New method of including QCD NLO corrections to hard process in Monte Carlo shower

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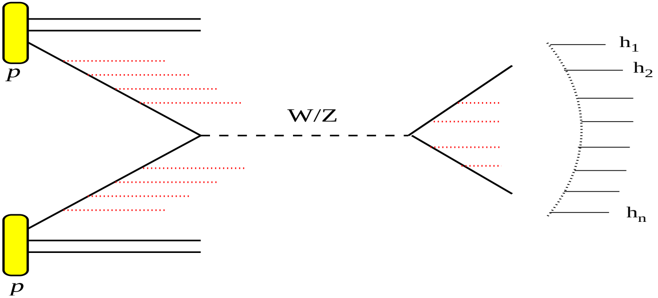
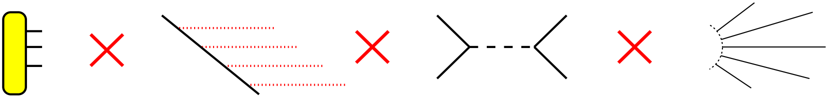
Outline

1. Introduction
2. Our project
   - Rebuilding LO MC shower
   - Including NLO corrections to hard process
   - Relation to other methods
   - Next step – fully NLO shower
3. Summary & outlook

LHC "event"





- *PDF* in proton
- fits to data at low scale + NLO (NNLO) evolution

- parton shower
- evolution eq.
- Monte Carlo (LO)

- hard process
- fixed order
- Monte Carlo (NLO)

- hadronization
- modelling

Factorization vs. Monte Carlo

Every Parton Shower Monte Carlo (MC) is based on the concept of *factorization*. For pp collisions it can be written as:

$$\sigma = \sum_{ij} \int dx_1 dx_2 \, f_i(x_1, \mu) f_j(x_2, \mu) \, \hat{\sigma}_i(p_1, p_2, \alpha_S(\mu), Q^2/\mu^2)$$

In MC we want to generate exclusive distributions (full 4-momenta dependence)

- Above formula is inclusive!
- It violates 4-momentum conservation ($k_\perp$ component);
- Features huge oversubtractions (time ordered exponential is constructed by geometrical series).

Factorization vs. Monte Carlo

Every Parton Shower Monte Carlo (MC) is based on the concept of *factorization*. For *pp* collisions it can be written as:

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In MC we want to generate exclusive distributions (full 4-momenta dependence)

- These problems have been solved *only* in LO approximation;
- Big progress towards NLO (POWHEG, MC@NLO);
- Still no solution for full NLO in the shower.

Aim

Ultimate aim

Construct MC with **NLO** parton shower and **NLO/NNLO** hard process

Steps on the way

- reformulate collinear factorization to exclusive (unintegrated) form;
- construct new LO shower;
- include NLO corrections to hard process;
- include NLO corrections to the shower itself (ladders);

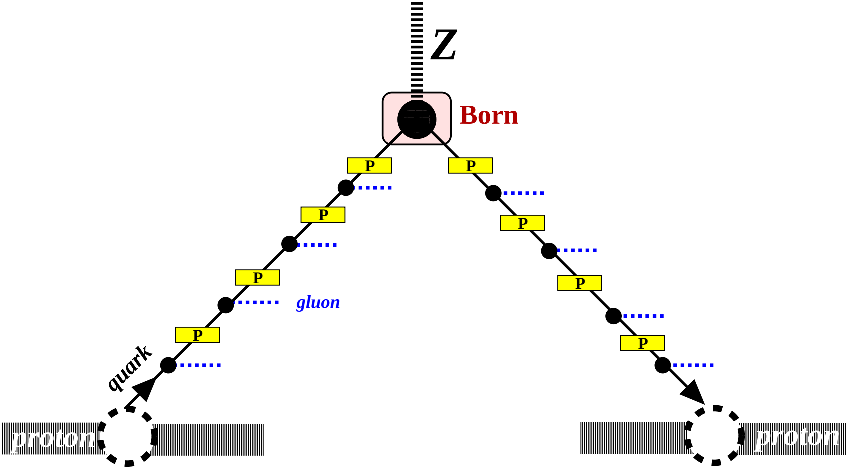
Outline

LO shower

Main features of new LO shower:

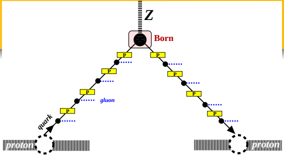
- based on modified collinear factorization;
  - conserving 4-momentum (also transverse component)
  - giving perturbative series directly in terms of exponent not geometrical series (avoiding oversubtractions)
  - real emissions regularized geometrically in 4 dimensions
  - subtractions defined in unintegrated form (corresponding to MC counterterms)
- angular ordering;
- cover the whole phase space – **no gaps**;

(re-)constructed LO shower



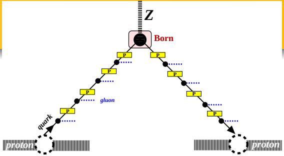
$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\{\sigma[C_0^{(0)}(\mathbb{P}'K_{0F}^{(1)})^{n_1}(\mathbb{P}''K_{0B}^{(1)})^{n_2}]\}_{T.O.}$$

(re-)constructed LO MC shower



$$
\begin{aligned}
\sigma_0 = &\int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B \\
&\times e^{-S_F} \int_{\Xi < \eta_{n_1}} \left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i) \theta_{\eta_i < \eta_{i-1}} \frac{2C_F \alpha_s}{\pi^2} \bar{P}(z_{Fi}) \right) \delta_{x_F = x_{0F} \prod_{i=1}^{n_1} z_{Fi}} \\
&\times e^{-S_B} \int_{\Xi > \eta_{n_2}} \left(\prod_{j=1}^{n_2} d^3\mathcal{E}(\bar{k}_j) \theta_{\eta_j > \eta_{j-1}} \frac{2C_F \alpha_s}{\pi^2} \bar{P}(z_{Bj}) \right) \delta_{x_B = x_{0B} \prod_{j=1}^{n_2} z_{Bj}} \\
&\times d\tau_2 (P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2) \; \frac{d\sigma_B}{d\Omega} (s x_F x_B, \hat{\theta})
\end{aligned}
$$

(re-)constructed LO MC shower



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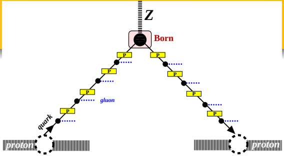
\end{aligned}
$$

- $d_0(\hat{t}_0, x_{0F})$ – initial PDF (preferably in MC scheme)

(re-)constructed LO MC shower



$$\sigma_0 = \int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B$$

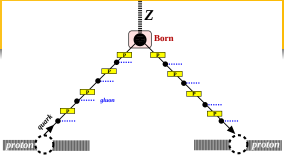
$$\times e^{-S_F} \int_{\Xi<\eta_{n_1}} \left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i) \theta_{\eta_i<\eta_{i-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Fi}) \right) \delta_{x_F = x_{0F} \prod_{i=1}^{n_1} z_{Fi}}$$

$$\times e^{-S_B} \int_{\Xi>\eta_{n_2}} \left(\prod_{j=1}^{n_2} d^3\mathcal{E}(\bar{k}_j) \theta_{\eta_j>\eta_{j-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Bj}) \right) \delta_{x_B = x_{0B} \prod_{j=1}^{n_2} z_{Bj}}$$

$$\times d\tau_2 (P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2) \; \frac{d\sigma_B}{d\Omega} (s x_F x_B, \hat{\theta})$$

- $d_0(\hat{t}_0, x_{0F})$ – initial PDF (preferably in MC scheme)
- S_F & S_B – Sudakov formfactors

(re-)constructed LO MC shower



$$\sigma_0 = \int dx_{0F} dx_{0B}\ \ d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F\ dx_B$$
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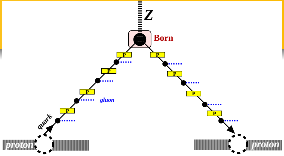
- $d_0(\hat{t}_0, x_{0F})$ – initial PDF (preferably in MC scheme)
- S_F & S_B – Sudakov formfactors
- LO DGLAP kernel $\bar{P}(z_{Bj}) = \frac{1}{2}(1 + z^2)$

(re-)constructed LO MC shower



$$\sigma_0 = \int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B$$

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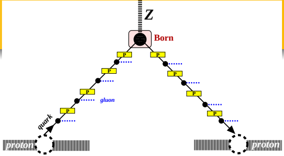
$$\times d\tau_2 (P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2) \; \frac{d\sigma_B}{d\Omega}(s x_F x_B, \hat{\theta})$$

- Ξ – Z-boson rapidity (division between F and B hemispheres)

(re-)constructed LO MC shower



$$\sigma_0 = \int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B$$

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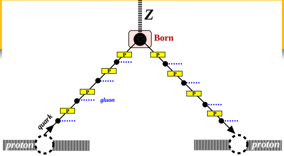
- $\eta_i = \frac{1}{2} \ln \frac{k^+}{k^-}$ – rapidity of emitted gluons ($\eta_{0F} > \cdots > \eta_{i-1} > \eta_i \cdots > \Xi$)

(re-)constructed LO MC shower



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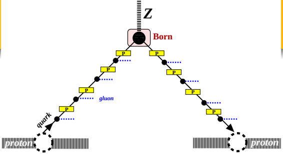
- $\eta_i = \frac{1}{2} \ln \frac{k^+}{k^-}$ – rapidity of emitted gluons $(\eta_{0F} > \cdots > \eta_{i-1} > \eta_i \cdots > \Xi)$
- $\bar{k}_i$ – rescaled 4-momenta

(re-)constructed LO MC shower



$$\sigma_0 = \int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B$$

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- $d^3\mathcal{E}(\bar{k}_j) = \frac{d^3k}{2k^0}\frac{1}{\mathbf{k}^2} = \pi \frac{d\phi}{2\pi}\frac{dk^+}{k^+} d\eta$ – eikonal phase-space for real gluon
- $\frac{d\sigma_B}{d\Omega}$ – LO hard process matrix-element

(re-)constructed LO shower

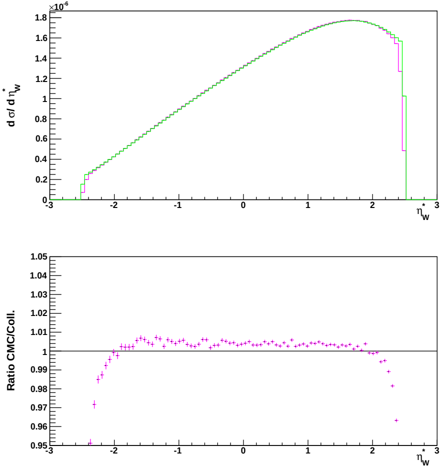
Analytical integration of LO MC distribution

$$\begin{aligned}\sigma_0 = &\int dx_{0F} dx_{0B}\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F\; dx_B \\ &\times e^{-S_F} \int_{\Xi<\eta_{n_1}} \Big(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i) \theta_{\eta_i<\eta_{i-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Fi}) \Big) \delta_{x_F = x_{0F} \prod_{i=1}^{n_1} z_{Fi}} \\ &\times e^{-S_B} \int_{\Xi>\eta_{n_2}} \Big(\prod_{j=1}^{n_2} d^3\mathcal{E}(\bar{k}_j) \theta_{\eta_j>\eta_{j-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Bj}) \Big) \delta_{x_B = x_{0B} \prod_{j=1}^{n_2} z_{Bj}} \\ &\times d\tau_2(P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2)\; \frac{d\sigma_B}{d\Omega}(s x_F x_B, \hat{\theta})\end{aligned}$$

over the multigluon phase-space gives the standard collinear formula:

$$\sigma_0 = \int_0^1 dx_F\; dx_B\; D_F(t, x_F)\; D_B(t, x_B)\; \sigma_B(s x_F x_B)$$

LO MC shower vs. collinear factorization

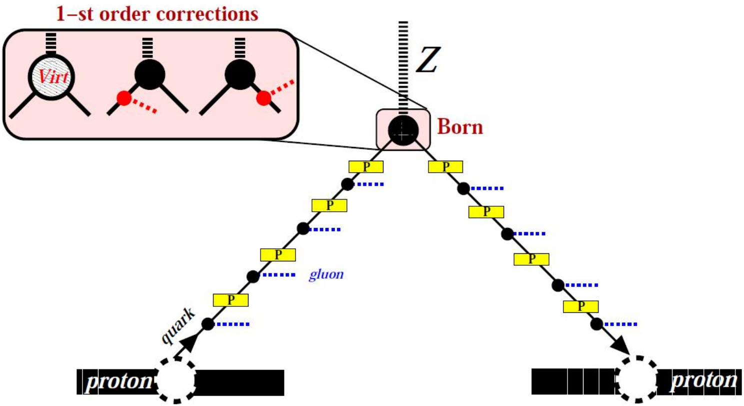


$$\eta^*_W = \frac{1}{2}\ln(x_F/x_B)$$

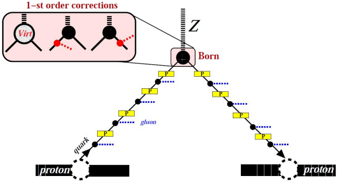
- η^*_W – in collinear limit – rapidity of W boson
- agreement of $< 0.5\%$

Outline

NLO correction to hard process



NLO correction to hard process



$$\sigma_0 = \int dx_{0F} dx_{0B} \;\; d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F \; dx_B$$

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$$\times d\tau_2(P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2) \; \frac{d\sigma_B}{d\Omega}(s x_F x_B, \hat{\theta}) \; W_{MC}^{NLO}$$

NLO correction to hard process

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

NLO correction to hard process

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- soft+virtual NLO correction (kinematics independent!)

$$\Delta_{V+S} = \frac{C_F \alpha_s}{\pi} \left(\frac{2}{3}\pi^2 - \frac{5}{4} \right)$$

NLO correction to hard process

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- soft+virtual NLO correction (kinematics independent!)

$$\Delta_{V+S} = \frac{C_F \alpha_s}{\pi} \left(\frac{2}{3}\pi^2 - \frac{5}{4} \right)$$

- real correction (with subtraction)

$$\tilde{\beta}_1(q_1, q_2, k) = \Big[\frac{(1-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_F) + \frac{(1-\alpha)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_B) \Big]$$
$$-\theta_{\alpha>\beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha<\beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}).$$

NLO correction to hard process

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

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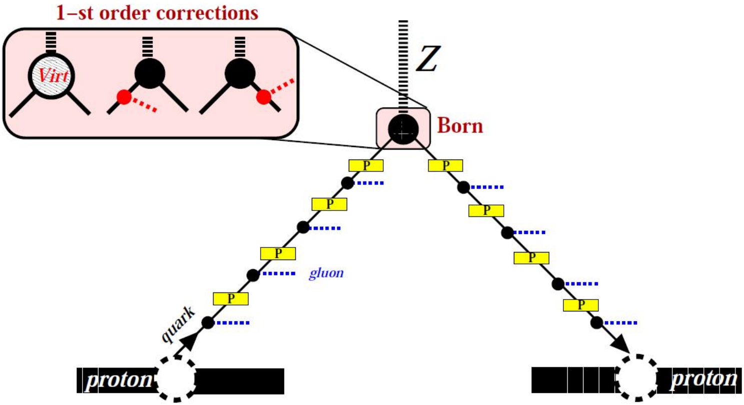
$$-\theta_{\alpha>\beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha<\beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}).$$

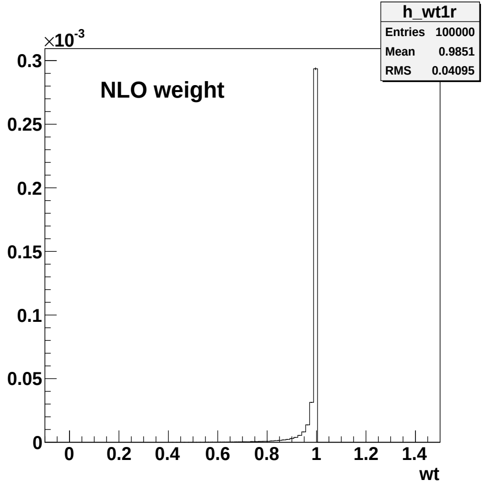
- summation over all partons!

Summation in NLO weight for hard process

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$



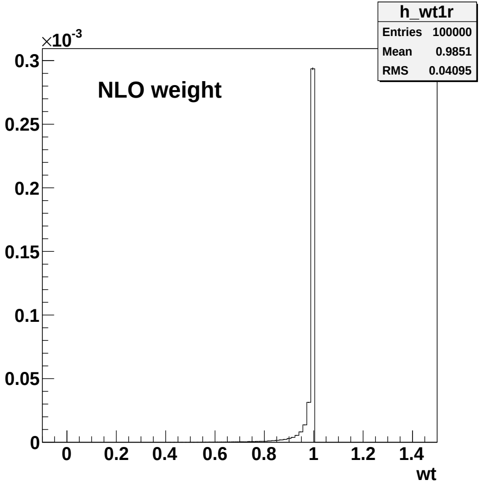
NLO weight distribution



W^{NLO}_{MC}

Correcting to NLO requires only reweighting with W^{NLO}_{MC} weight:

- strongly peaked
- positive
- without long-ranged tails



W_{MC}^{NLO}

Correcting to NLO requires only reweighting with W_{MC}^{NLO} weight:

- strongly peaked
- positive
- without long-ranged tails

Good behavior of W_{MC}^{NLO} is a profit form reformulating the collinear factorization and rebuilding LO shower.

NLO correction to hard process

As in LO, the NLO MC distribution can be integrated analytically over the multigluon phase space giving standard collinear formula:

$$\sigma_1 = \int_0^1 dx_F\, dx_B\, dz\, D_F(t, x_F)\, D_B(t, x_B)\, \sigma_B(szx_Fx_B)\{\delta_{z=1}(1 + \Delta_{S+V}) + C_{2r}(z)\}$$

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- "coefficient function": $C_{2r}(z) = -\frac{C_F\alpha_s}{\pi}\,(1-z)$

  is different then the $\overline{\text{MS}}$ one: $C_{2r}^{\overline{\text{MS}}}(z) = \frac{C_F\alpha_s}{\pi}\,\frac{1+z^2}{1-z}\,[2\ln(1-z)-\ln(z)]$

- singular logarithmic terms $\frac{\ln(1-z)}{1-z}$ of $\overline{\text{MS}}$ are absent
- we have a build-in resummation of $\frac{\ln^n(1-z)}{1-z}$ terms

NLO correction to hard process – proof of concept

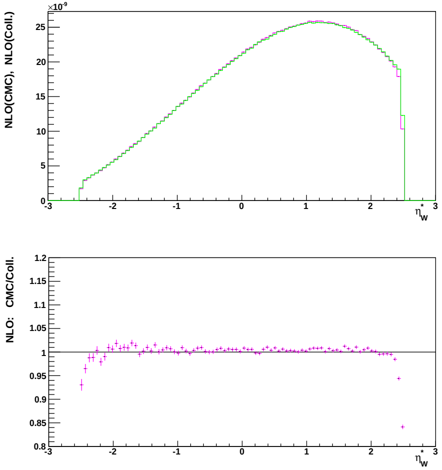
To test concept of our method we compare numerically inclusive distributions obtained from the full scale MC generation (4-momenta conserving)

$$\sigma_0 = \int dx_{0F} dx_{0B}\ d_0(\hat{t}_0, x_{0F}) d_0(\hat{t}_0, x_{0B}) \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dx_F\ dx_B$$
$$\times e^{-S_F} \int_{\Xi<\eta_{n_1}} \Big(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i) \theta_{\eta_i<\eta_{i-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Fi}) \Big) \delta_{x_F = x_{0F} \prod_{i=1}^{n_1} z_{Fi}}$$
$$\times e^{-S_B} \int_{\Xi>\eta_{n_2}} \Big(\prod_{j=1}^{n_2} d^3\mathcal{E}(\bar{k}_j) \theta_{\eta_j>\eta_{j-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Bj}) \Big) \delta_{x_B = x_{0B} \prod_{j=1}^{n_2} z_{Bj}}$$
$$\times d\tau_2(P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2)\ \frac{d\sigma_B}{d\Omega}(s x_F x_B, \hat{\theta})\ W_{MC}^{NLO}$$

and from the collinear formula

$$\sigma_1 = \int_0^1 dx_F\ dx_B\ dz\ D_F(t, x_F)\ D_B(t, x_B)\ \sigma_B(s z x_F x_B) \{ \delta_{z=1}(1 + \Delta_{S+V}) + C_{2r}(z) \}$$

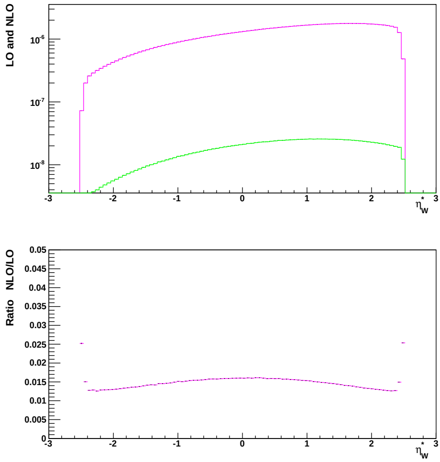
NLO correction to hard process – proof of concept



- η^*_W – in collinear limit – rapidity of W boson
- agreement of $< 1\%$

$$\eta^*_W = \frac{1}{2}\ln(x_F/x_B)$$

NLO correction to hard process – proof of concept



$$\eta^*_W = \frac{1}{2}\ln(x_F/x_B)$$

- η^*_W – in collinear limit – rapidity of W boson
- NLO correction is only $\sim 1.5\%$ of LO!

Summary of presented scheme

- NLO corrections to hard process added on top of the LO MC with a simple, positive weight;
- There is a built-in resummation of $\frac{\ln^n(1-x)}{1-x}$ terms;
- Virtual+soft corrections Δ_{V+S} are completely kinematics independent (all the complicated $d\Sigma^{c\pm}$ contributions of MC@NLO scheme are absent);
- There is no need to correct for the difference in the collinear counter-terms of the MC and $\overline{\text{MS}}$ scheme

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- NLO corrections to hard process added on top of the LO MC with a simple, positive weight;
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but there is a price for it:

- we needed to rebuild LO shower
- deviations from $\overline{\text{MS}}$ factorization scheme means different coefficient function and PDFs (this is well controlled)

Outline

Relation to other methods

There are two well established methods for including NLO corrections to hard process in MC shower:

- POWHEG
- MC@NLO

There is also an effort of Japanese group of H. Tanaka (arXiv:1106.3390)

In the following I show some of the conceptual differences between our scheme and POWHEG and MC@NLO methods (concentrating on POWHEG).

Relation to other methods

Main differences with respect to POWHEG and MC@NLO are:

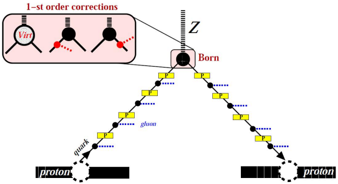
- "Democratic" summation over all emitted gluons, without deciding explicitly which gluon is involved in hard process

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j \in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j \in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

- differences related to modified *factorization scheme* (like absence of $(1/(1-z))_+$ distributions in the real part of the NLO corrections)

Here I concentrate only on the first point.

Relation to other methods

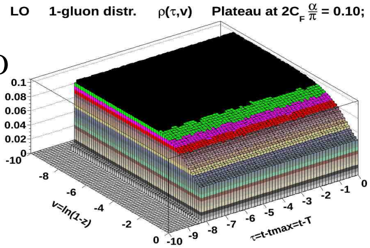


For better visualization we restrict to one "ladder" and a simplified weight:

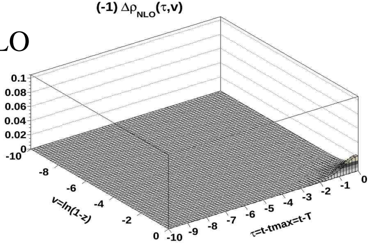
$$W_{MC}^{NLO} = 1 + \sum_{j \in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

Relation to other methods

LO



NLO



Inclusive distribution of gluons on the log Sudakov plane of:

- rapidity $t = \xi$
- $v = \ln(1 - z)$

NLO correction located near the rapidity of the hard process $t = \xi = t_{\max}$

- complete phase space near $(z = 0, t = t_{\max})$ corner
- standard LO MCs feature empty "dead zone" (used by POWHEG and MC@NLO)

k_T–ordering & hardest emission

We can anticipate that the dominant contribution to

$$W_{MC}^{NLO} = 1 + \sum_{j \in F} W_j^{NLO}$$

comes from the gluon with the highest

$$\ln k_j^T \sim t_j + \ln(1 - z_j)$$

that is the closest to the hard process corner ($z = 0$, $t = t_{\max}$).

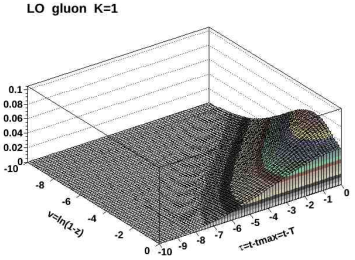
We can easily relabel gluons generated in MC ($\sum_j \to \sum_K$), so they are ordered in transverse momentum

$$\kappa_{K+1} < \kappa_K, \qquad \kappa_K \sim \ln k_K^T$$

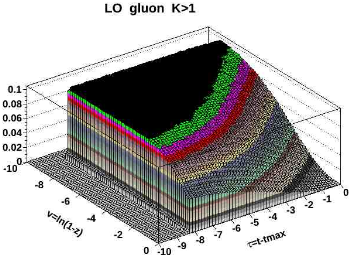
with $K = 1$ being the hardest one.

k_T–ordering & hardest emission

hardest in k^T gluon

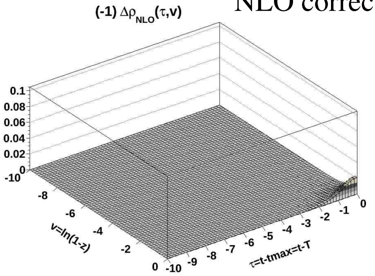


rest



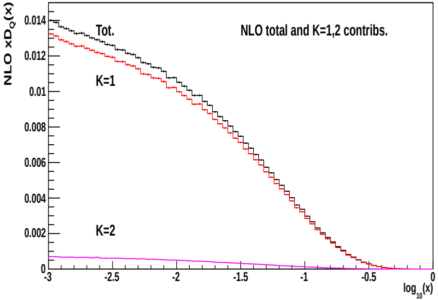
- $K = 1$ component reproduces the LO distribution over all the region where NLO correction is non-negligible (hard process corner).
- This represents the hardest emission of POWHEG.

NLO correction



k_T–ordering & hardest emission

Comparison of the full NLO correction: $\sum_j W_j^{NLO}$ and its two hardest (in k^T) components $W_{K=1}^{NLO}$, $W_{K=2}^{NLO}$:



$$x = \prod_j z_j$$

- $K = 1$ saturates the entire sum very well
- $K = 2$ component is small (additional NNLO terms)

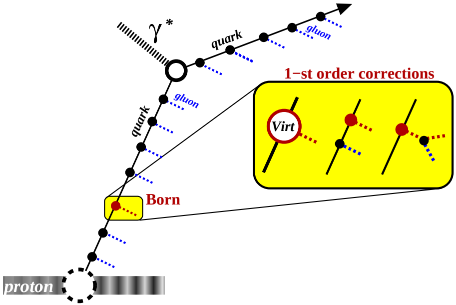
Ordering & hardest emission – summary

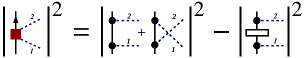
- We can keep the sum $\sum_j W_j^{NLO}$ or restrict to $W_{K=1}^{NLO}$.
- We use angular ordering but no vetoed/truncated showers are needed (we just relable already generated gluons)
- In POWHEG scheme $K = 1$ gluon is generated separately in the first step, requiring the use of vetoed/truncated showers in case of angular-ordered LO MC shower.

Outline

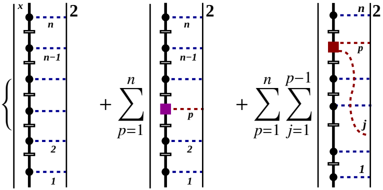
1. Introduction
2. Our project
   - Rebuilding LO MC shower
   - Including NLO corrections to hard process
   - Relation to other methods
   - Next step – fully NLO shower
3. Summary & outlook

NLO corrections in the MC ladder (gluons out of quarks)





$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Big\{ \;[\ldots]\; + \sum_{p=1}^{n} \;[\ldots]\; + \sum_{p=1}^{n}\sum_{j=1}^{p-1} \;[\ldots]\; \Big\} =$$



$$= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho^{(1)}_{1B}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \right\}.$$

Outline

Summary/Prospects

The "proof of concept" of the methodology is done!

- ✓ we extended collinear factorization to be suitable for MC;
- ✓ we constructed a new LO Parton Shower;
- ✓ method for implementing NLO corrections in *hard process* is working;
- ✓ we have a method for including NLO corrections in the shower *ladders*;

Summary/Prospects

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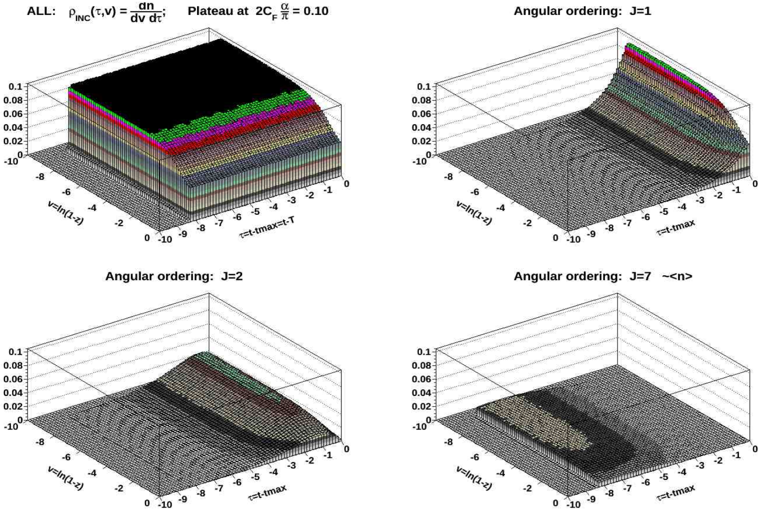
- ✓ we constructed a new LO Parton Shower;
- ✓ method for implementing NLO corrections in *hard process* is working;
- ✓ we have a method for including NLO corrections in the shower *ladders*;

but... this is done for *non-singlet* case (gluonstrahlung), now we need some more work to go to the practical level

- ✗ include *singlet* diagrams (easier)
- ✗ optimize technique of adding NLO corrections to the ladders;
- ✗ work on implementation of W/Z production at LHC (first: LO shower + NLO hard process);

BACKUP SLIDES

Angular ordered shower



Mapping (rescalling)

- We order all gluons by the distance form Ξ – the rapidity of the produced Z boson. We define permutation π:
  $|\eta_{\pi_i} - \Xi| > |\eta_{\pi_{i-1}} - \Xi|, \quad i \in F, B$
- Mapping is now defined recursively as:

$$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}, \qquad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2(P - \sum_{j=1}^{i-1} k_{\pi_j}) \cdot \bar{k}_{\pi_i}}, \qquad i = 1, 2, ..., n_1 + n_2.$$

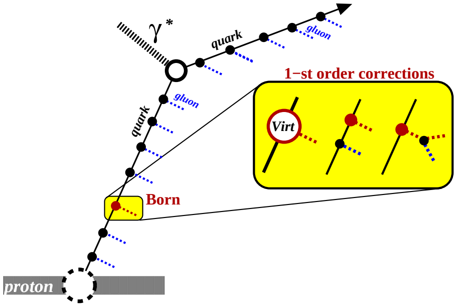
Features of the mapping:

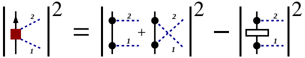
- pure rescaling
- preserves angels
- preservers soft factors $(d\alpha/\alpha)$
- no gaps in phase-space

LO MC algorithm

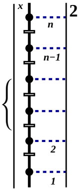
1. variables $\hat{z}_F$ and $\hat{z}_B$ are generated by the FOAM MC Sampler;
2. four-momenta $\bar{k}_i^\mu$ are generated separately in the F and B parts of the phase space with the constraints $\sum_{j\in F} \hat{\alpha}_j = 1 - \hat{z}_F$ and $\sum_{j\in B} \hat{\beta}_j = 1 - \hat{z}_B$;
3. double-ordering permutation π is established;
4. rescaling parameter λ_1 is calculated; $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set, such that $(P - k_{\pi_1})^2 = sx_1$;
5. parameter λ_2 is calculated and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set, such that $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$ and so on... ;
6. in the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ four-momenta q_1^μ and q_2^μ are generated according to the Born angular distribution.

NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

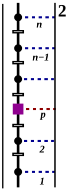




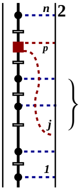
$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$



$+ \sum_{p=1}^{n}$



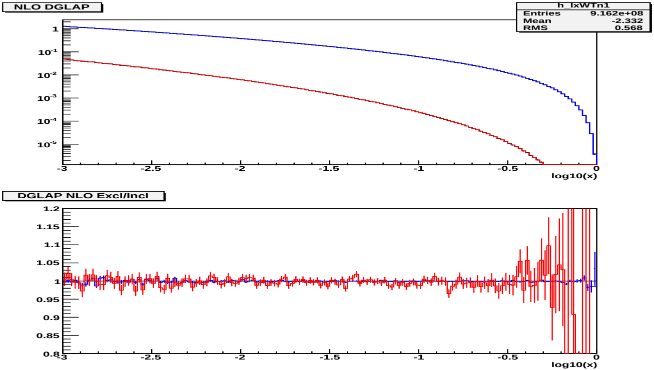
$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$



$\Bigg\} =$

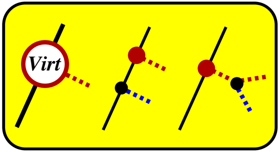
$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho_{1B}^{(1)}(k_i) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

Numerical test of ISR pure C_F^2 NLO MC

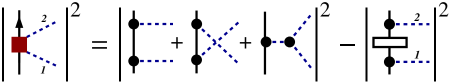


Numerical results for $D(x, Q)$ from **two** Monte Carlos inclusive and exclusive.
Blue curve is single NLO insertion, red curve is double insertion component.
Evolution 10GeV→1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

Gluon pair component of the NLO kernel, $\sim C_F C_A$ (FSR)



Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:

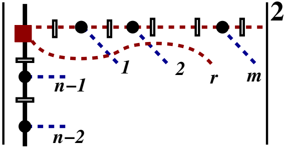
SOLUTION:

Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

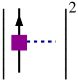
NLO FSR correction at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR correction at the end of the ladder:

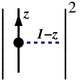
$$e^{-S_{ISR} - S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



where Sudakov S_{FSR} is subtracted in the virtual part:

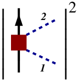


$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$



.

and FSR counterterm is subtracted in the 2-real-gluon part:



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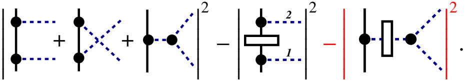


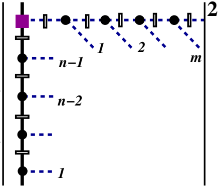
.

The miracle: both are free of any collinear or soft divergences!!!

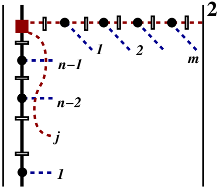
ISR+FSR NLO corrections at end of the ladder

$$\bar{D}^{[1]}_{NS}(x,Q) =$$

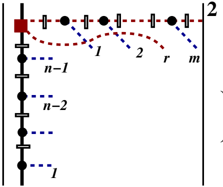
$$e^{-S} \sum_{n,m=0}^{\infty} \Bigg\{$$



$$+ \sum_{j=1}^{n-1}$$



$$+ \sum_{r=1}^{m}$$

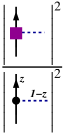


$$\Bigg\}$$

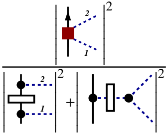
$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho^{(1)}_{1B}(k_i) \Bigg) e^{-S_{FSR}} \sum_{m=0}^{\infty} \Bigg(\prod_{j=1}^{m} \int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j \; \rho^{(1)}_{1V}(k'_j) \Bigg)$$

$$\times \Bigg[\beta^{(1)}_0(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^{m} W(\tilde{k}_n, \tilde{k}'_r) \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

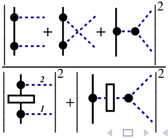
$$\beta^{(1)}_0 \equiv$$



$$, \quad W(k_2, k_1) \equiv$$



$$=$$



$$- 1.$$