Heavy Quark Production at Higher Orders

Ideas, Issues and Intricacies

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Conspirators:
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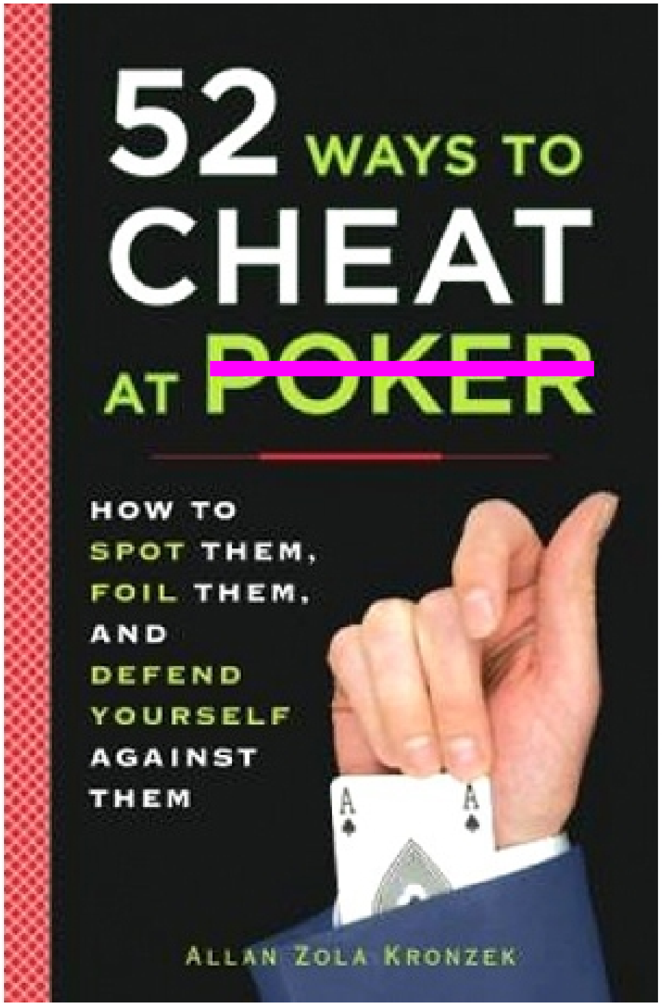
T.P. Stavreva, J. Owens, C. Keppel, D. Soper ...

LCWS'12
25 October 2012



QCD

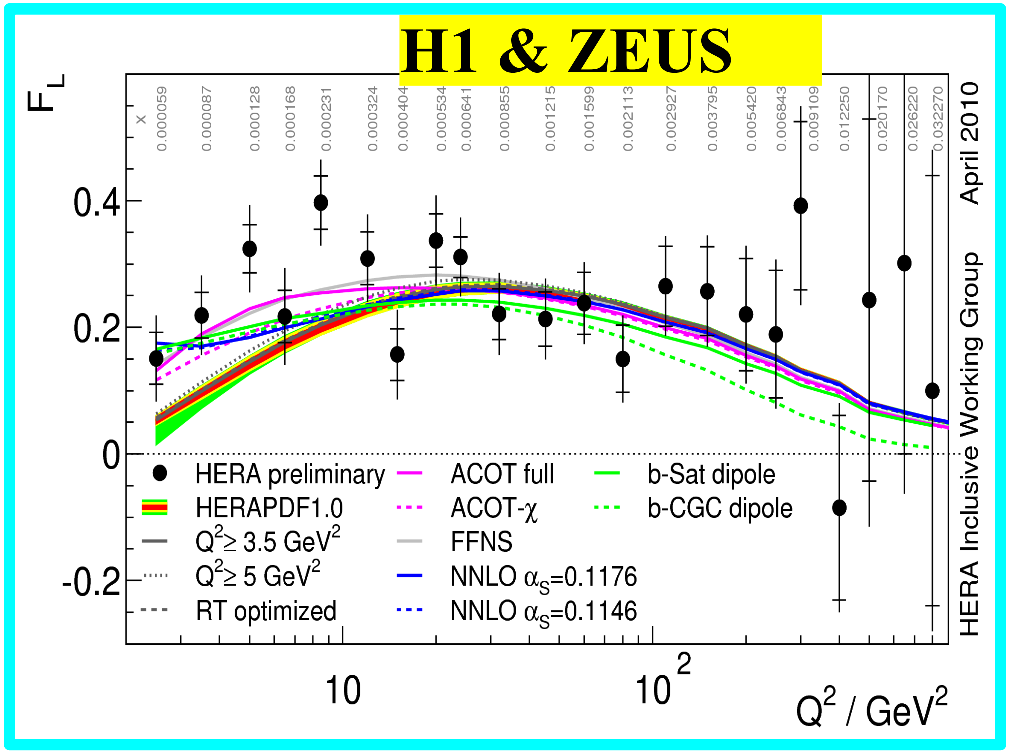
DISCONTINUITIES

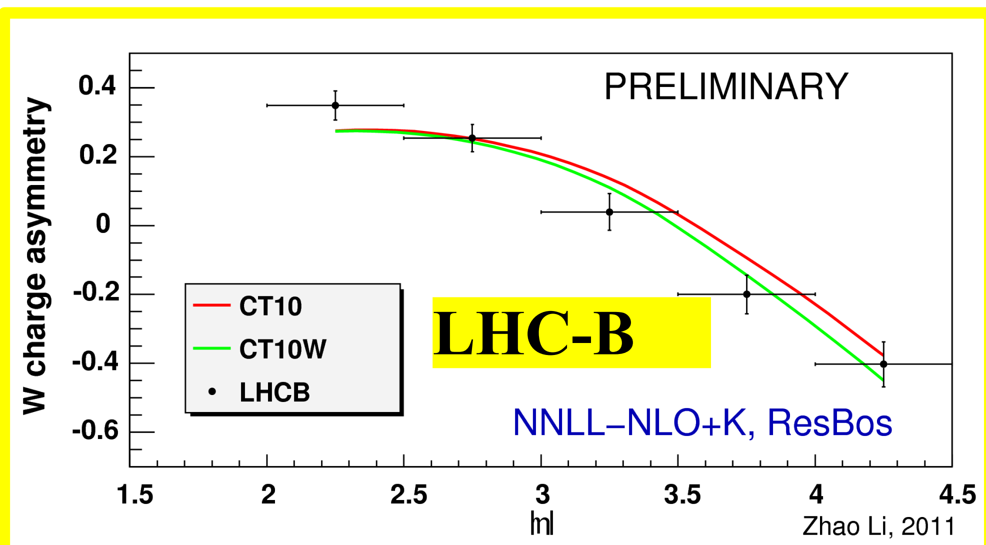
Precision Experimental Measurements

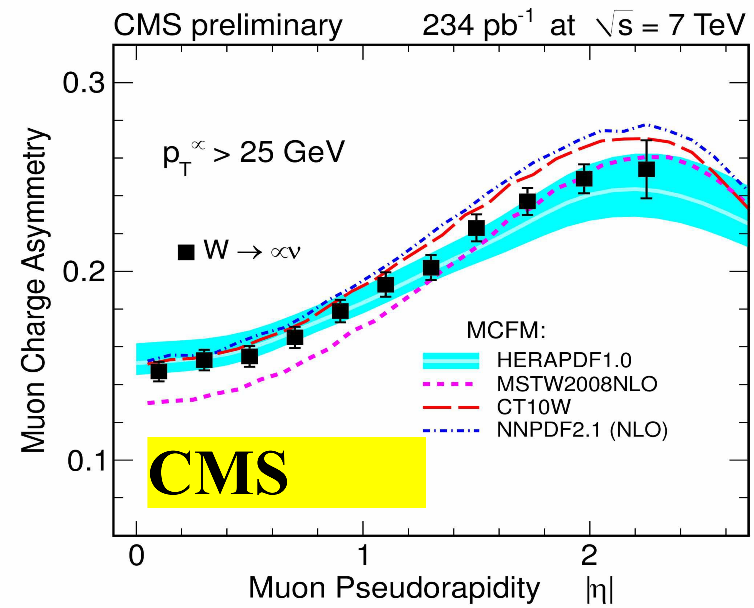
Require

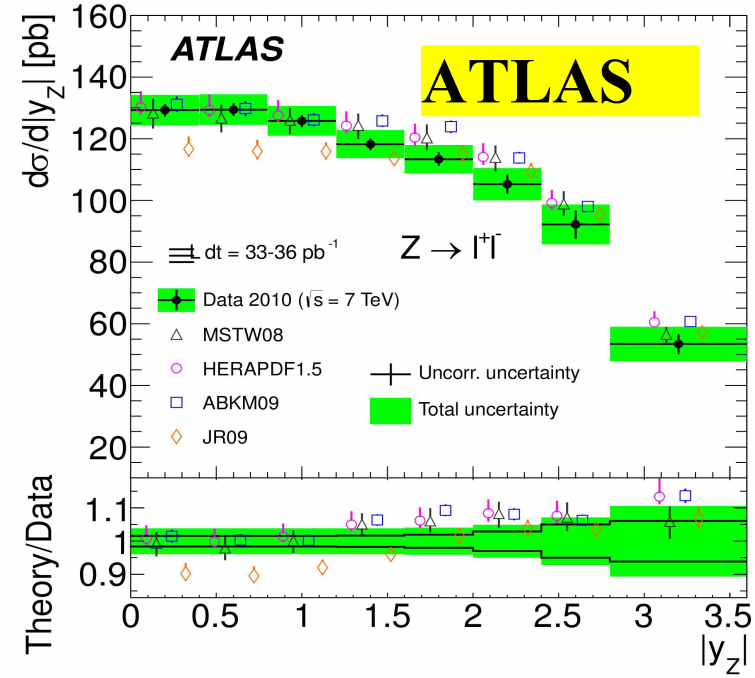
Precision Theoretical Predictions

Motivation: Recent Measurements Requires Theoretical Improvements



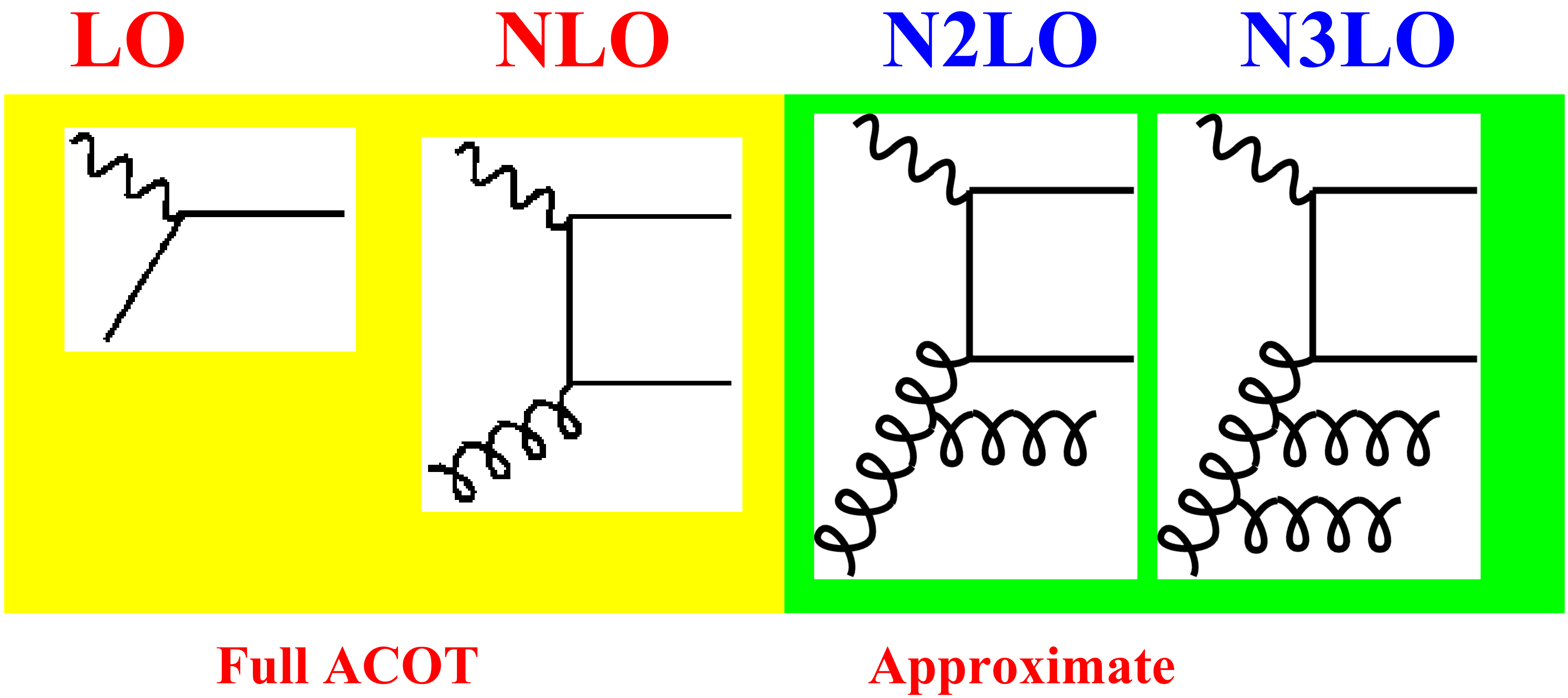






Jan Kretzschmar, 7.3.2012 – p.5

Heavy Quark Production: Extension to Higher Orders



Based on the Collins-Wilczek-Zee (CWZ) Renormalization Scheme
... hence, extensible to all orders

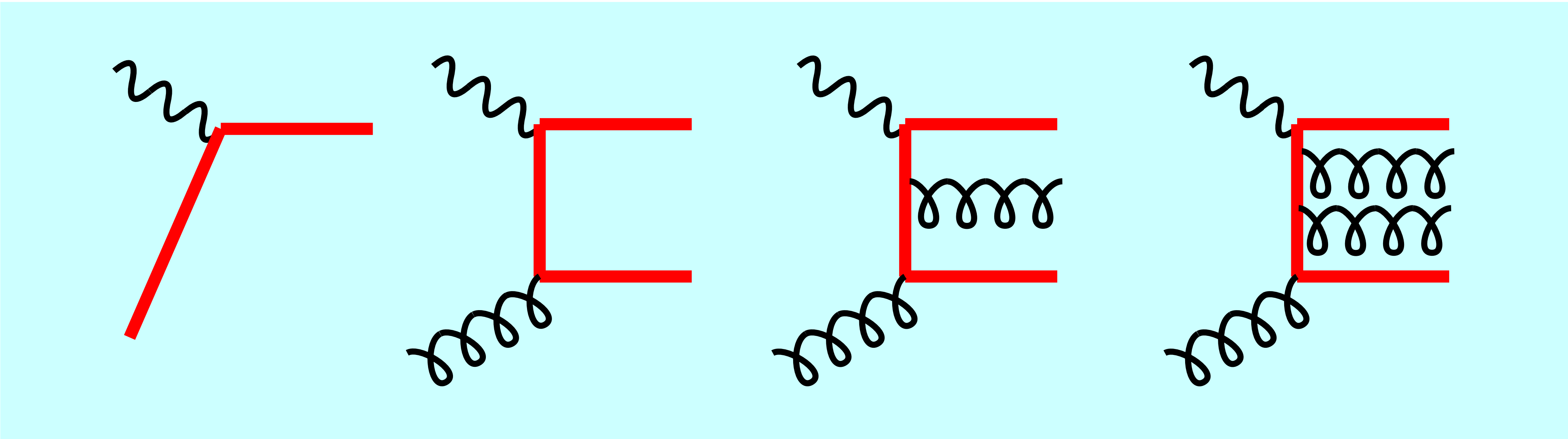
DGLAP kernels & PDF evolution are pure MS-Bar
Subtractions are MS-Bar

ACOT: $m \to 0$ limit yields MS-Bar
with no finite renormalization

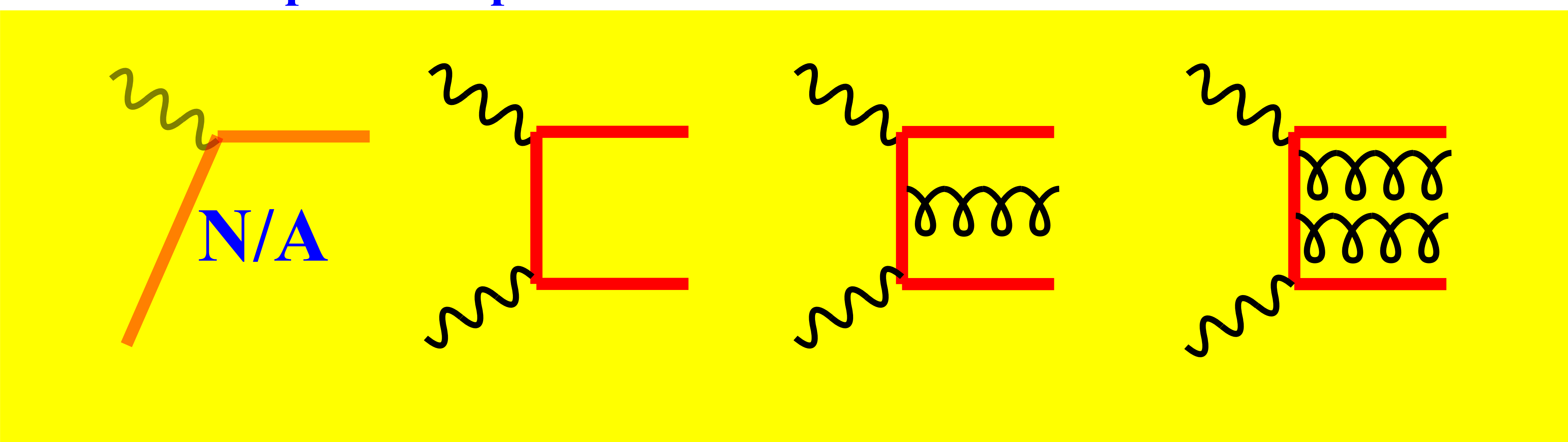
Stavreva, Olness, Schienbein, Jezo, Kusina, Kovarik, Yu
PhysRevD.85.114014

Heavy Quark Production: Extension to Higher Orders

Lepton—Hadron Collider



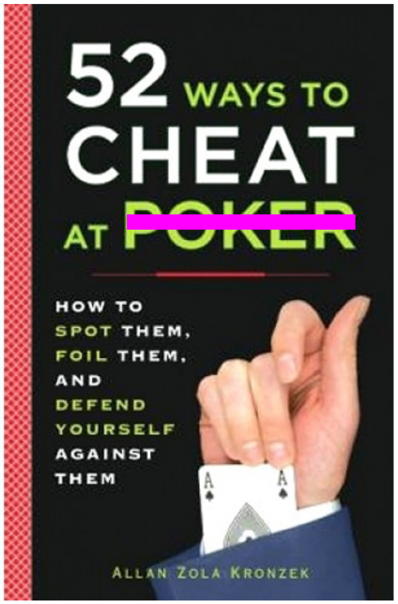
Lepton—Lepton Collider



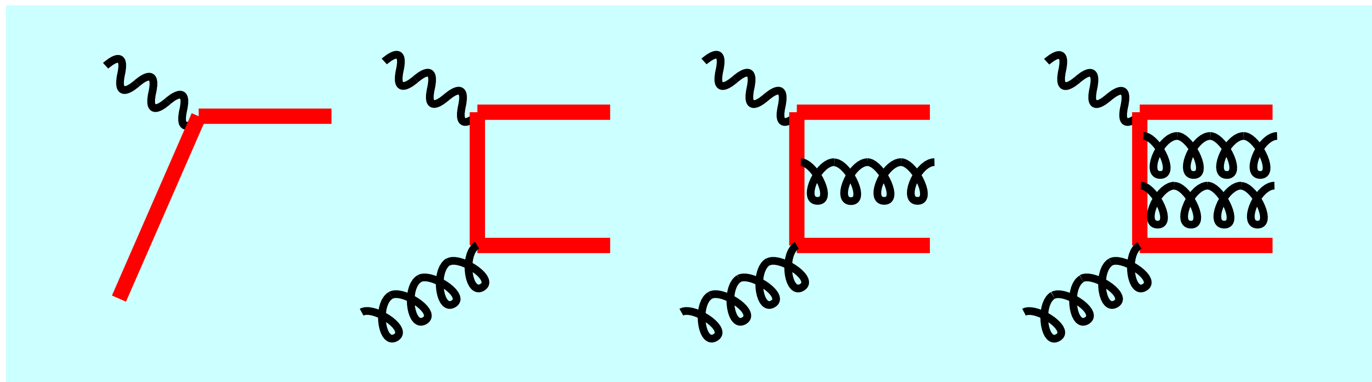
S-ACOT: *Can set initial state to m=0*

This is not an approximation; it is a choice of Renormalization Scheme

Can we "guess" the mass effects???

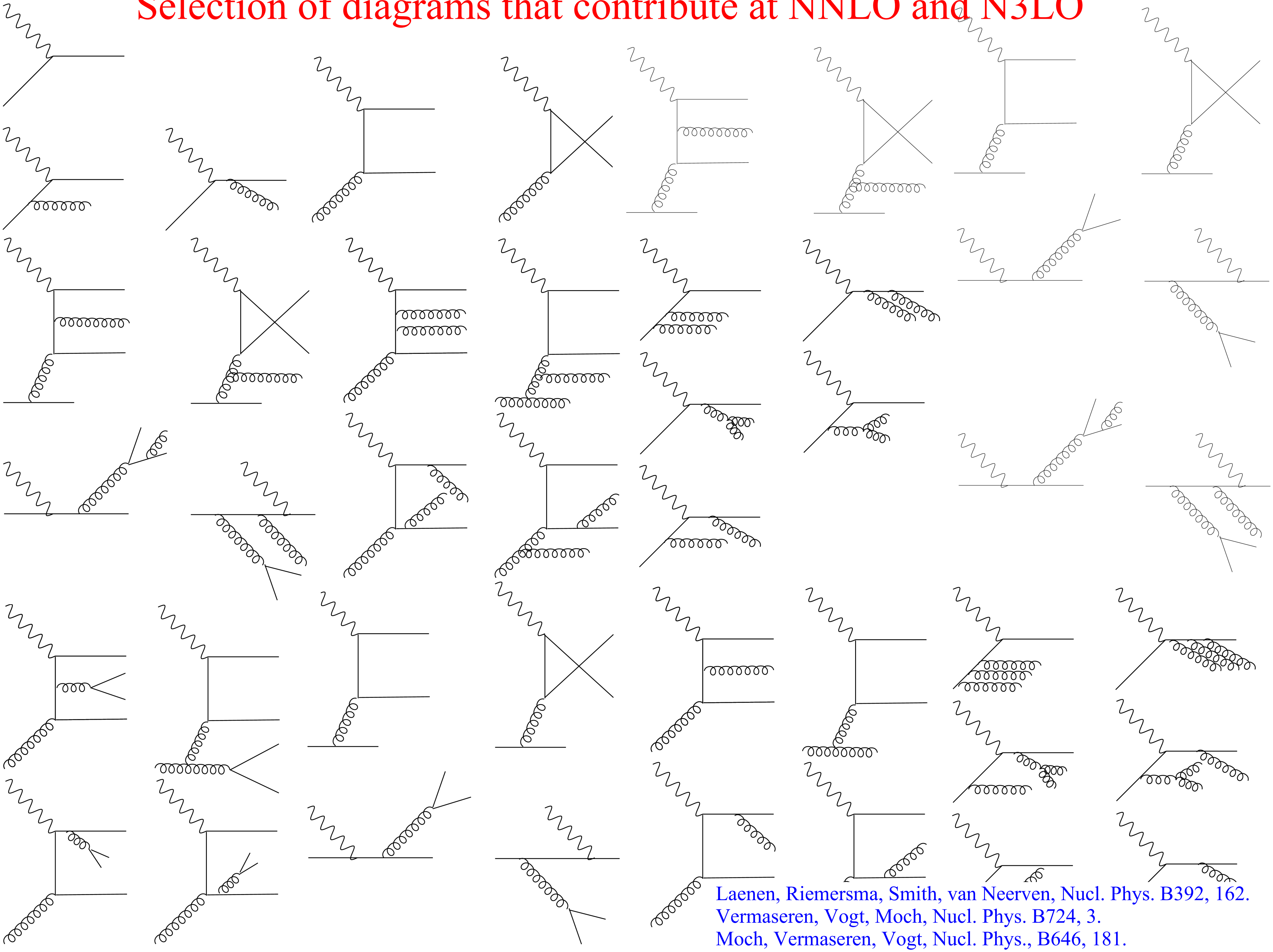


QCD



... or do we need to do the full calculation???

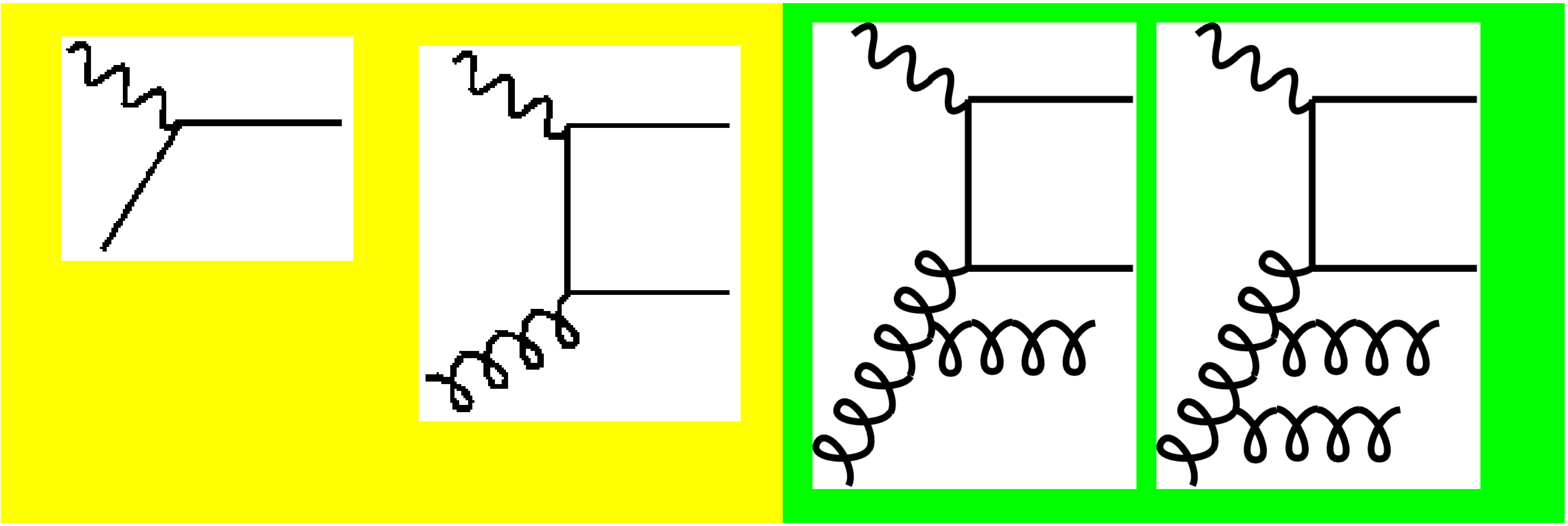
Selection of diagrams that contribute at NNLO and N3LO



Laenen, Riemersma, Smith, van Neerven, Nucl. Phys. B392, 162.
Vermaseren, Vogt, Moch, Nucl. Phys. B724, 3.
Moch, Vermaseren, Vogt, Nucl. Phys., B646, 181.
Moch, Vermaseren, Vogt, Phys. Lett., B606, 123.
Blumlein, Hasselhuhn, Kovacikova, Moch, Phys.Lett. B700, (2011) 294.

Heavy Quark Production: Extension to Higher Orders

LO **NLO** **N2LO** **N3LO**



Full ACOT

Extensible to any order

$$\sigma \sim \int |\mathcal{M}(m_{dyn})|^2 \, d\Gamma(m_{ps})$$

$$\text{factor} \sim \left(1 + \left[\frac{n\, m_{ps}}{Q}\right]^2\right)$$

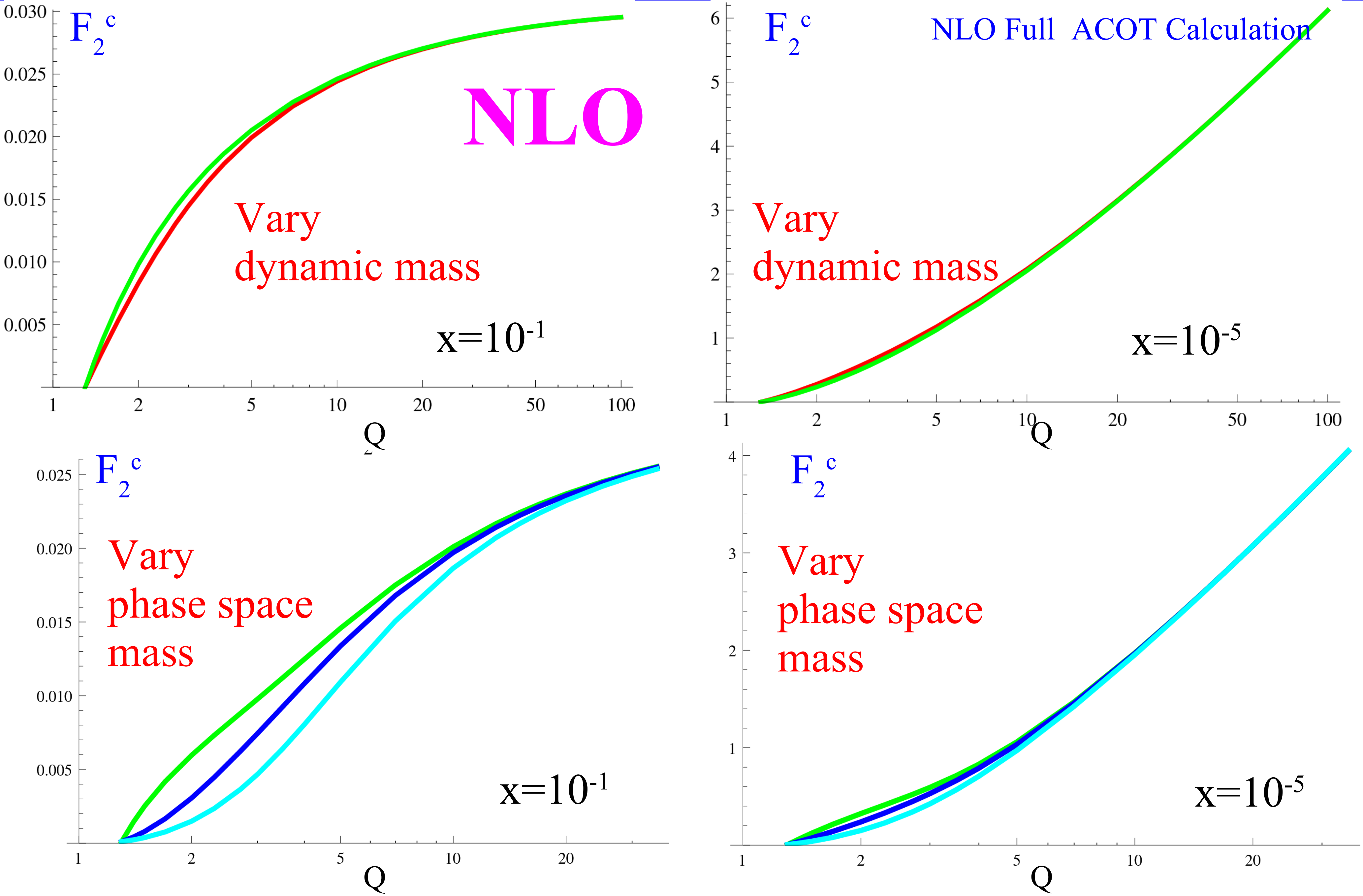
$$n = \{0, 1, 2\}$$

Distinguish:
"phase space" mass
"dynamic" mass

PLAN:

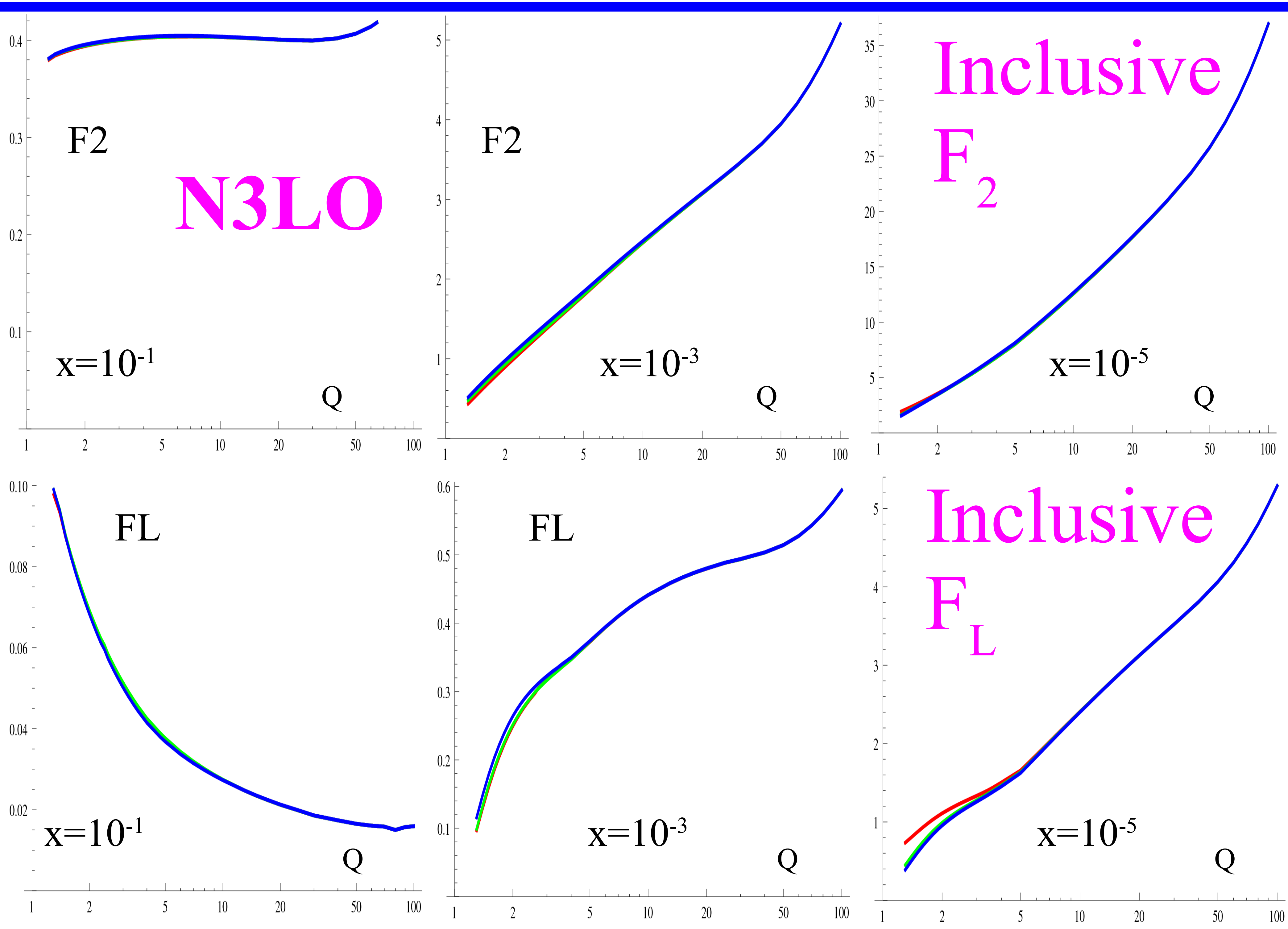
1) **Hope PS mass dominates**
2) **Neglect Dynamic mass**

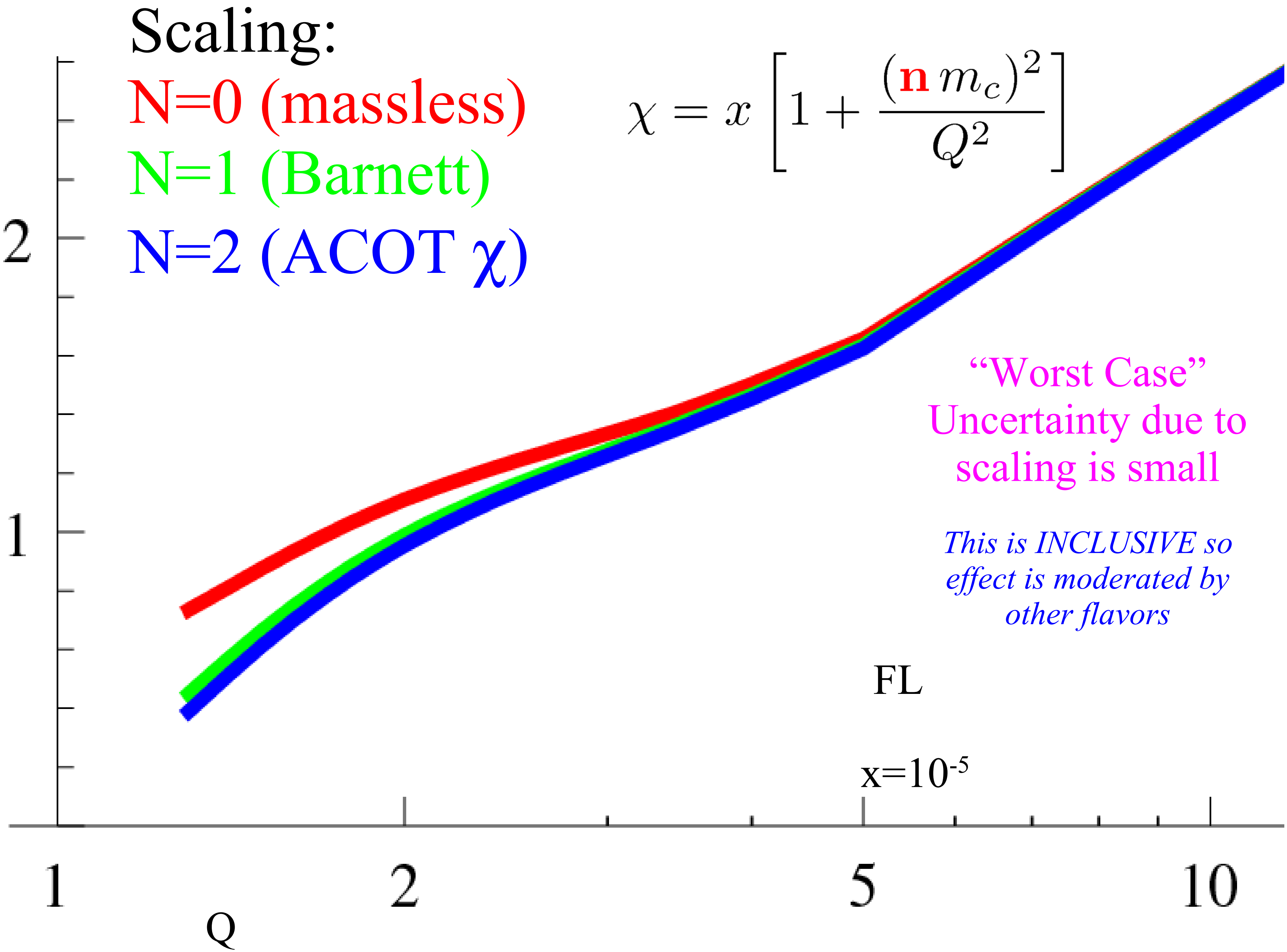
Identify Two Types of Mass Depencence: "dynamic" & "phase space"



"phase space" mass yields larger variation. Not a proof, but ….

EFFECT OF MASS SCALING @ N3LO (Phase Space Mass)



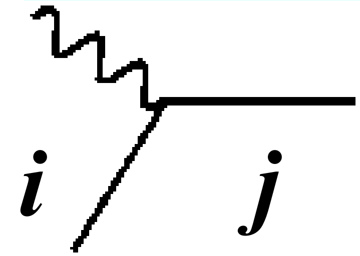

Scaling:
N=0 (massless)
N=1 (Barnett)
N=2 (ACOT χ)
$\chi = x\left[1 + \frac{(\mathbf{n}\, m_c)^2}{Q^2}\right]$
"Worst Case" Uncertainty due to scaling is small
This is INCLUSIVE so effect is moderated by other flavors
FL
x=10^-5
2
1
1
2
5
10
Q

Master formula for decomposing the flavor components

T.P. Stavreva, I Schienbein

$$F = \sum_{i,j}^{6} F^{ij}$$

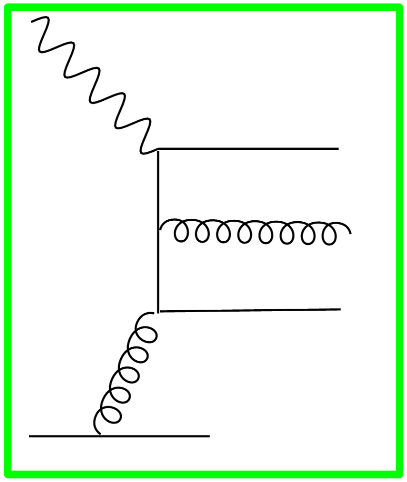
The Goal: Convert from {s, ns, ps} to {q,g, ...}



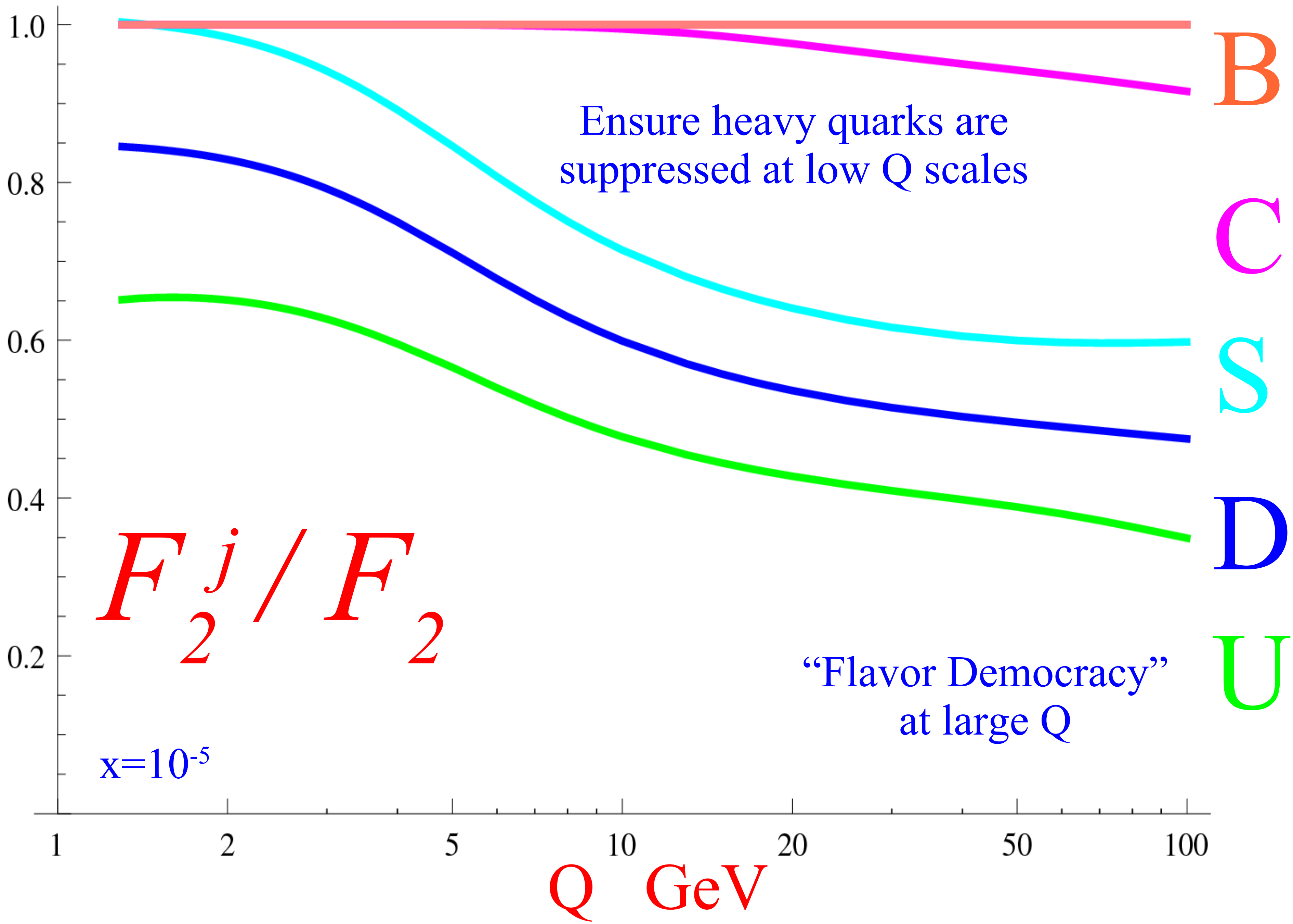
$$x^{-1} F_a^{ij} = q_i^+ \otimes \Big\{ e_i^2 \Big[C_{a,q}^{\rm ns}(n_f = 0)\ \delta_{ij} + C_{a,q}^{\rm ns}(j) - C_{a,q}^{\rm ns}(j-1) \Big] - \langle e^2 \rangle^{(j)}\ C_{a,q}^{\rm ps}(j) - \langle e^2 \rangle^{(j-1)}\ C_{a,q}^{\rm ps}(j-1) \Big\}$$

Issues: Flavor separation:
New diagrams at this order

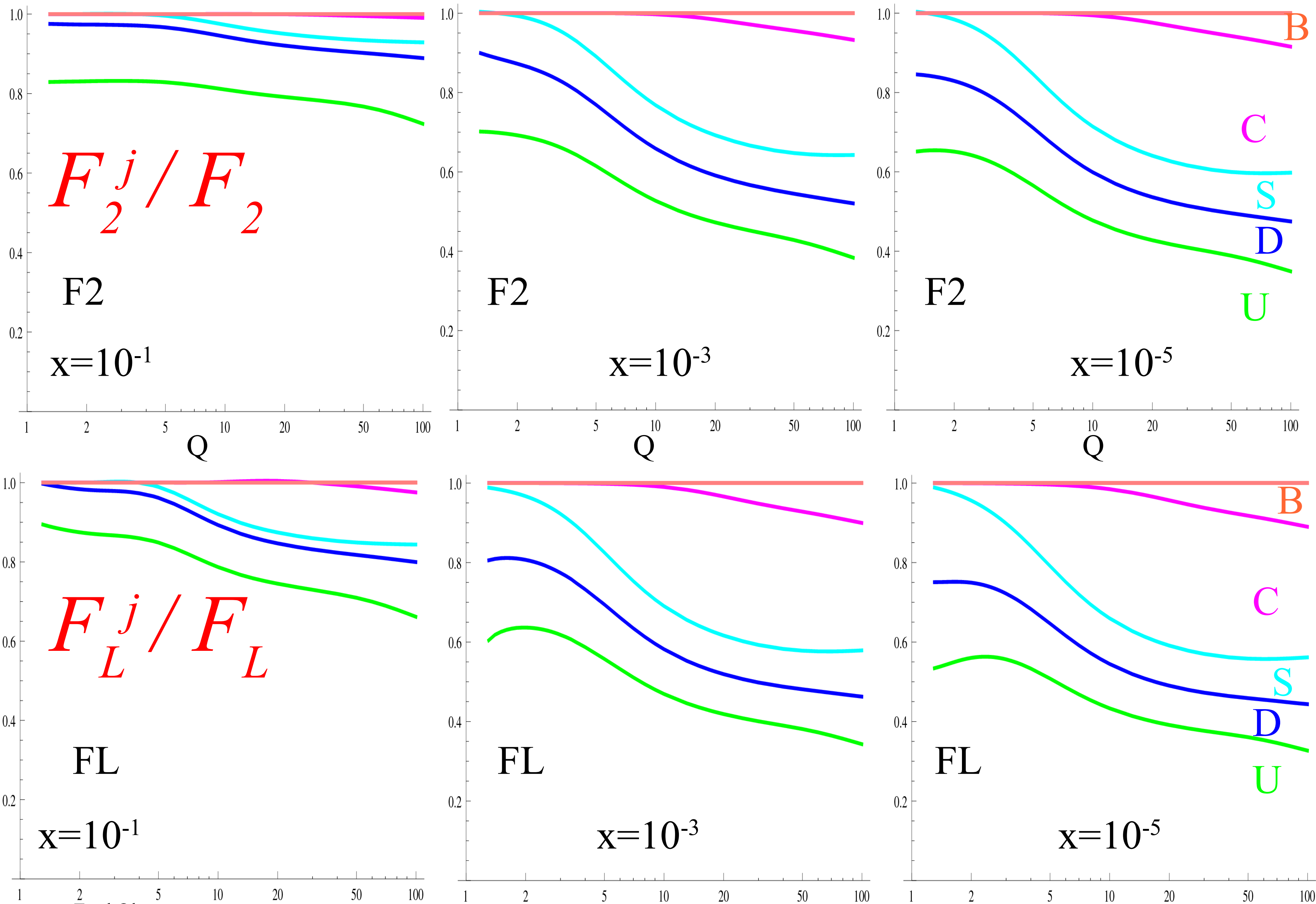
- c,b, goes down beam pipe
- both c & b in final state



FLAVOR DECOMPOSITION

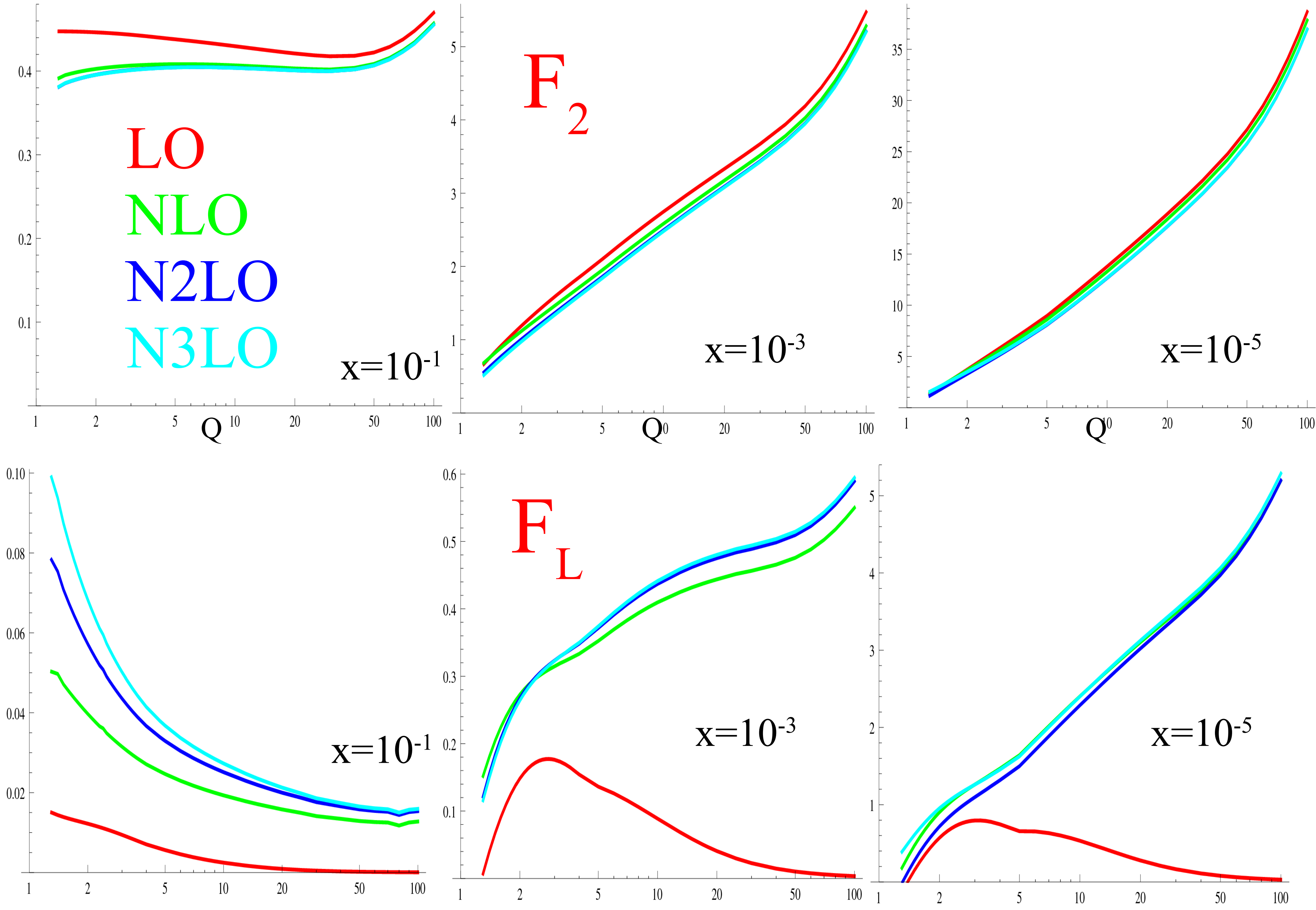


FLAVOR DECOMPOSITION: Final State Quark:



$F_{2,L}$ @ N3LO

$F_{2,L}$ @ N3LO



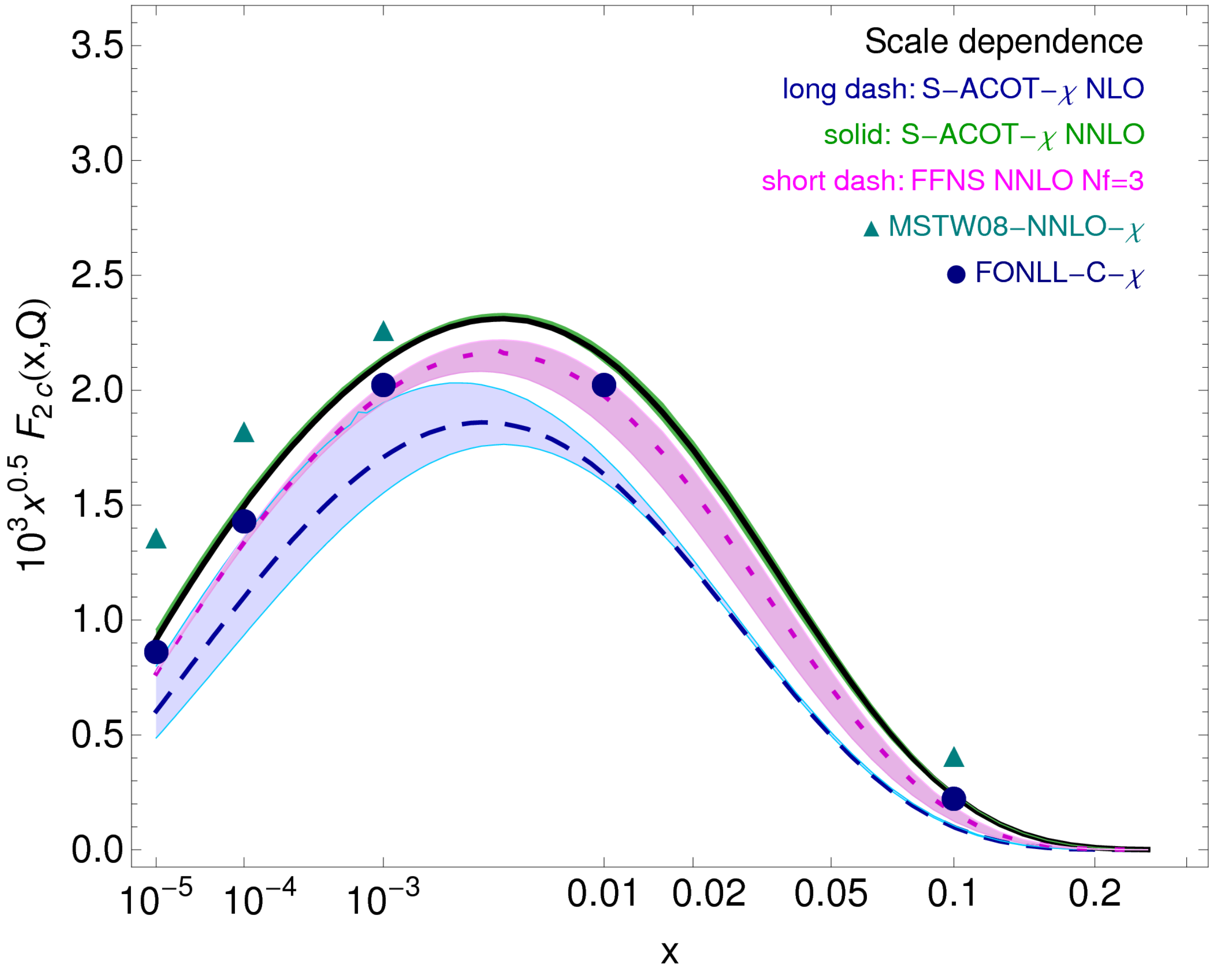
A Complementary Approach

S-ACOT χ at NNLO

led by Marco Guzzi and Pavel Nadolsky

Drastic μ_F-scale reduction in $F_2^c(x, Q^2)$ at NNLO

LH PDFs Q=2 GeV, m_c=1.41 GeV



S-ACOT χ

for Neutral Currents

based on Smith vanNeerven

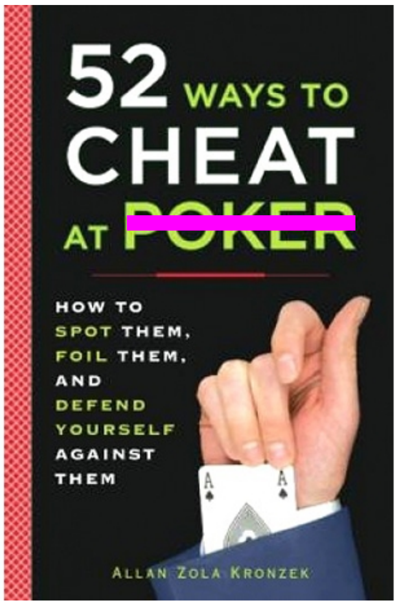
key step for NNLO PDFs

(see NNLO CT10 PDFs)

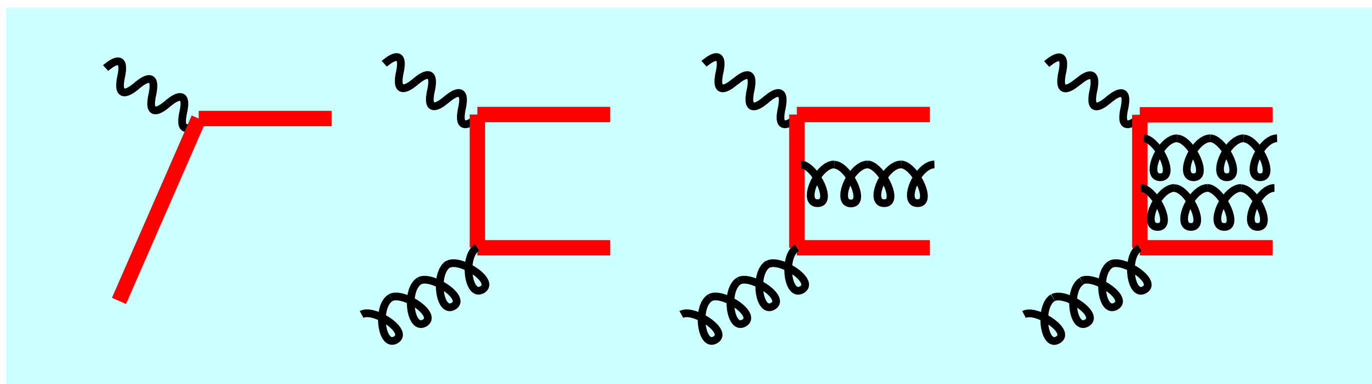
Comparison in progress by B. Clark

By using S-ACOT-χ we obtain a drastic reduction of the theoretical errors compared to the NLO computation.

Can we "guess" the mass effects???



QCD



DISCONTINUTIES

α_s is Discontinuous At Higher-Orders

The β-function coefficients, the b_i, are given for the coupling of an *effective theory* in which n_f of the quark flavors are considered light ($m_q \ll \mu_R$), and in which the remaining heavier quark flavors decouple from the theory. One may relate the coupling for the theory with $n_f + 1$ light flavors to that with n_f flavors through an equation of the form

$$\alpha_s^{(n_f+1)}(\mu_R^2) = \alpha_s^{(n_f)}(\mu_R^2) \left(1 + \sum_{n=1}^{\infty} \sum_{\ell=0}^{n} c_{n\ell} \, [\alpha_s^{(n_f)}(\mu_R^2)]^n \ln^\ell \frac{\mu_R^2}{m_h^2} \right) , \qquad (9.4)$$

where m_h is the mass of the $(n_f+1)^{\text{th}}$ flavor, and the first few $c_{n\ell}$ coefficients are $c_{11} = \frac{1}{6\pi}$, $c_{10} = 0$, $c_{22} = c_{11}^2$, $c_{21} = \frac{19}{24\pi^2}$, and $c_{20} = -\frac{11}{72\pi^2}$ when m_h is the $\overline{\text{MS}}$ mass at scale m_h ($c_{20} = \frac{7}{24\pi^2}$ when m_h is the pole mass — mass definitions are discussed below and in the review on "Quark Masses"). Terms up to $c_{4\ell}$ are to be found in Refs. 11, 12. Numerically, when one chooses $\mu_R = m_h$, the matching is a modest effect, owing to the zero value for the c_{10} coefficient. Relations between n_f and (n_f+2) flavors where the two heavy flavors are close in mass are given to three loops in Ref. 13.

$$\alpha_S^{N_F+1}(m_q) \sim \alpha_S^{N_F}(m_q) \left(1 + c \left[\alpha_S^{N_F}(m_q) \right]^2 \right)$$

These discontinuities are real, and they persist at all orders!!!

Issues at Higher Orders: Discontinuities

$$\alpha_S^{N_F+1}(m_q) \sim \alpha_S^{N_F}(m_q) \left(1 + c\left[\alpha_S^{N_F}(m_q)\right]^2\right)$$

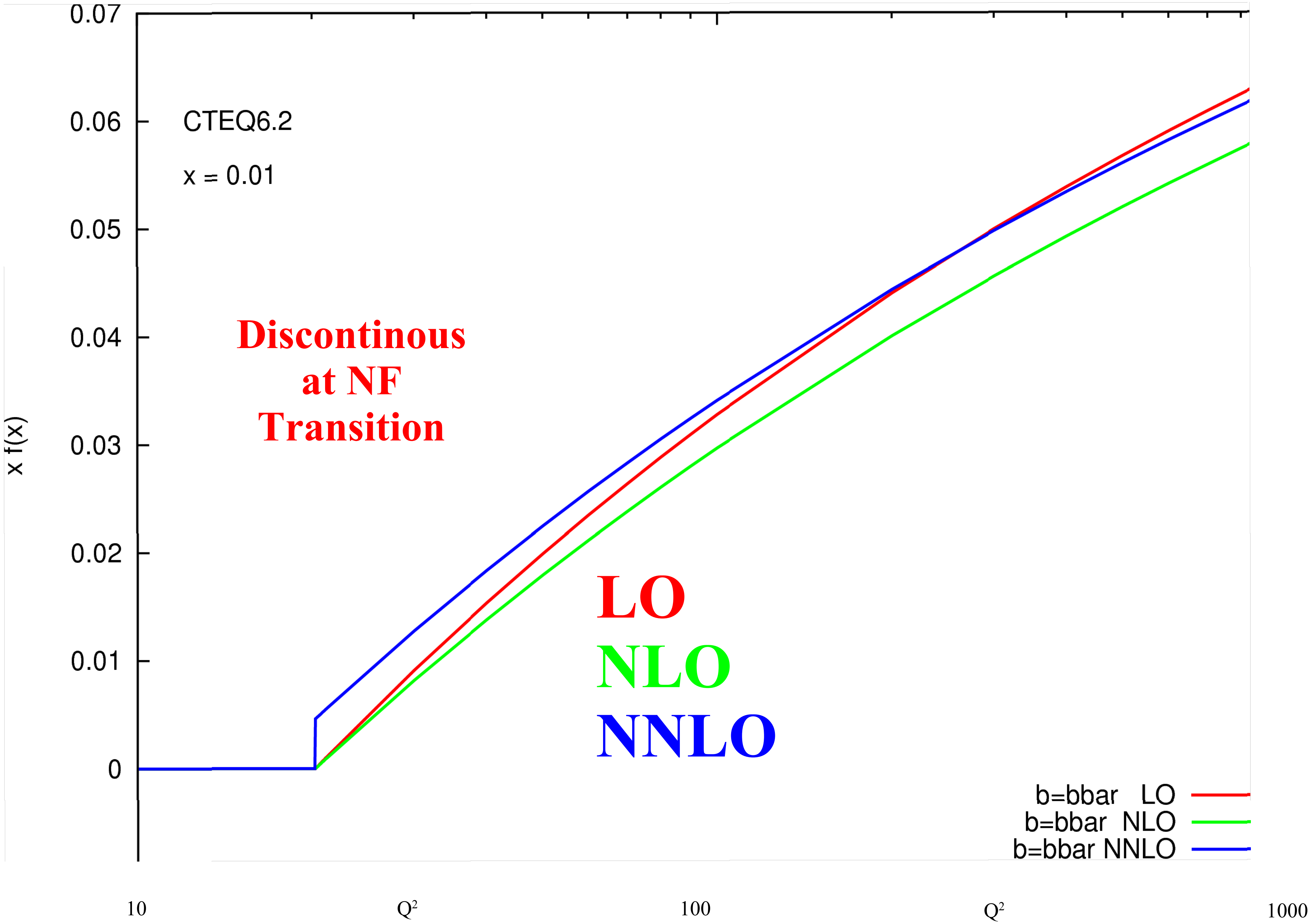
Note: These discontinuities do not go away; they persist at all higher orders

BUT, Physical quantities ($d\sigma$, F_i) will be continuous to the order of the perturbation theory.

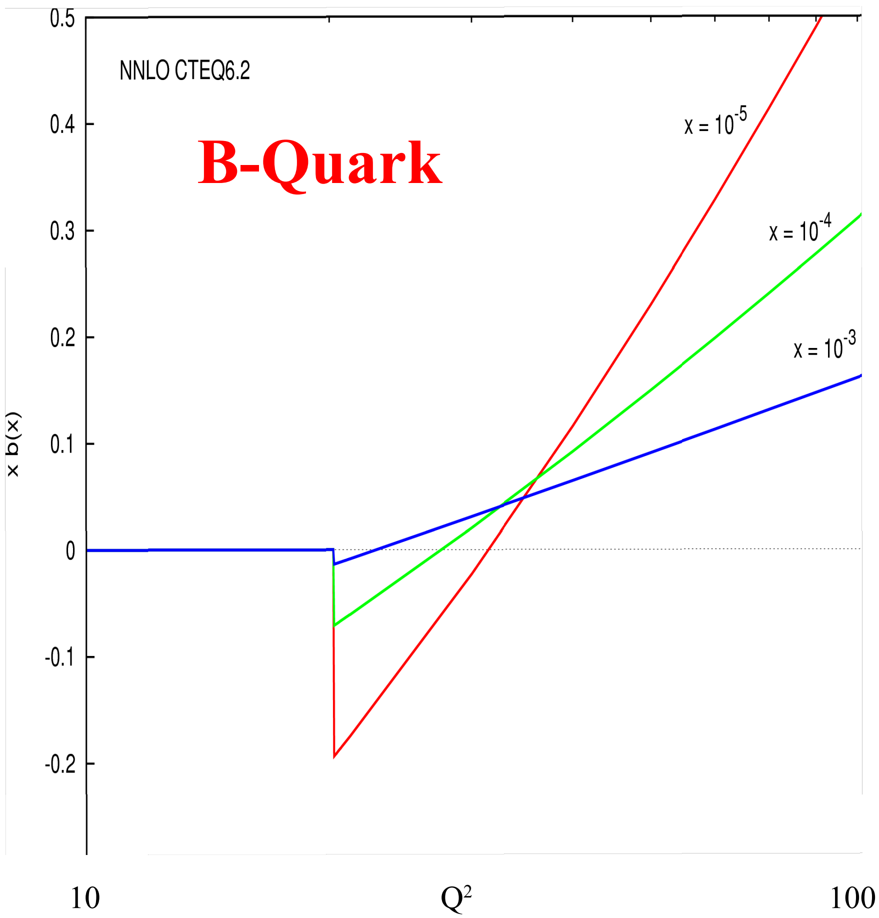
$$\sigma^{n_F=N}_{\mathcal{O}(\alpha_S^{137})} = \sigma^{n_F=N+1}_{\mathcal{O}(\alpha_S^{137})} + \mathcal{O}(\alpha_S^{138})$$

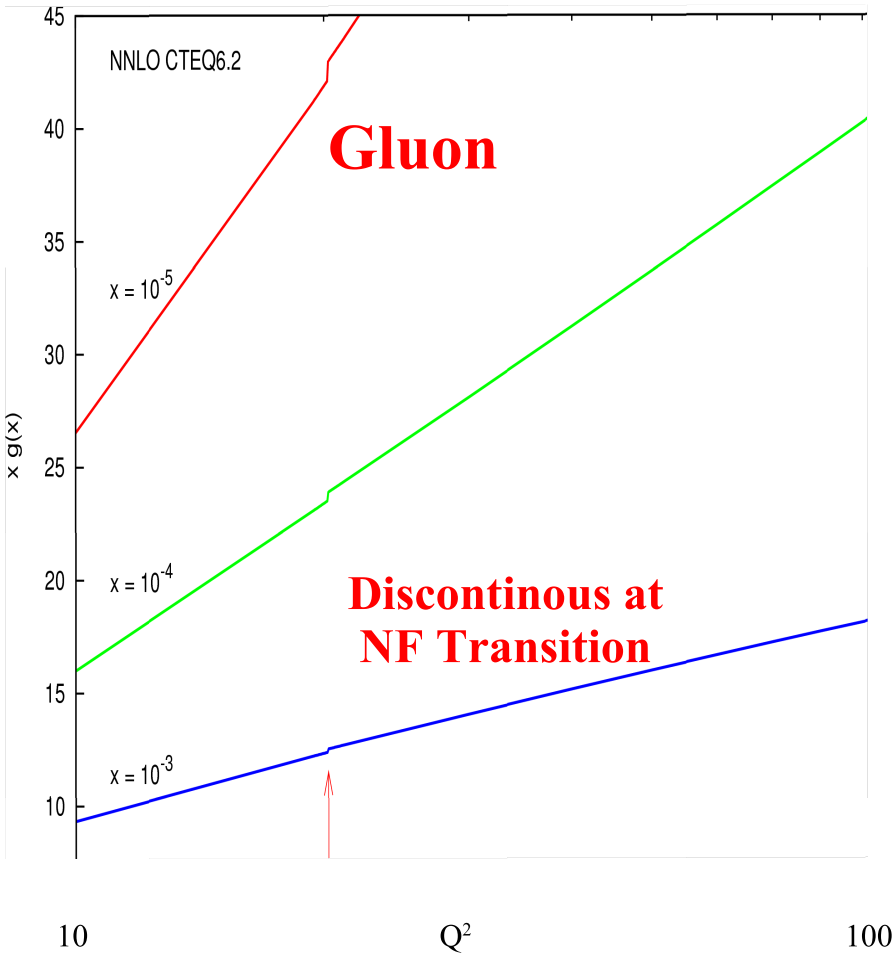
QCD Really Works!!!

B-Quark PDF vs. Q^2



Discontinuity of the PDFs At Higher-Orders

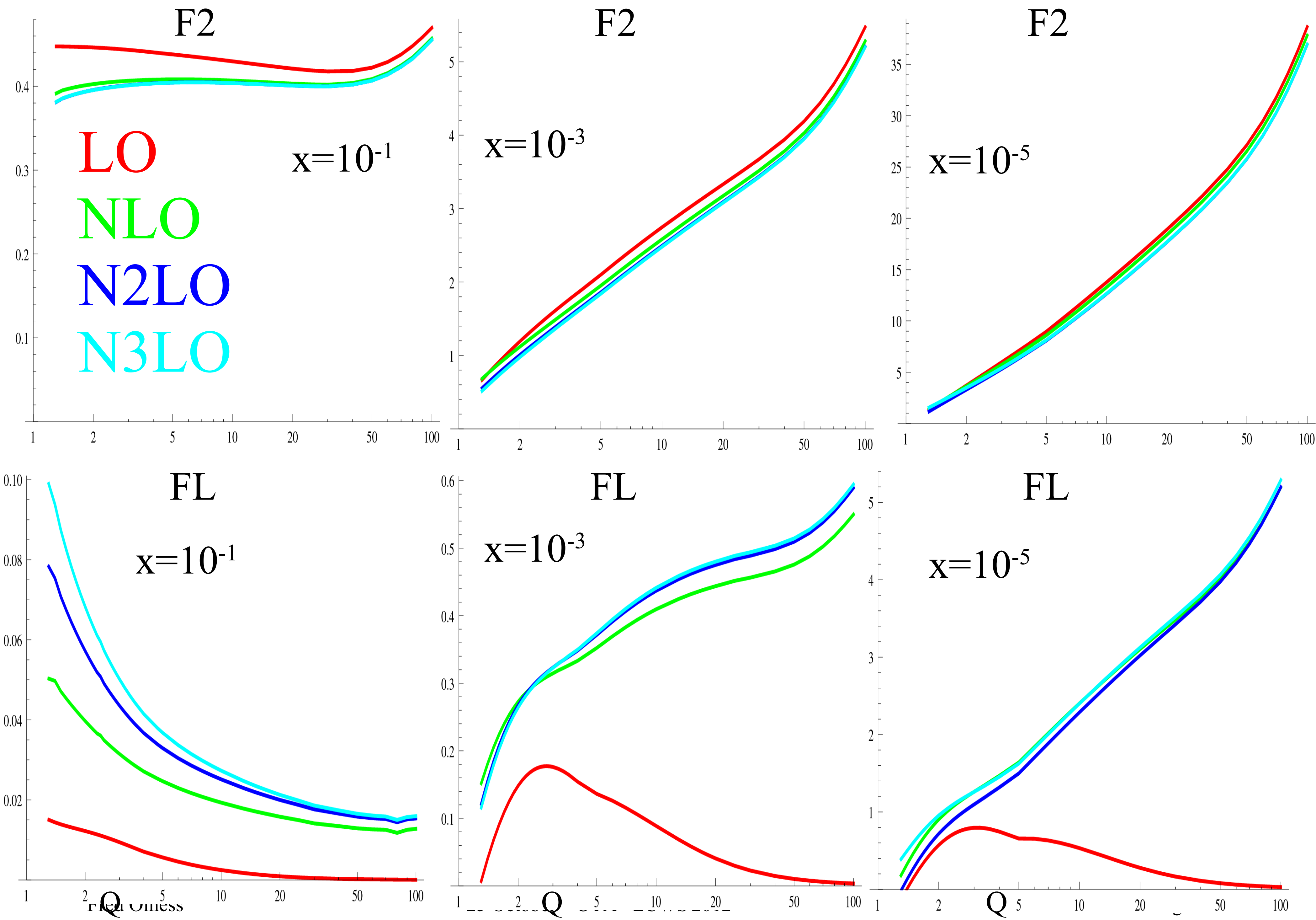




Note, discontinuity is in opposite directions; consequence of the sum rule

$F_{2,L}$ @ N3LO

F_L & F_2 @ N3LO

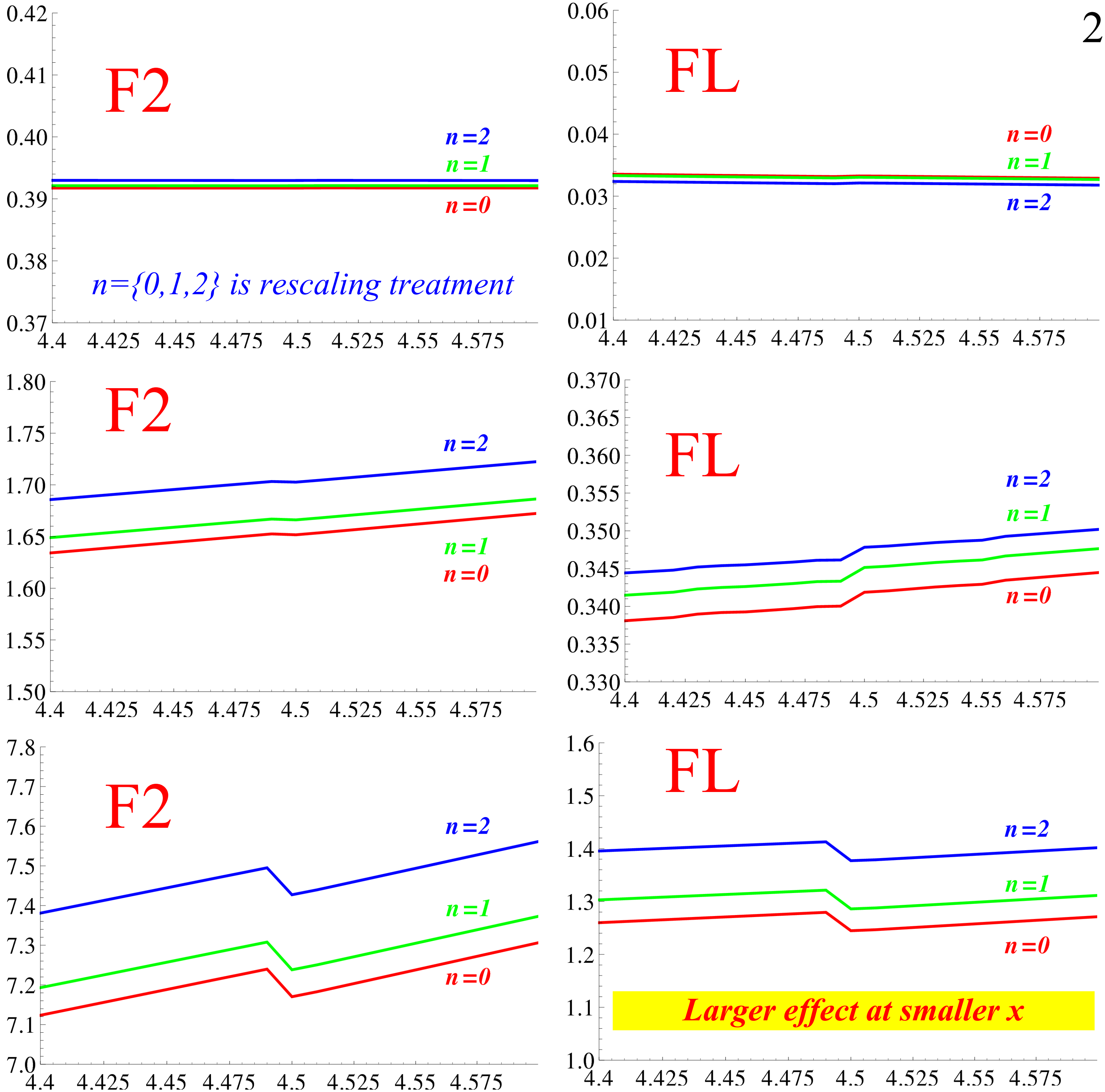

F2
F2
F2
LO
NLO
N2LO
N3LO
x=10-1
x=10-3
x=10-5
FL
FL
FL
x=10-1
x=10-3
x=10-5
Q
Q
Q

Enlarge Q Scale

$x=10^{-1}$

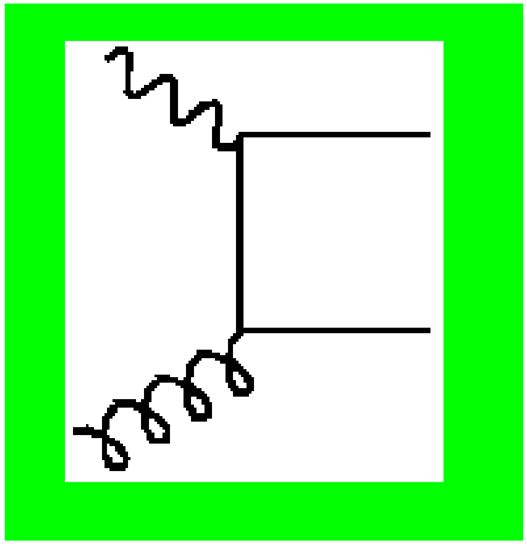
$x=10^{-3}$

$x=10^{-5}$

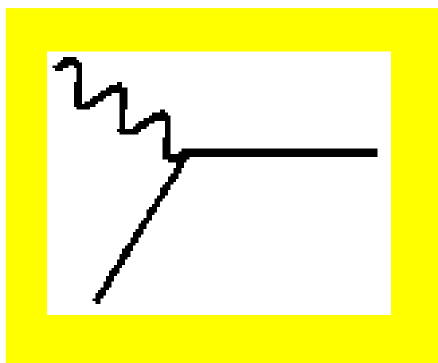


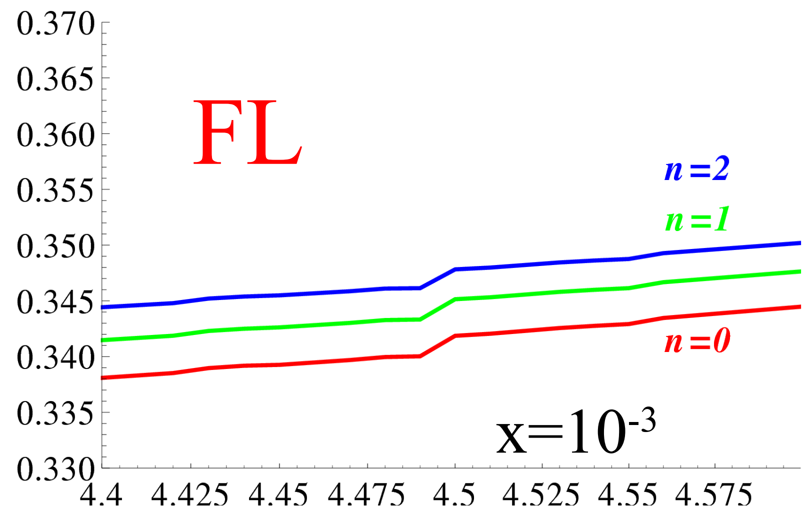
Fred Olness

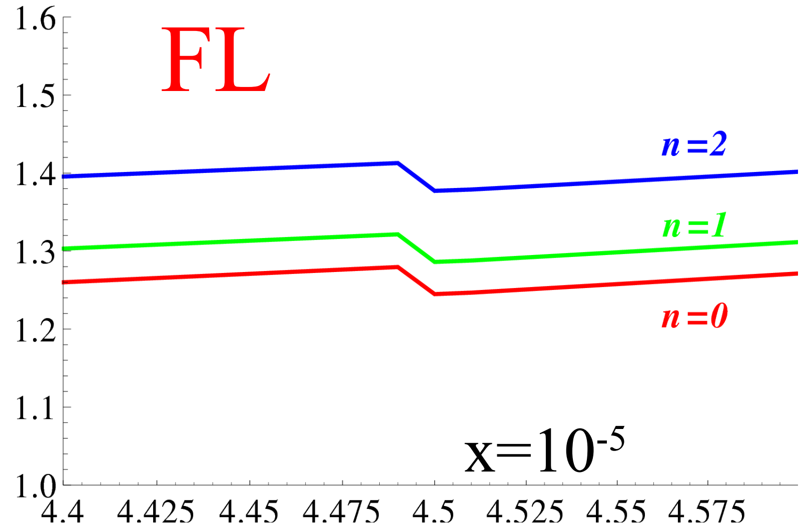
QCD Compensates!!!



Quark & Gluon have opposite discontinuities







$$\sigma_{TOT}^{N_F+1} = \sigma_{TOT}^{N_F} + \mathcal{O}(\alpha_S^{m+1})$$

… this is really cool!!!

QCD Compensates: *the details*

$$f_b^5 = \left\{0 + \frac{\alpha_s}{2\pi} P_{qg}\,(L + a_{qg}) + O(\alpha_s^2)\right\} \otimes f_g^4$$

$$f_g^5 = \left\{1 + \frac{\alpha_s}{2\pi} P_{gg}\,(L + a_{gg}) + O(\alpha_s^2)\right\} \otimes f_g^4$$

$$L = \ln(\mu^2/m_b^2)$$

$$\sigma_{LO} = C^0 \otimes f_b^5 \simeq C^0 \otimes \left\{0 + \frac{\alpha_s}{2\pi} P_{qg}\,(L + a_{qg})\right\} \otimes f_g^4$$

$$\sigma_{NLO} = C^1 \otimes f_g^5 \simeq C^1 \otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg}\,(L + a_{gg})\right\} \otimes f_g^4$$

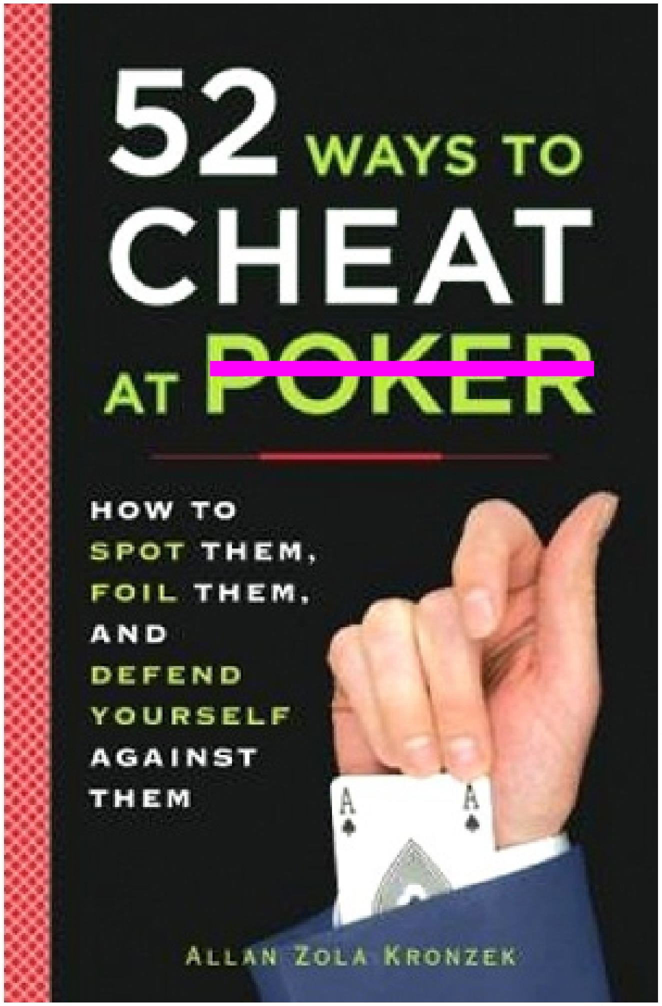
$$\sigma_{SUB} = C^0 \otimes \widetilde{f}_{g\to q} \otimes f_g^5 \simeq C^0 \otimes \left\{\frac{\alpha_s}{2\pi} P_{qg}\,(L + a_{qg})\right\} \otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg}\,(L + a_{gg})\right\} \otimes f_g^4$$

$$\sigma_{TOT}^{N_F=5} = \sigma_{LO} + \sigma_{NLO} - \sigma_{SUB} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

$$\sigma_{TOT}^{N_F=4} = \sigma_{NLO} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

$$\sigma_{TOT}^{N_F=5} = \sigma_{TOT}^{N_F=4} + O(\alpha_s^2)$$

Conclusion



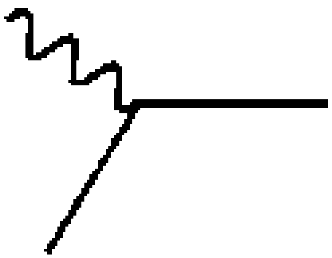
QCD

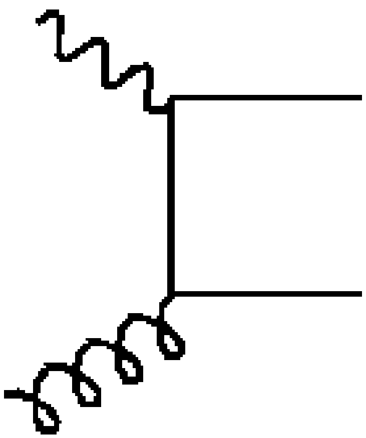
DISCONTINUITIES

Leftover

SCALING RELATIONS:

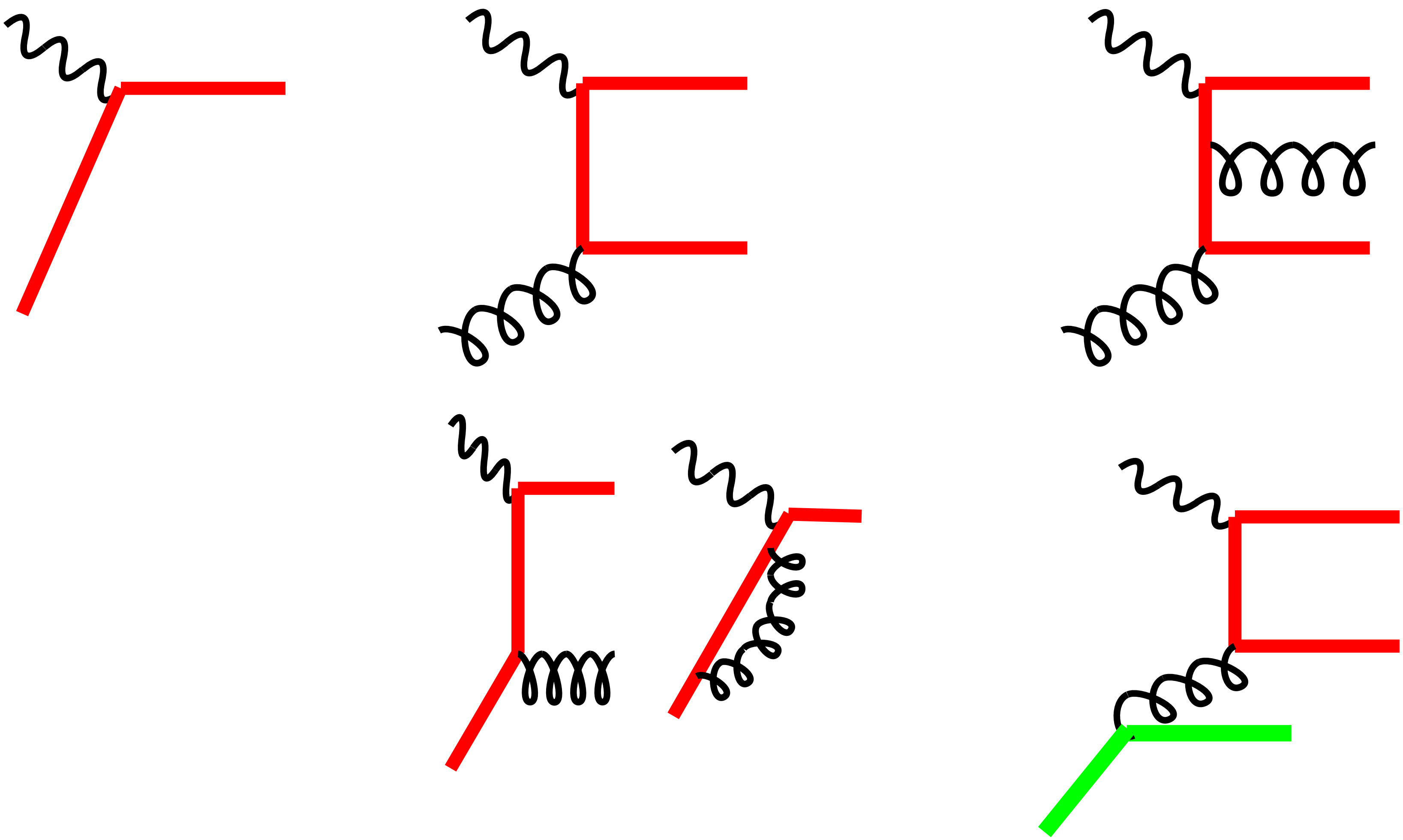
ξ	General	$m_1 = 0$	$m_1 = m_2 = m$	χ-scheme:
$2 \to 1$	$\eta \left[\frac{Q^2 - m_1^2 + m_2^2 + \Delta[-Q^2, m_1^2, m_2^2]}{2Q^2}\right]$	$\eta \left[1 + \frac{m_2^2}{Q^2}\right]$	$\eta \left[1 + \frac{m^2}{Q^2}\right]$	$\eta \left[1 + \frac{(2m)^2}{Q^2}\right]$
$2 \to 2$	$\eta \left[1 + \left(\frac{m_1 + m_2}{Q}\right)^2\right]$	$\eta \left[1 + \frac{m_2^2}{Q^2}\right]$	$\eta \left[1 + \frac{(2m)^2}{Q^2}\right]$	$\eta \left[1 + \frac{(2m)^2}{Q^2}\right]$





$$\chi = x \left[1 + \frac{(\mathbf{n}\, m_c)^2}{Q^2}\right]$$

LO **NLO** **NNLO**



$$F_{123} \sim q(x) \otimes C_q^0 + \alpha_S \left\{ g(x) \otimes C_g^1 + q(x) \otimes C_q^1 \right\}$$

Issues at Higher Orders: Discontinuities

OUTLINE:

Machinery to deal with this is already built into HERA-Fitter & QCDNUM

- PDFs Discontinuous at N2LO
- α_s Discontinuous at α_s^3

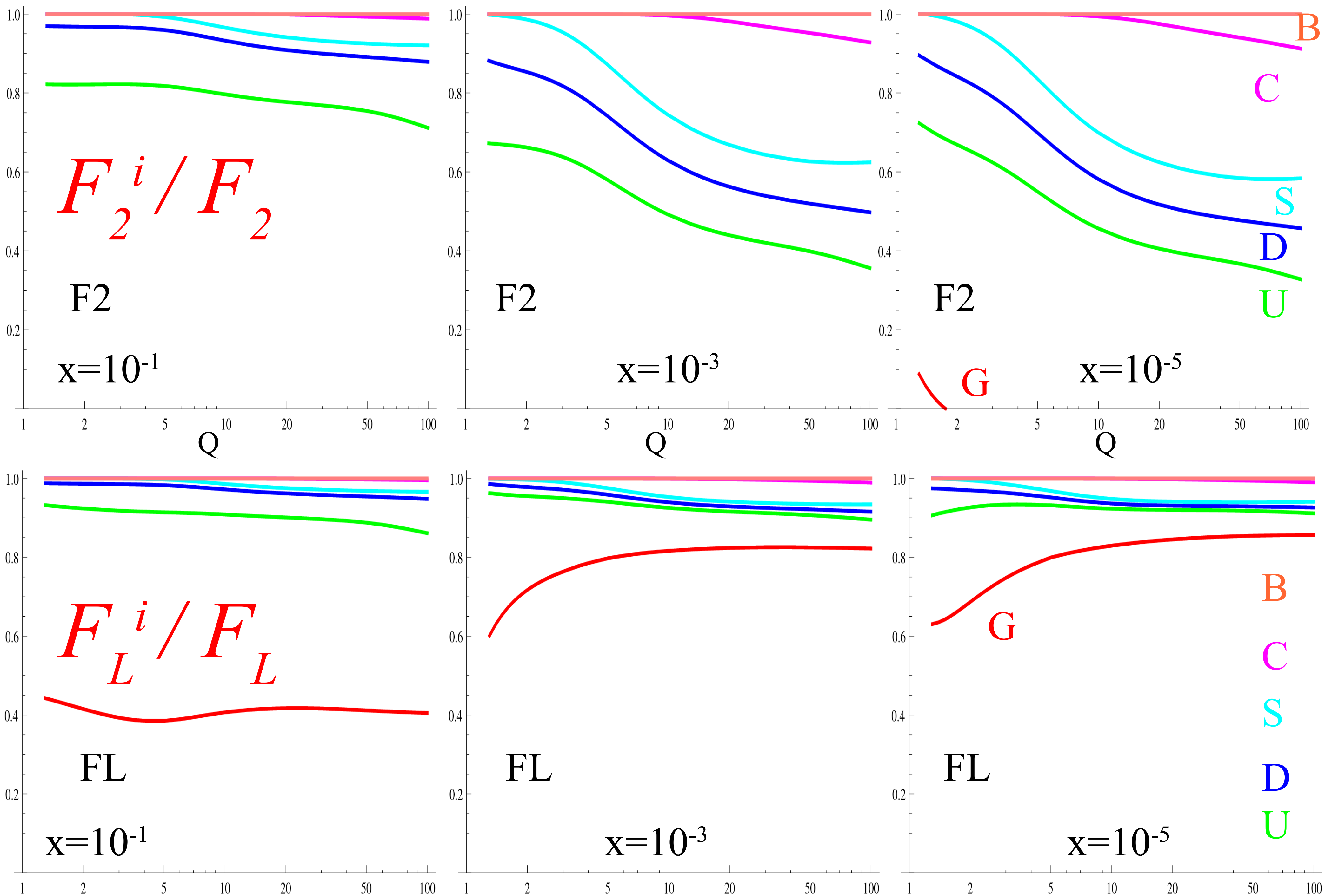
Note: These discontinuities do not go away; they persist at all higher orders

BUT, Physical quantities (dσ, F_i) will be continuous to the order of the perturbation theory.

$$\sigma^{N_F=N}_{\mathcal{O}(\alpha_S^{137})} = \sigma^{N_F=N+1}_{\mathcal{O}(\alpha_S^{137})} + \mathcal{O}(\alpha_S^{138})$$

QCD Really Works!!!

FLAVOR DECOMPOSITION: Initial State Quark:



ACOT at NNLO and N3LO

This technique provides an NNLO & N3LO extension of ACOT

"Phase space" mass is included via rescaling
Dominant effect for LO & NLO

F2: Stable.
LO and NLO have full m-dependence
N2LO and N3LO very similar

FL: More complex as NLO corrections are large (Callan-Gross)
N2LO and N3LO terms converge

Heavy quark terms vanish for low Q;
this moderates mass effects

Thanks to: K. Kovarik, A. Kusina, T.P. Stavreva I Schienbein, J.-Y. Yu,
P. Nadolsky, M. Guzzi, J. Owens, J. Morfin, C. Keppel, D. Soper …
& the HERA-PDF Working Group