Top Quark Antenna Splitting Function

work in progress with
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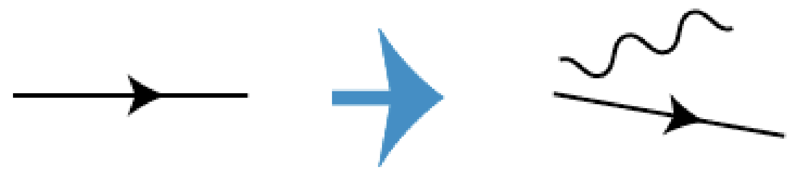
Parton showers were developed in the 1980’s to model the properties of QCD jets over a large dynamic range. They are the basic ingredients of the Monte Carlo generators PYTHIA and HERWIG.

Today, there is a renewed interest in parton showers. For analyses at the LHC, we would like to exploit more subtle effects of QCD: color correlations, spin dynamics. Can we produce parton showers that model these effects more accurately ?

The standard PYTHIA/HERWIG shower is based on a 1 > 2 parton splitting.



The probability assigned to this splitting has the form

$$\frac{\alpha_s}{\pi}\int\frac{dp_T}{p_T}\int dz\ P(z)$$

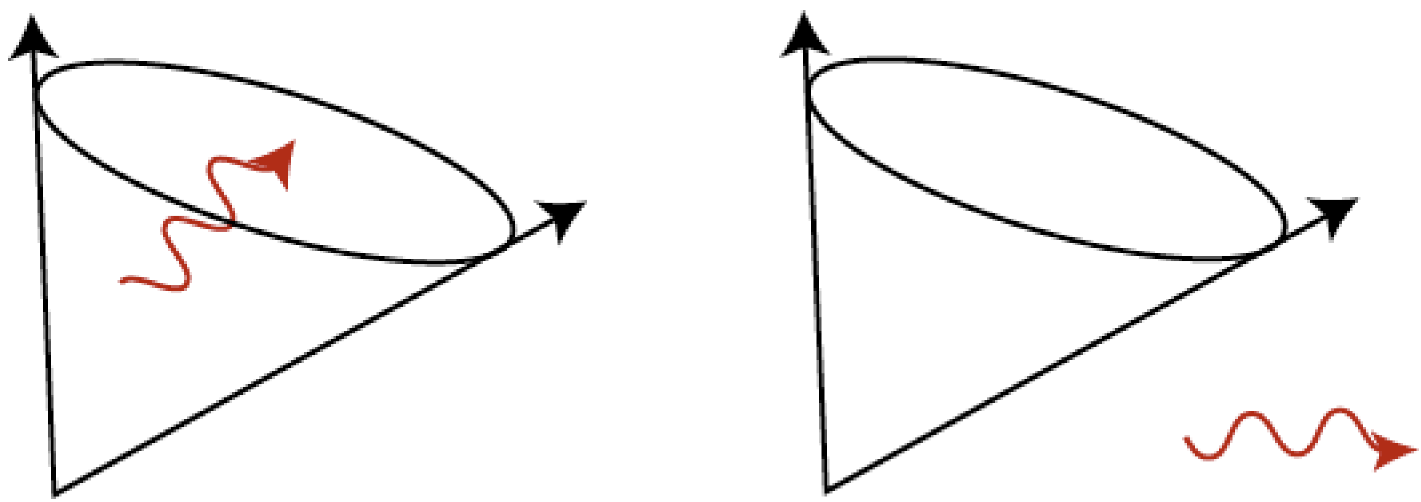
where P(z) is the Altarelli-Parisi function. In the soft parton emission limit

$$\frac{\alpha_s}{\pi}\int d\cos\theta\int dz\ \frac{1}{z(1-\cos\theta)}$$

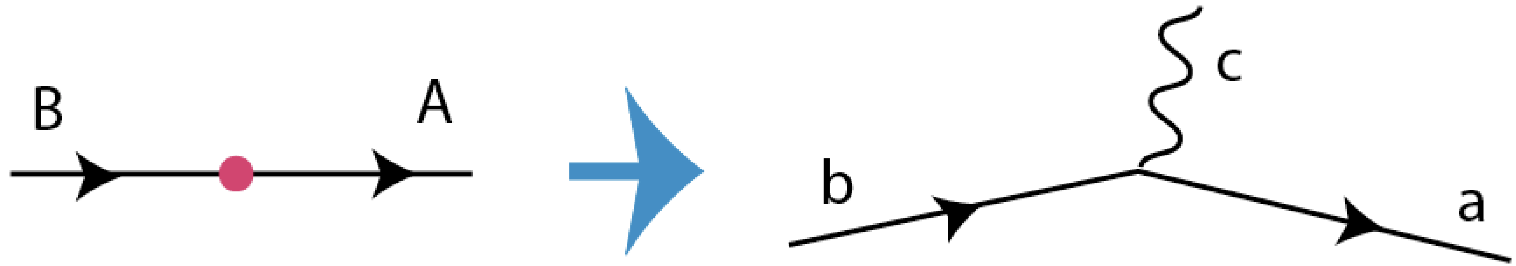
The evolution spans a 2-dimensional space. HERWIG covers this space with angular (θ) ordering. PYTHIA uses pT ordering, but rejects emissions that do not satisfy angular ordering.

Why angular ordering ?   This encodes color correlations.

We can think of QCD emissions as radiated from color dipoles. The radiation is large inside the dipole, and small outside the dipole where the color sources destructively interfere.



Another approach to modelling this physics is to use an antenna shower, that is, a shower in which the basic splitting is a 2 > 3 process



ARIADNE
Andersson, Gustafson, Lonnblad, Pettersson

VINCIA
Giele, Kosower, Skands

The probability assigned to this splitting can be written

$$\frac{\alpha_s}{\pi} \int dz_a dz_b \frac{\mathcal{N}(z_a, z_b, z_c)}{y_{ab} y_{bc} y_{ca}}$$

where

$$z_a = \frac{2k_a \cdot Q}{Q^2} \qquad y_{bc} = \frac{s_{bc}}{Q^2} = (1 - z_a)$$

In the soft limit.

$$\frac{\alpha_s}{\pi} \int dz_a dz_b \frac{1}{y_{bc} y_{ca}}$$

This expression is the correct soft limit of a color dipole and thus incorporates color interference.

Altarelli and Parisi wrote down spin-dependent splitting functions.

Recently, Larkoski and I worked out spin-dependent antenna splitting functions for use in antenna showers:

$$\mathcal{N}(z_a, z_b, z_c) =$$

	$+++$	$++-$	$+-+$	$-++$	$--+$	$-+-$	$+--$	$---$
$g_+g_+ \to ggg$	1	y_{ac}^4	y_{ab}^4	y_{bc}^4	0	0	0	0
$g_-g_+ \to ggg$	0	0	y_{bc}^4	z_a^4	z_b^4	y_{ac}^4	0	0
$g_+g_+ \to \bar{q}qg$	-	-	$y_{ab}^3 y_{bc}$	$y_{ab} y_{bc}^3$	-	0	0	-
$g_-g_+ \to \bar{q}qg$	-	-	$y_{ab} y_{bc}^3 z_b^2$	$z_a^2 z_b^2 y_{ab} y_{bc}$	-	0	0	-
$q_-\bar{q}_+ \to qg\bar{q}$	-	-	-	$y_{ab} z_a^2$	$y_{ab} z_b^2$	-	-	-
$q_-\bar{q}_- \to qg\bar{q}$	-	-	-	-	-	y_{ab}^3	-	y_{ab}
$q_-g_- \to qgg$	-	-	-	0	y_{ac}^4	$y_{ab}^3 z_b$	-	z_a
$q_-g_+ \to qgg$	-	-	-	z_a^3	$y_{ab} z_b^3$	y_{ac}^4	-	0
$q_-g_- \to q\bar{q}q$	-	-	-	-	$y_{ab} y_{ac}^3$	$y_{ab}^2 y_{ac} z_b$	-	-
$q_-g_+ \to q\bar{q}q$	-	-	-	-	$z_a y_{ab} y_{ac} z_b^2$	$z_a y_{ab} y_{ac}^3$	-	-

So far, all of these results are for all massless particles.

However, at the LHC we will have many energetic top quarks, and it will be interesting to study gluon radiation from top quarks.

How do we do this in an antenna framework ?

Massive Altarelli-Parisi splitting function $t \to gt$

(Catani-Dittmaier-Trocsanyi-Seymour)

$$P(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z} - \frac{m^2}{k_t \cdot k_g}\right]$$

To understand this, it is interesting to compare to the spin-dependent gluon emission amplitudes:

$$\begin{aligned}
i\mathcal{M}(t_L \to g_L t_L) &= \sqrt{2} i g \frac{p_T}{z(1-z)^{1/2}} \cdot 1 \\
i\mathcal{M}(t_L \to g_R t_L) &= \sqrt{2} i g \frac{p_T}{z(1-z)^{1/2}} \cdot (1-z) \\
i\mathcal{M}(t_L \to g_L t_R) &= \sqrt{2} i g \frac{m}{z(1-z)^{1/2}} \cdot z^2 \\
i\mathcal{M}(t_L \to g_R t_R) &= \sqrt{2} i g \frac{m}{z(1-z)^{1/2}} \cdot 0
\end{aligned}$$

Square and integrate with phase space and the massive denominator $1/(P^2 - m^2)^2$, with

$$P^2 - m^2 = \frac{p_T^2 + z^2 m^2}{z(1-z)}$$

and make use of $$2g^2 \int \frac{d^3p}{(2\pi)^3 2E} = \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \int dp_T^2$$

Then we find for the emission probability,

$$\frac{\alpha_s}{2\pi} \int dz \int \frac{dp_T^2}{[p_T^2 + z^2 m^2]^2} \left(\frac{p_T^2 (1 + (1-z)^2)}{z} + \frac{m^2 z^4}{z} \right)$$

$$= \frac{\alpha_s}{2\pi} \int dz \int \frac{dp_T^2}{[p_T^2 + z^2 m^2]} \left(\frac{1 + (1-z)^2}{z} - \frac{2m^2}{p_T^2 + z^2 m^2} \right)$$

The first of these formulae is more illuminating. There is an extra term specifically from helicity-flip emission, and it shows a "dead cone" for soft radiation from slow top quarks.

Can we find analogous formulae in for antennae ?

Our strategy in the earlier paper was to consider the antenna as generated by a local operator. Then the antenna splitting function is

$$Q^2 \left| \frac{\mathcal{M}(\mathcal{O} \to 3)}{\mathcal{M}(\mathcal{O} \to 2)} \right|^2$$

to be integrated over 3-particle phase space.

By considering chiral operators and polarized 3-particle final states, we derive the spin-dependent splitting functions.

The computation of heavy quark amplitudes is easier if we use spinor products. For heavy quarks, we use the Schwinn-Weinzierl representation of the massive spinors,

$$\langle t_L| = \frac{[q(k+m)}{[qk^\flat]} \qquad \langle t_R| = \frac{\langle q(k+m)}{\langle qk^\flat\rangle}$$

with
$$q^\flat = k - \frac{m^2}{2k\cdot q} q$$

q is a massless reference vector. It is most convenient to choose q to be the massless vector in the backwards direction to k . We call this $k^\sharp$. Then

$$k = (E,0,0,k)\ , \quad k^\flat = \frac{E+k}{2}(1,0,0,1)\ , \quad k^\sharp = \frac{E+k}{2}(1,0,0,-1)$$

With this definition, the above spinors correspond to the standard helicity states.

Analyze the simplest case of a $t\overline{q}$ dipole, where q is a light quark. This has spin 0 and spin 1 cases; I will do only the spin 0 case.

The chiral local operator generating this dipole is $t_L\overline{q}_L$. The 2-body matrix elements $\mathcal{O} \to t(A)\overline{q}(B)$ are

$$\begin{aligned} i\mathcal{M}(\mathcal{O} \to t_L\overline{q}_L) &= \langle A^\flat B\rangle \\ i\mathcal{M}(\mathcal{O} \to t_R\overline{q}_L) &= m\langle qB\rangle/\langle qA^\flat\rangle \end{aligned}$$

Taking $q = A^\sharp$, and recognizing that $A^\sharp$ is parallel to B, we have

$$\langle A^\sharp B\rangle = 0$$

Then we have the standard helicity rule that a spin zero state has only $t_L\overline{q}_L$ and no $t_R\overline{q}_L$.

Here are the 3-body matrix elements $\mathcal{O} \to t(a)g(c)\overline{q}(b)$ of this operator:

$$
\begin{aligned}
i\mathcal{M}(t_L g_L \overline{q}_L) &= -\sqrt{2} i g \Big\{ \frac{\langle a^\flat b\rangle}{[a^\sharp c]} \Big(\frac{[a^\sharp a c\rangle}{s_{ac} - m^2} - \frac{[a^\sharp b c\rangle}{s_{bc}} \Big) \\
&\qquad + \langle a^\flat c\rangle \langle c b\rangle \Big(\frac{1}{s_{ac} - m^2} + \frac{1}{s_{bc}} \Big) \Big\} \\
i\mathcal{M}(t_L g_R \overline{q}_L) &= \sqrt{2} i g \frac{\langle a^\flat b\rangle [c a b\rangle}{\langle b c\rangle} \frac{1}{s_{ac} - m^2} \\
i\mathcal{M}(t_R g_L \overline{q}_L) &= -\sqrt{2} i g \frac{m}{\langle a^\sharp a^\flat\rangle [a^\flat c]} \Big\{ \frac{\langle a^\sharp c\rangle [a^\flat (a + c) b\rangle}{s_{ac} - m^2} \\
&\qquad + \frac{\langle b c\rangle [a^\flat (b + c) a^\sharp\rangle}{s_{bc}} \Big\} \\
i\mathcal{M}(t_R g_R \overline{q}_L) &= \sqrt{2} i g \frac{m}{\langle a^\sharp a^\flat\rangle \langle b c\rangle} \langle a^\sharp b\rangle \frac{[c a b\rangle}{s_{ac} - m^2}
\end{aligned}
$$

The splitting functions are given by these functions, multiplied by

$$\frac{Q}{\langle A^\flat B\rangle}$$

The functions on the previous page are not simple, but they are not so difficult to evaluation.

The main problem is the presence of $a^\flat$ and $a^\sharp$. However, we will generate phase space by working with massless vectors, balancing momentum, and then rescaling so that we have the correct energy including the top quark mass. The vectors $a^\flat$ and $a^\sharp$ arise naturally as the massless vectors in this process.

Larkoski and I feel that this is a promising strategy for generating spin-aware antenna showers with top quarks. Much work remains to be done. Wish us luck !