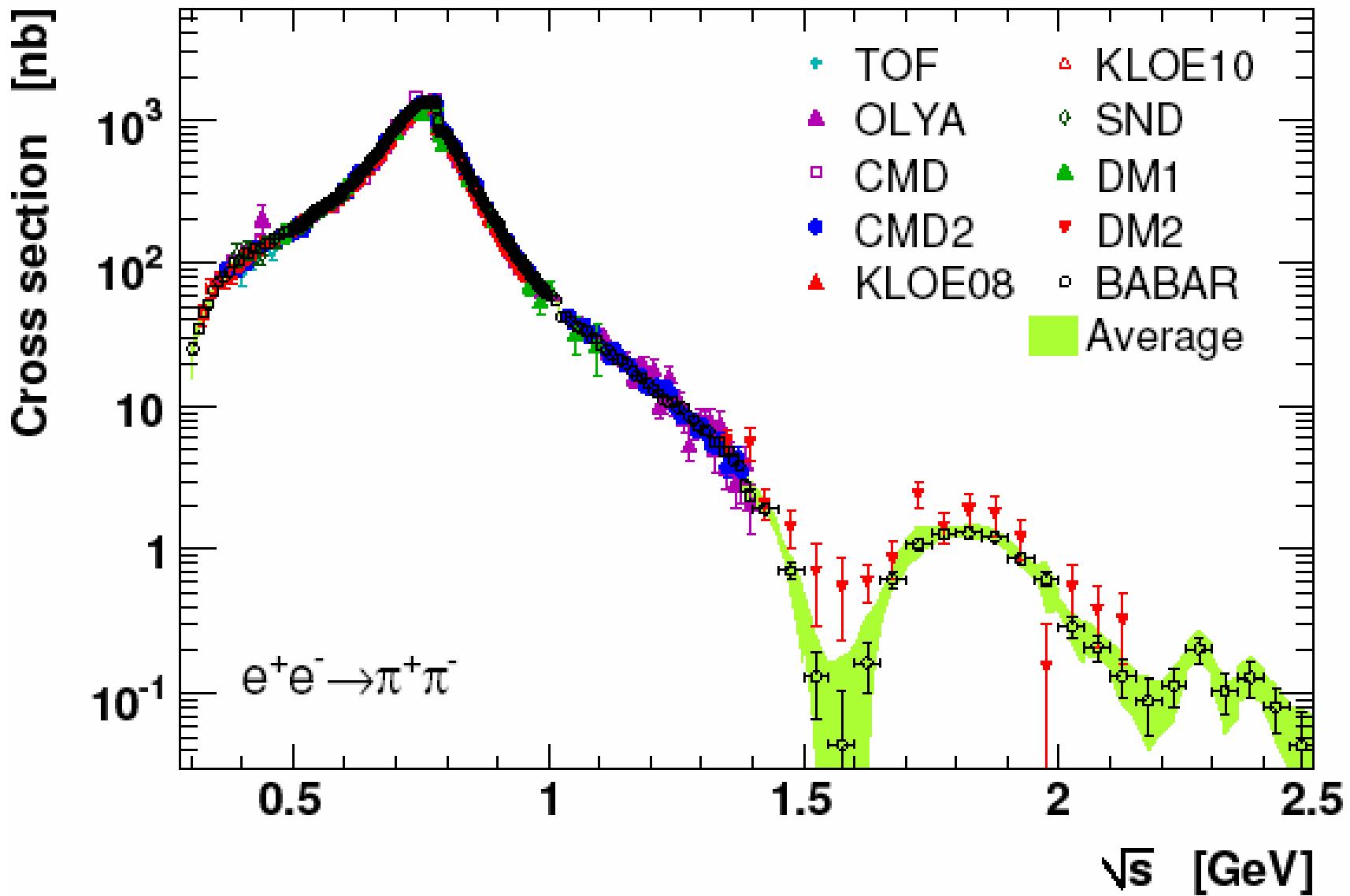
Performed using PDG prescription

Weights of experiments in average versus mass

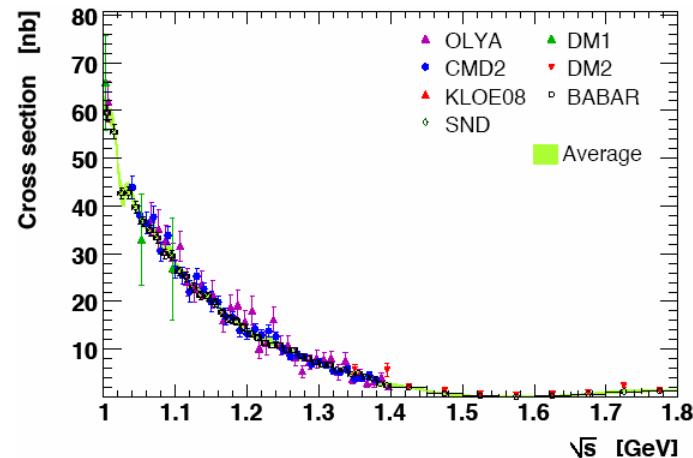
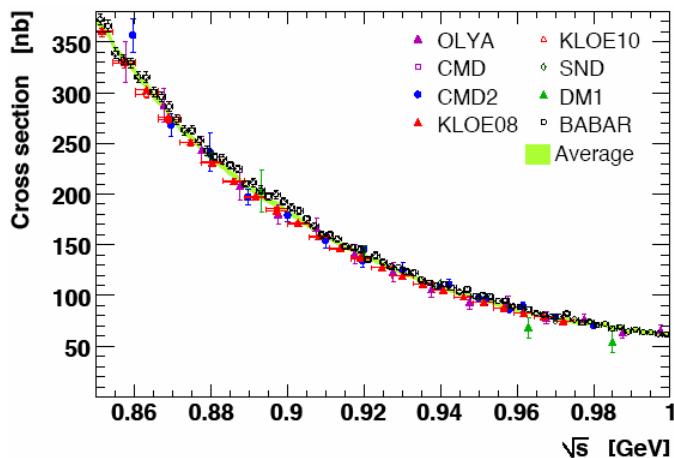
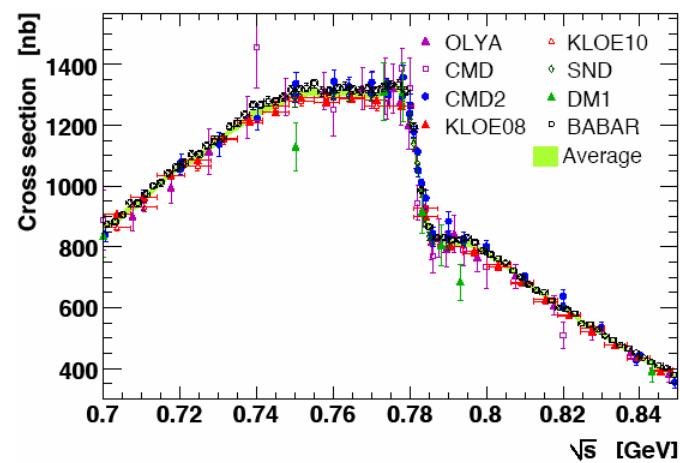
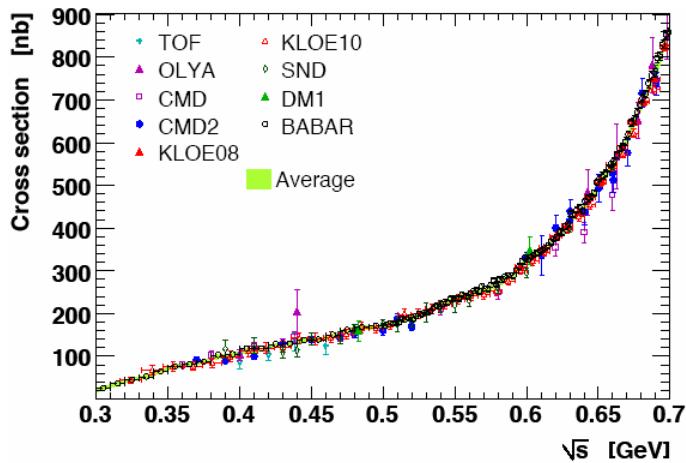
BABAR dominates everywhere, except for KLOE 08 between 0.8 and 0.93 GeV



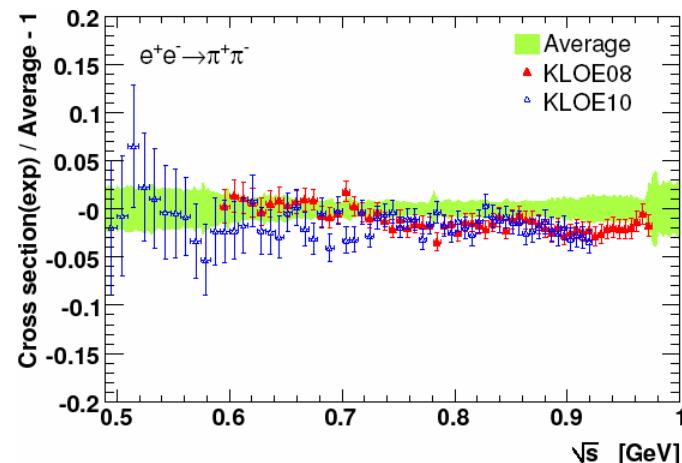
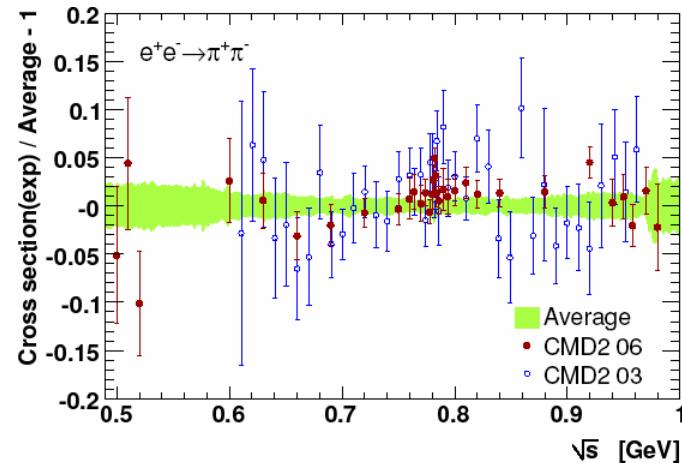
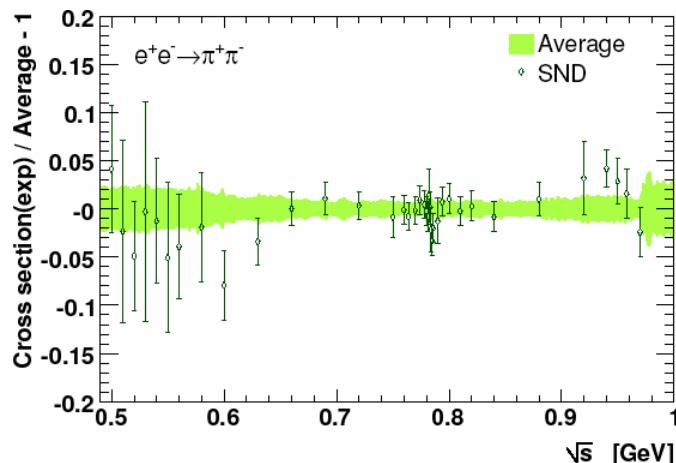
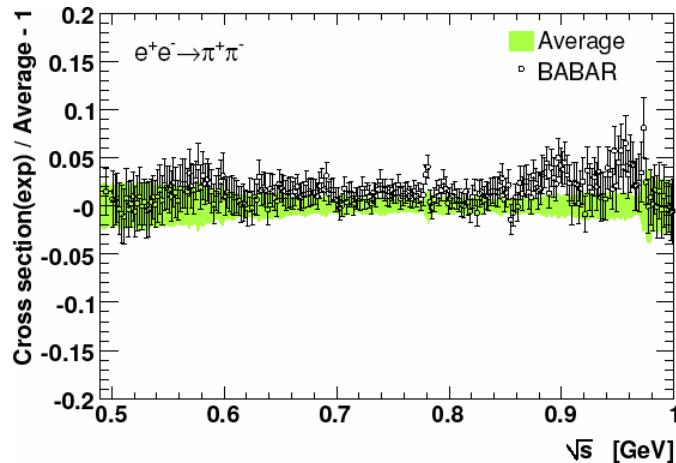
2π combined



2π combined

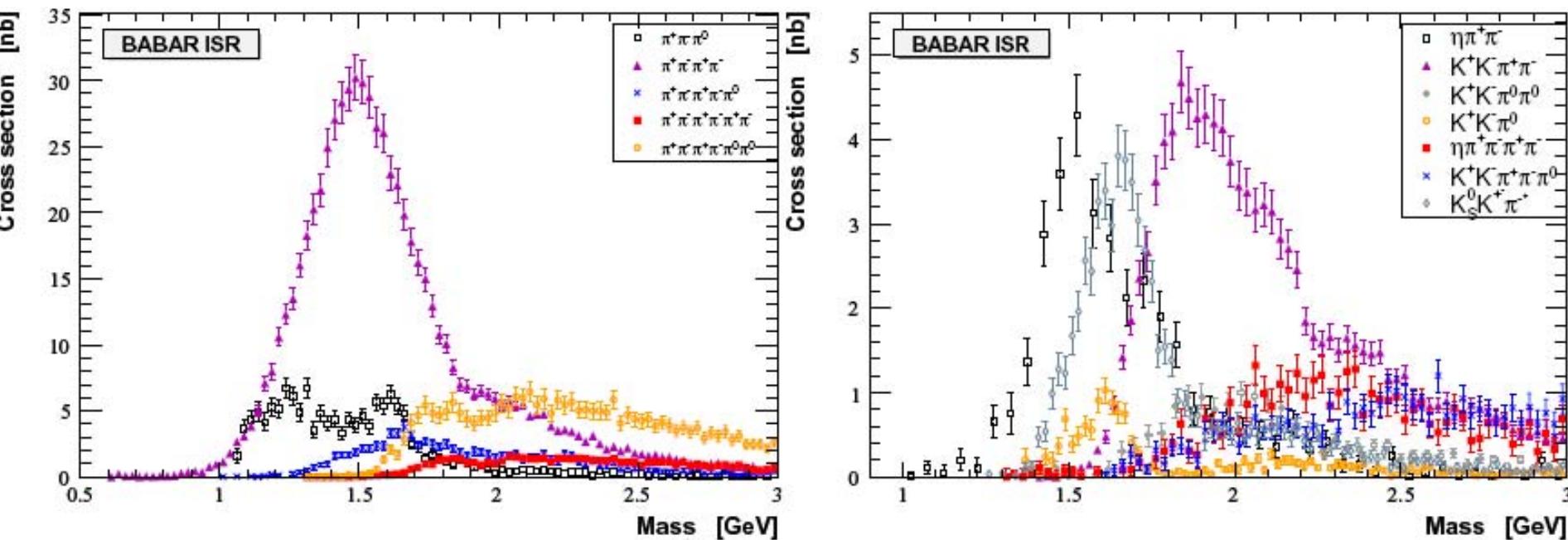


2π experiments compared to combined



Multihadronic channels from BABAR

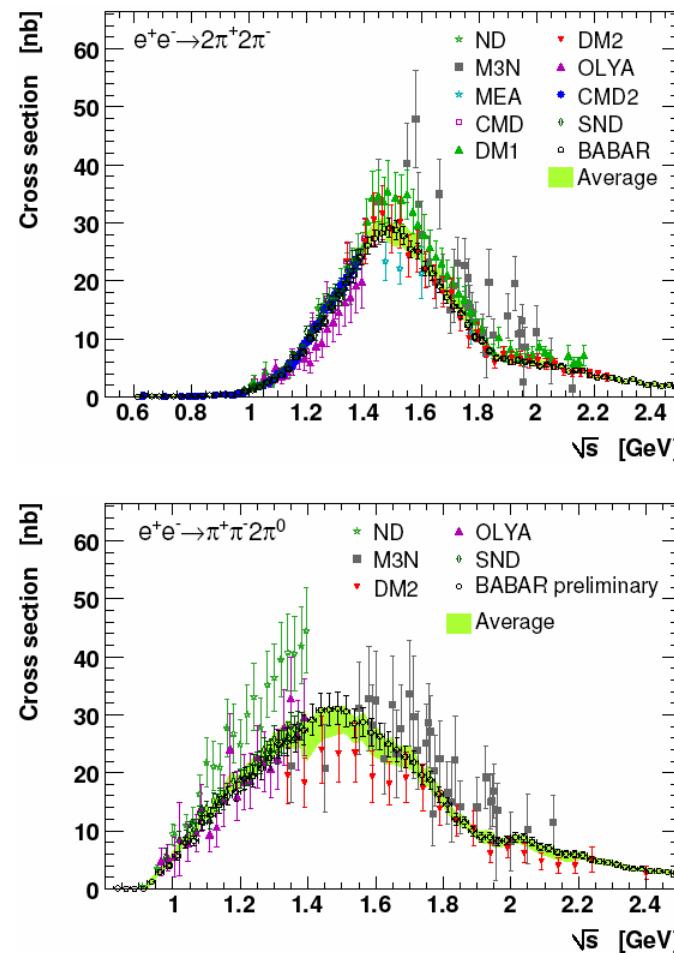
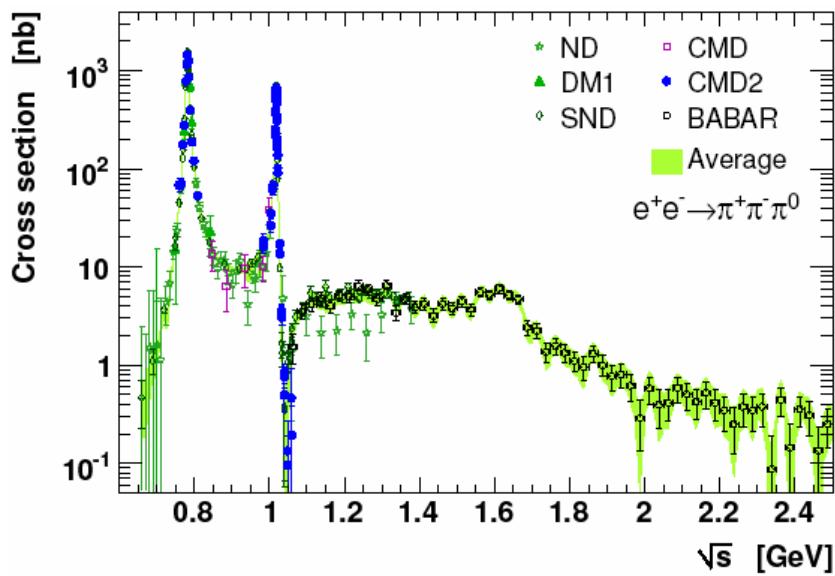
Statistical + systematic errors



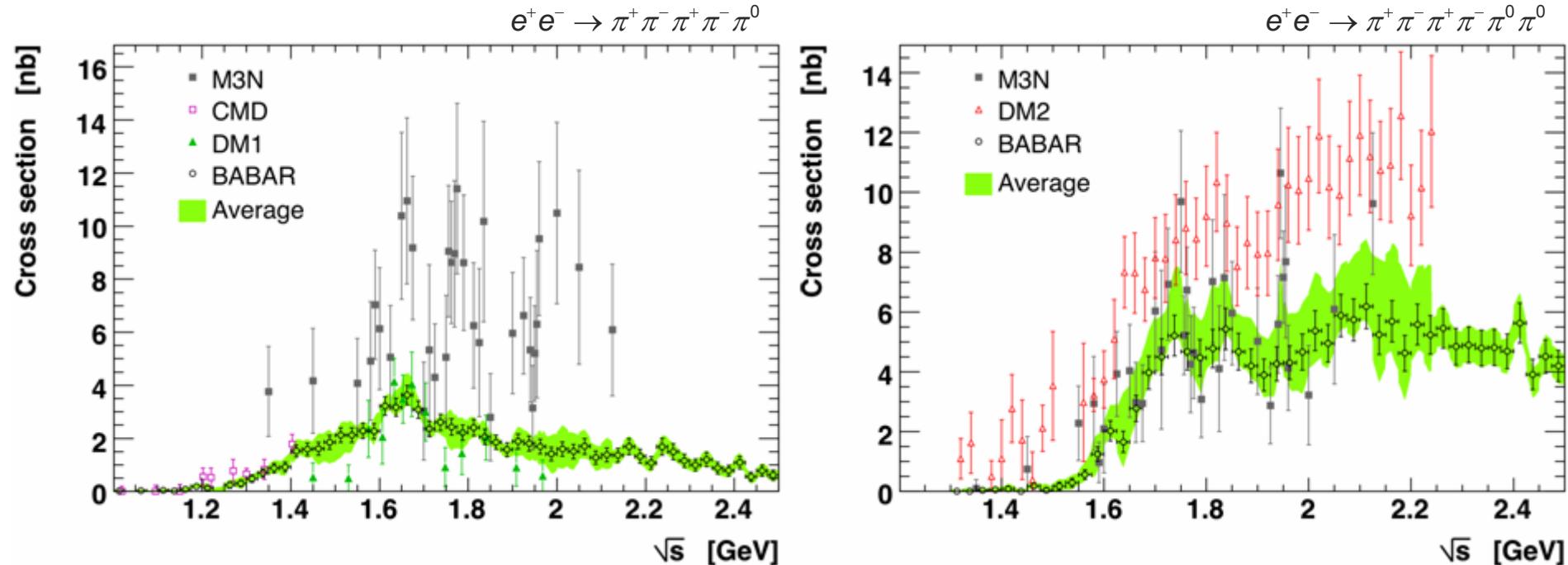
Still more channels under analysis: K^+K^- , $KK\pi\pi$ with $K^0, \pi^+\pi^-2\pi^0$

Also for each channel: dynamics (resonance production) has been studied
⇒ important to estimate missing channels through isospin relations

3π , 4π , $2\pi 2\pi^0$ combined



Some changes



In several cases, earlier poor measurements have lead to overestimated cross sections

Results

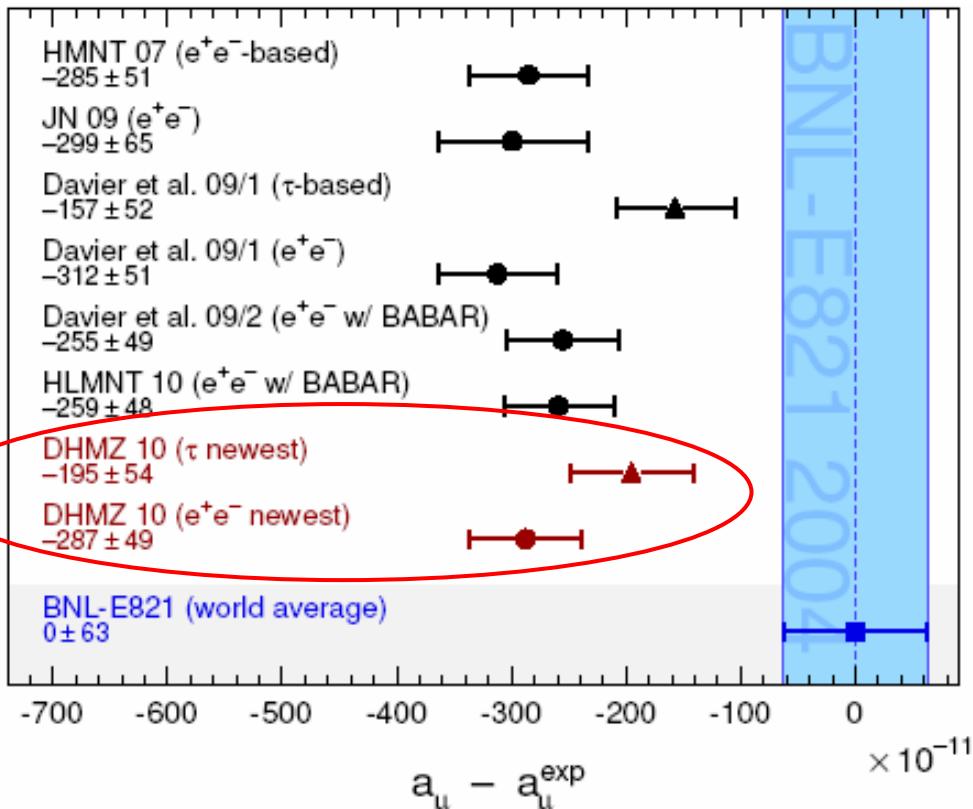
Channel	$a_\mu^{\text{had}} (e^+e^-)$	$a_\mu^{\text{had}} (\tau)$
$\pi^0\gamma$	$4.4 \pm 0.1_{\text{stat}} \pm 0.2_{\text{syst}}$	
$\pi^+\pi^-$	$507.8 \pm 1.2 \pm 2.6$	$515.2 \pm 3.0 \pm 1.9_{\text{IB}}$
$\pi^+\pi^-\pi^0$	$46.0 \pm 0.4 \pm 1.4$	
$2\pi^+2\pi^-$	$13.4 \pm 0.1 \pm 0.5$	$11.9 \pm 1.4 \pm 0.3_{\text{IB}}$
$\pi^+\pi^-2\pi^0$	$18.0 \pm 0.1 \pm 1.2$	$21.2 \pm 1.3 \pm 0.5_{\text{IB}}$
KK	$34.6 \pm 0.3 \pm 0.9$	
$KK\eta\pi$	$3.7 \pm 0.1 \pm 0.4$	
Other exclusive modes	$6.0 \pm 0.3 \pm 0.3$	
Charm region (3.7 – 5 GeV)	$7.3 \pm 0.1 \pm 0.3$	
$J/\psi, \psi(2S)$	7.9 ± 0.2	
R [QCD]	$43.3 \pm 0.3_{\text{theo}}$	<i>Error includes size of 4-loop term + FOPT/CIPT ambiguity</i>
Sum	$692.3 \pm 1.4 \pm 3.9$	

$\times 10^{-10}$

Full a_μ^{SM}

$$a_\mu^{\text{SM}}(\text{ee}) = (11\ 659\ 180.2 \pm 4.2_{\text{had LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+EW}}) \times 10^{-10}$$

DHMZ 2010



Observed Difference with Experiment:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29.6 \pm 8.1) \times 10^{-10}$$

→ 3.6 "standard deviations"

τ -based (2π , 4π , $2\pi 2\pi^0$) estimate
also decreased (multihadronic
modes) ⇒ 2.4σ

Summarizing the changes

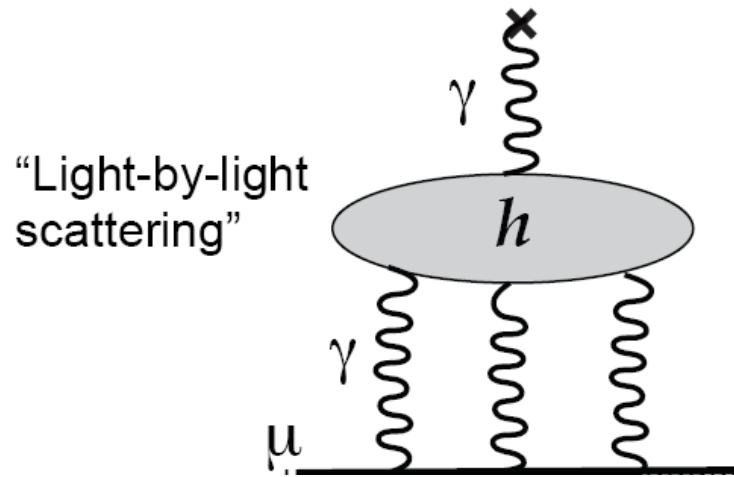
The Tau-2010 DHMZ result is -3.2×10^{-10} smaller than that of 2009

Origins of main changes:

$\pi^+\pi^-$	-0.6	New KLOE 2010 data
$\pi^+\pi^-\pi^0\pi^0$	+0.4	Use preliminary BABAR data
Other exclusive modes and resonances	-2.5	New BABAR data, improved isospin constraints on unknown modes
QCD	-0.5	4-loop pQCD coefficient

Many modes also have been computed for the first time with HVPTools featuring a more precise interpolation, and better error propagation than our previous software

A possible show-stopper: HLBL contribution



- “consensus” (de Rafael, Prades, Vainhstein)
 $+(10.5 \pm 2.6) 10^{-10}$
- estimated error more controversial (Nyffeler)
- beware of double counting
- cannot be obtained from data
- need models
- eventful track record: sign, magnitude
- identified contributions:
 - meson poles (π^0, η, η')
 - meson loops
 - quark loops
- several groups \Rightarrow consistent results

Running of α_{QED}

Same principle as for $g - 2$: energy-dependent vacuum polarisation effects screen the bare electromagnetic coupling. Leptonic contributions computed via QED, hadronic contributions obtained from dispersion relation:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)} \quad \text{with: } \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s) = -4\pi\alpha \operatorname{Re}[\Pi_\gamma(s) - \Pi_\gamma(0)]$$

$$\Delta\alpha_{\text{lep}}^{\text{3-loop}}(M_Z^2) = 0.031497686$$

Steinhauser, hep-ph/9803313 (1998)

$$\Delta\alpha_{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \int_{m_{\pi^0}^2}^{\infty} \frac{R(s)}{s(s - M_Z^2) - i\varepsilon} ds = 0.02768(22)_{\text{had (5)}} - 0.000073(2)_{\text{top}}$$



Integration kernel more
“democratic” than for $g - 2$
(influence of tau data less pronounced)

New evaluation of $\alpha(M_Z)$

Also the hadronic contribution to $\alpha_{\text{QED}}(M_Z)$ has been re-evaluated:

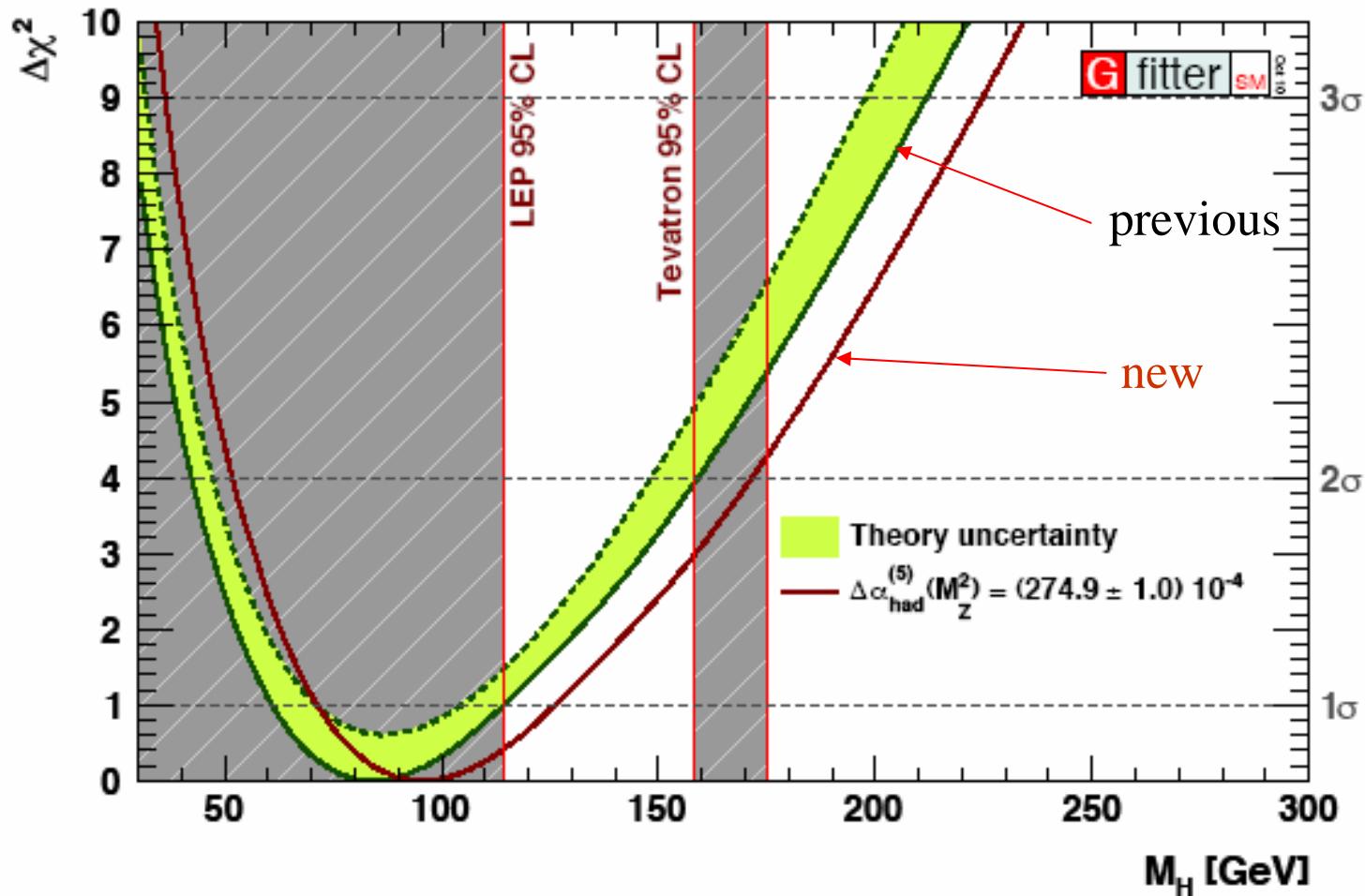
$$\Delta\alpha_{\text{had}}^{(5)} = (274.9 \pm 0.2_{\text{stat}} \pm 0.9_{\text{syst}} \pm 0.5_{\text{QCD}} (\pm 1.0_{\text{tot}})) \cdot 10^{-4}$$

$$\alpha^{-1}(M_Z) = 128.962 \pm 0.014$$

The better precision of the DHMZ value with respect to HMNT (± 1.5) is because of the use of QCD instead of BES data between 1.8 and 3.7 GeV (HMNT employs QCD central values, but with BES errors)

Due to the -40% correlation between $\Delta\alpha_{\text{had}}(M_Z)$ and M_H in the global electroweak fit, the change in the central value should increase M_H and reduce tension between fit and direct searches !

Global EW fit with new value of $\alpha(M_Z)$



\Rightarrow (indirect) M_H increased from $(84 \pm {}^{30}_{23})$ GeV to $(96 \pm {}^{31}_{24})$ GeV
 reduces tension with direct-search lower limit

Conclusions and Perspectives (1)

Recent progress in evaluating hadronic contribution to muon ($g-2$)

- better data to compute dispersion integral
 2π (CMD2, SND, KLOE, BABAR) multihadrons (BABAR)
- multihadronic dynamics (BABAR) to estimate un-measured modes
- still some disagreement for 2π (KLOE/BABAR), but reduced
- better τ data (ALEPH, CLEO, BELLE)
- better estimates of isospin-breaking corrections to τ
- ee/ τ disagreement reduced (+ BABAR), still problem with KLOE
- KLOE/(BABAR- τ) discrepancy must be understood

Conclusions and Perspectives (2)

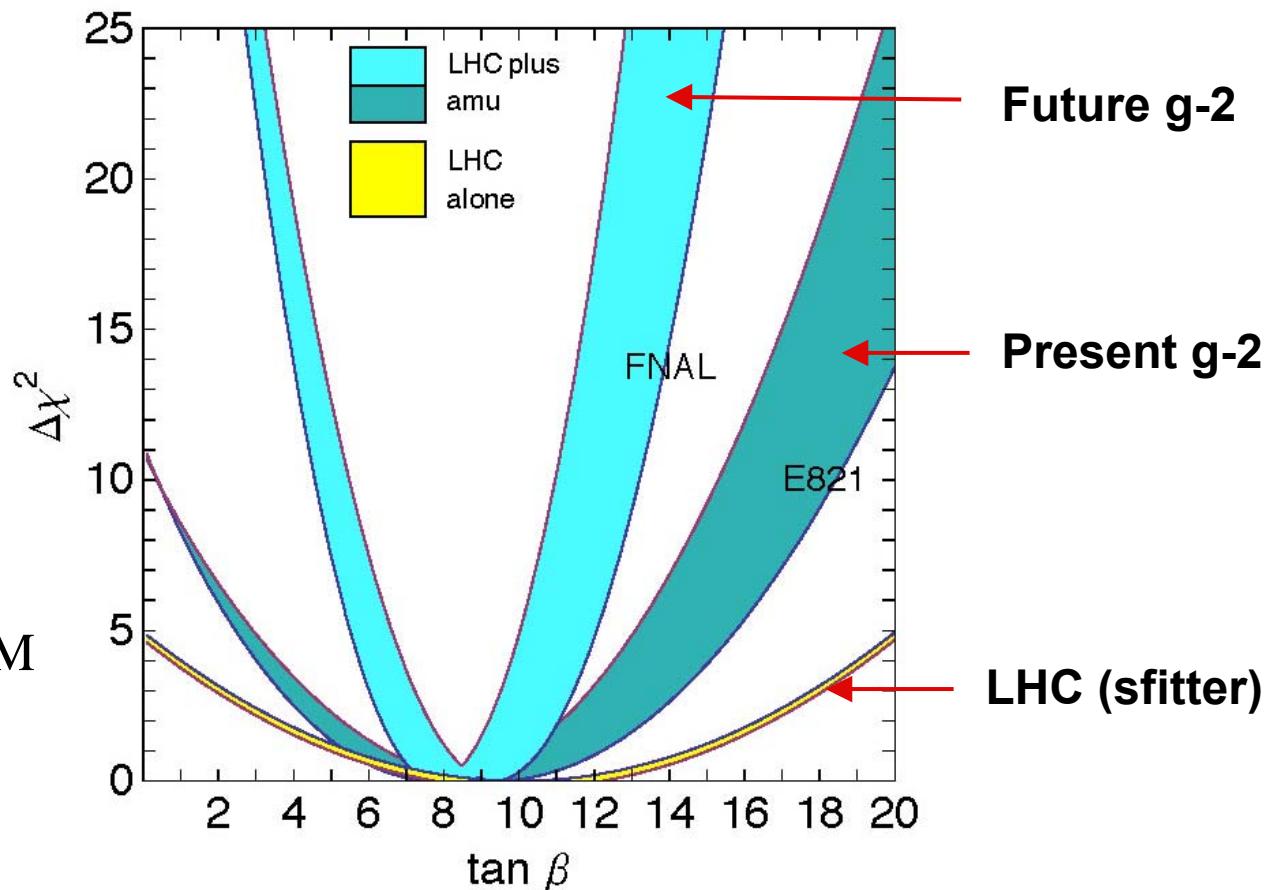
- most recent/complete evaluation (DHMZ) increases SM/exp discrepancy on a_μ to 3.6σ
- progress will continue on LO HVP contribution (VEPP-2000)
- NLO contribution from hadronic light-by-light still a weak spot will soon limit the SM prediction uncertainty
⇒ more work needed
- SM uncertainty (4.9) smaller than present experimental error (6.3)
⇒ need a new experiment (FNAL, JPARC)
- new evaluation of $\alpha(M_Z)$ is more precise and increases (indirect) Higgs mass by 12 GeV

Impact on new physics : ex. SUSY

LHC: direct search for SUSY partners

difficult to measure couplings and disentangle between models (ILC)

g-2 measurement + theory prediction: sensitivity to couplings



Backup Slides

The E-821 a_μ Measurement at BNL Updated

a_μ measured from a ratio of frequencies

$$\omega_a = \omega_{\text{precession}} - \omega_{\text{cyclotron}}$$

$$\omega_{\text{precession}} = \omega_L + \omega_T$$

$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\tilde{\omega}_p}{\omega_L/\tilde{\omega}_p - \omega_a/\tilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

$\lambda = \omega_L / \omega_p = \mu_\mu / \mu_p$ from muonium hyperfine splitting

value used by E-821 3.18334539(10)

new value 3.183345137(85) Mohr et al., RMP 80 (2008) 633

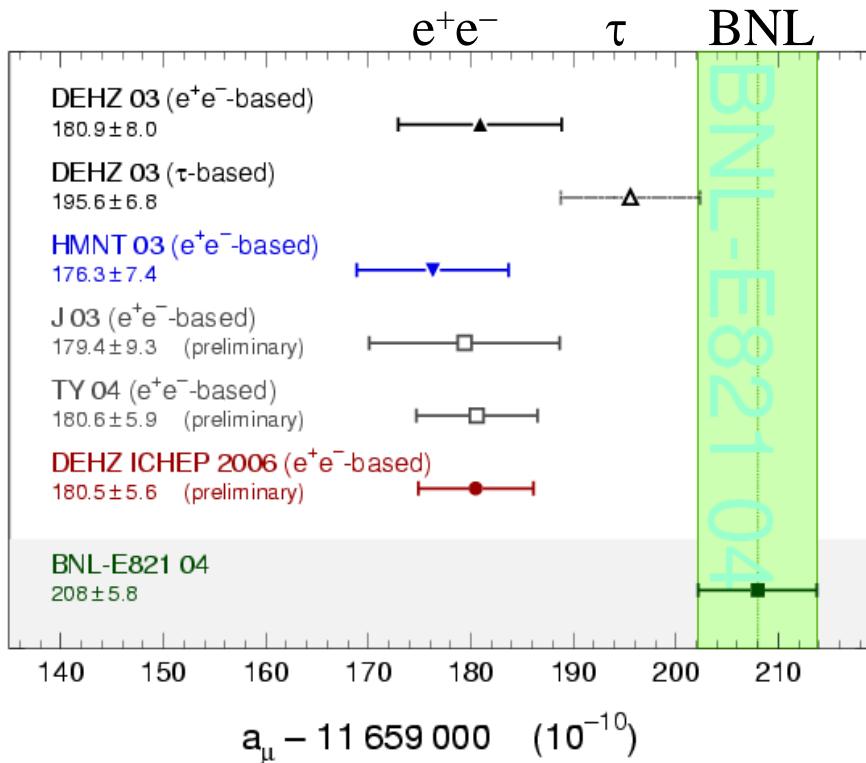
⇒ change in a_μ (+0.92 10^{-10})
(review in RPP2009 (Höcker-Marciano))

$a_\mu^{\text{exp}} = (11\ 659\ 208.9 \pm 5.4 \pm 3.3) \ 10^{-10}$ updated
 (± 6.3) (0.54 ppm)

Situation at ICHEP'06 / 08

$$a_\mu^{\text{had}} [\text{ee}] = (690.9 \pm 4.4) \times 10^{-10}$$

$$a_\mu [\text{ee}] = (11\ 659\ 180.5 \pm 4.4_{\text{had}} \pm 3.5_{\text{LBL}} \pm 0.2_{\text{QED+EW}}) \times 10^{-10}$$



Hadronic HO	$-(9.8 \pm 0.1) \times 10^{-10}$
Hadronic LBL	$+(12.0 \pm 3.5) \times 10^{-10}$
Electroweak	$(15.4 \pm 0.2) \times 10^{-10}$
QED	$(11\ 658\ 471.9 \pm 0.1) \times 10^{-10}$

Knecht-Nyffeler (2002), Melnikov-Vainhstein (2003)

Davier-Marciano (2004)

Kinoshita-Nio (2006)

Observed Difference with BNL using e⁺e⁻:

$$a_\mu [\text{exp}] - a_\mu [\text{SM}] = (27.5 \pm 8.4) \times 10^{-10}$$

→ 3.3 „standard deviations“

But estimate using τ data consistent with E-821 !

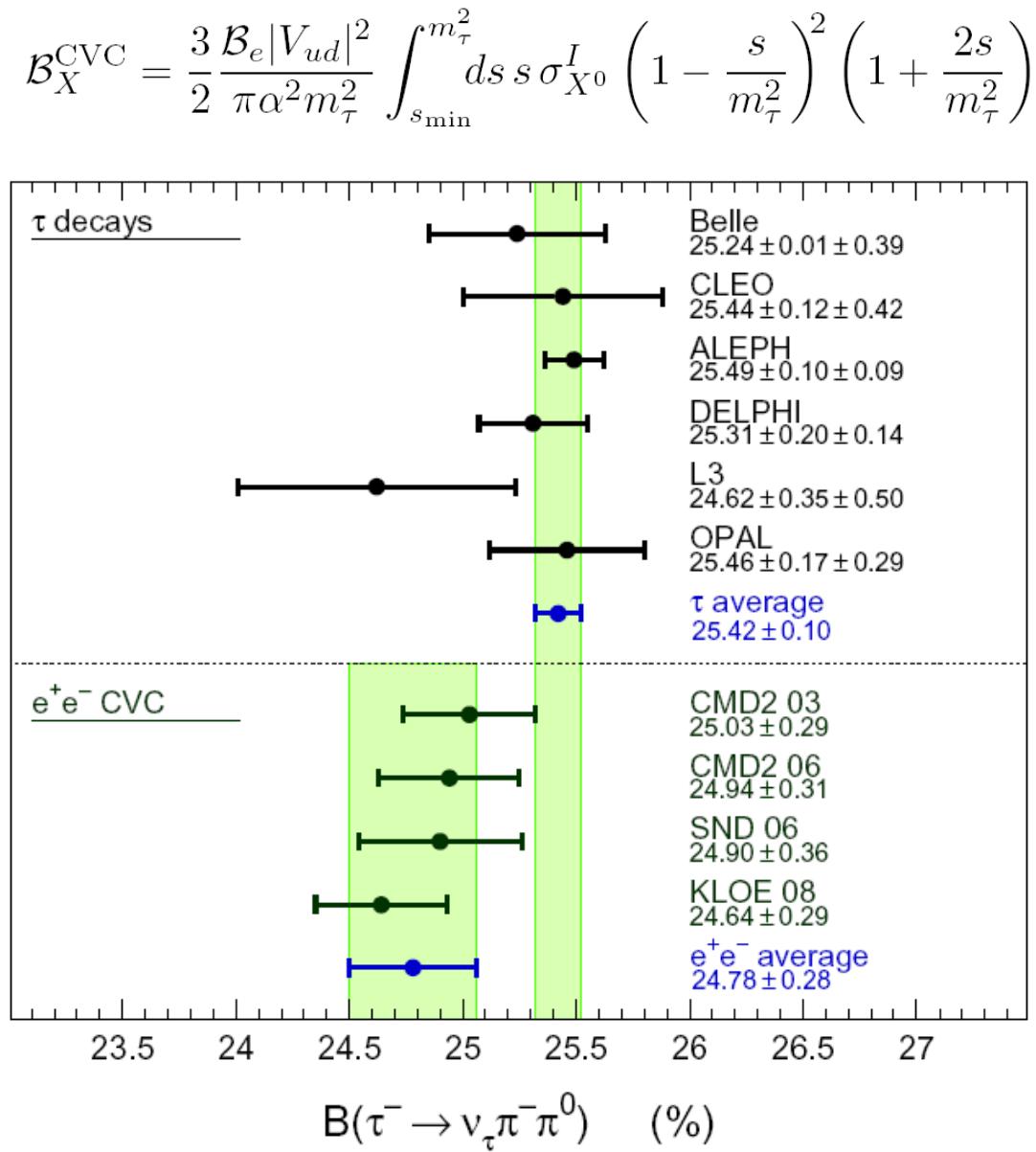
Comparison of ee and τ Data Revisited (2)

Global test of spectral functions:
prediction of τ BR using ee data

⇒ apply to $\pi\pi^0$ channel

IB corrections applied to ee data
this time

- data from CMD2-SND overconsistent ?
- fair agreement CMD2-SND with τ
- larger disagreement with KLOE



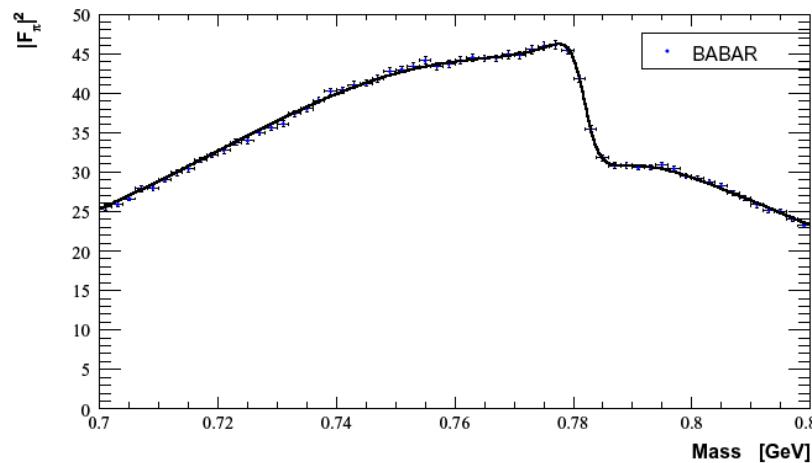
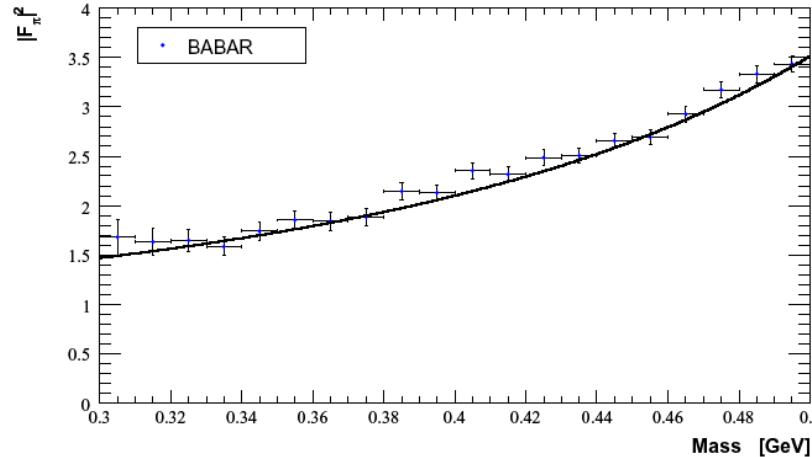
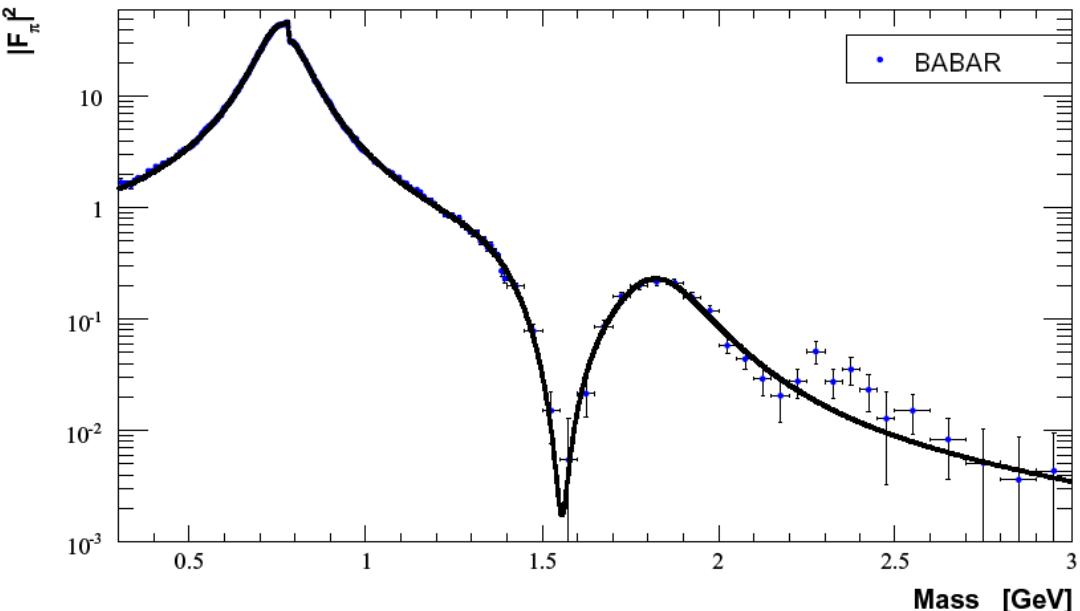
VDM Fit of the BaBar Pion Form Factor

$$F_\pi(s) = \frac{BW_\rho^{GS}(s, m_\rho, \Gamma_\rho) \frac{1+\alpha BW_\omega^{KS}(s, m_\omega, \Gamma_\omega)}{1+\alpha} + \beta BW_{\rho'}^{GS}(s, m_{\rho'}, \Gamma_{\rho'}) + \gamma BW_{\rho''}^{GS}(s, m_{\rho''}, \Gamma_{\rho''})}{1 + \beta + \gamma}$$

$$|F_\pi|^2(s') = \frac{3s'}{\pi \alpha^2(0) \beta_\pi^3} \sigma_{\pi\pi}(s')$$

$$\sigma_{\pi\pi}(s') = \frac{\sigma_{\pi\pi(\gamma)}^0(s')}{1 + \frac{\alpha}{\pi} \eta(s')} \left(\frac{\alpha(s')}{\alpha(0)} \right)^2$$

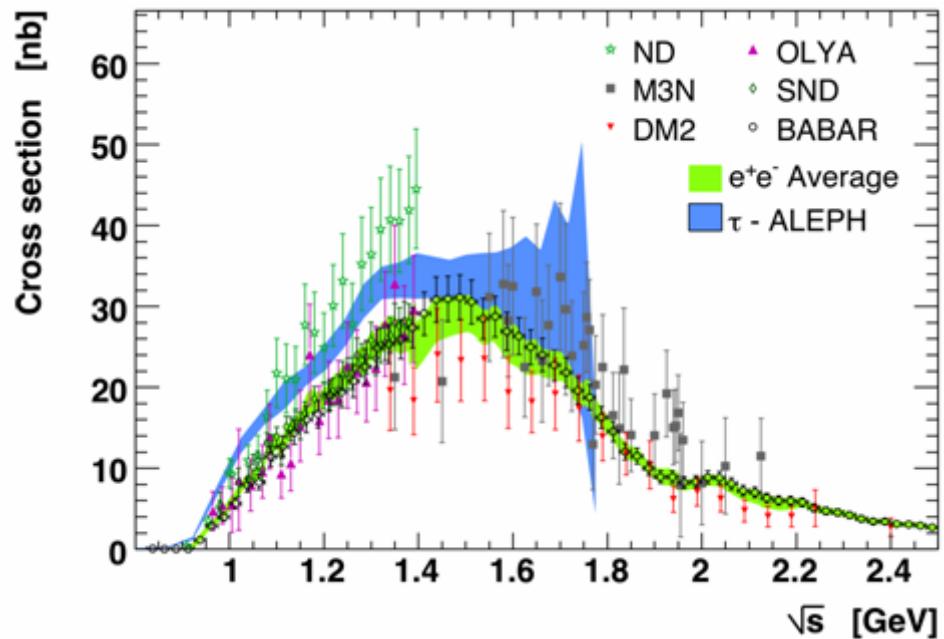
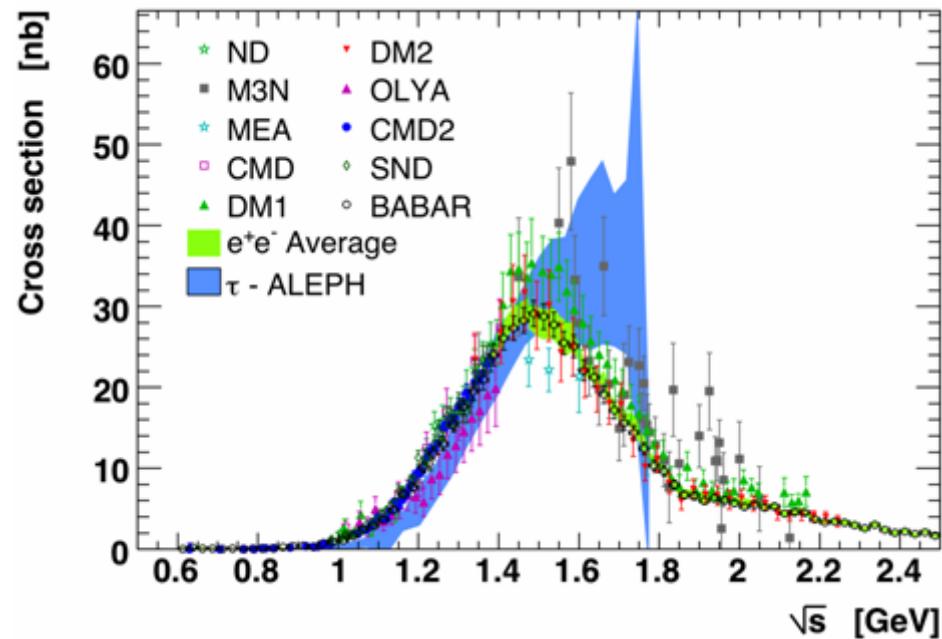
add. FSR α Running (VP)



Multihadron channels: 4-pion (ee and τ)

Recall 4π isospin relations: $\sigma_{2\pi^+ 2\pi^-}^{(I=1)} = 2 \cdot \frac{4\pi\alpha^2}{s} v_{\pi^- 3\pi^0 \nu_\tau}$

$$\sigma_{\pi^+ \pi^- 2\pi^0}^{(I=1)} = \frac{4\pi\alpha^2}{s} \left(v_{2\pi^- \pi^+ \pi^0 \nu_\tau} - v_{\pi^- 3\pi^0 \nu_\tau} \right)$$

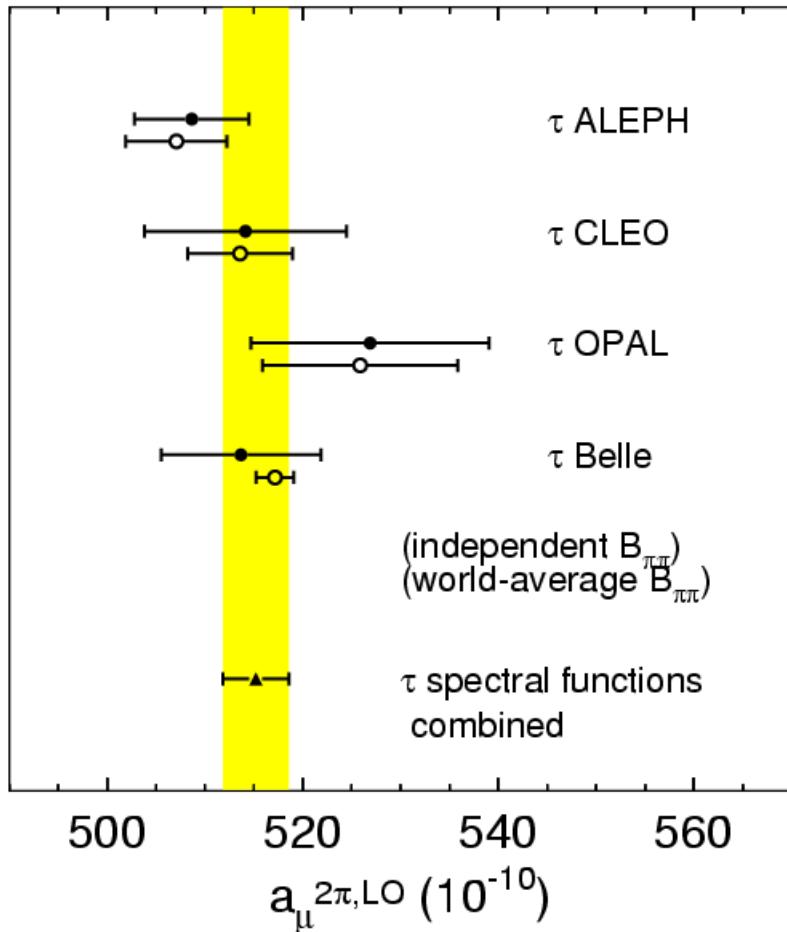


Revisited Analysis τ Data: new IB corrections

Source	$\Delta a_\mu^{\text{had,LO}}[\pi\pi, \tau] (10^{-10})$	
	GS model	KS model
S_{EW}		-12.21 ± 0.15
G_{EM}		-1.92 ± 0.90
FSR		$+4.67 \pm 0.47$
ρ - ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ		-7.88
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\text{bare}}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
		-16.07 ± 1.85

⇐ large change
since DEHZ (2003)

Consistency of τ Data: Dispersion Integrals



- using BR from each experiment makes results independent from each other
 - consistent results (*disagreement with Benayoun et al. arXiv:09075603v1*)
 - using WA BR checks consistency for the spectral function shapes
 - WA BR + combined spectral function \Rightarrow
- $$a_\mu^{2\pi, \text{LO}} = (515.2 \pm 2.0_{\text{exp}} \pm 0.9_{\text{Be}} \pm 2.1_{\text{B}\pi\pi} \pm 1.6_{\text{IB}}) 10^{-10}$$
- 0.7% precision