

# Renormalization of the Higgs sector in the triplet model

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# The Higgs Triplet Model

The Higgs triplet field  $\Delta$  is added to the SM.

|          | SU(2) <sub>I</sub> | U(1) <sub>Y</sub> | U(1) <sub>L</sub> |
|----------|--------------------|-------------------|-------------------|
| $\Phi$   | 2                  | 1/2               | 0                 |
| $\Delta$ | 3                  | 1                 | -2                |

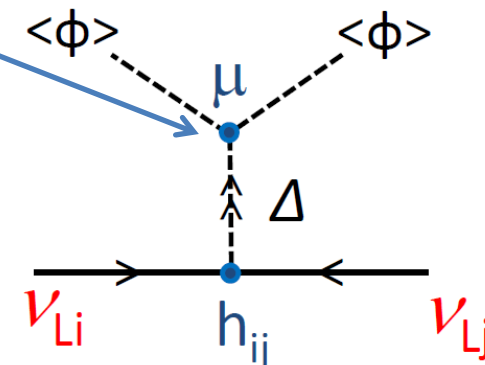
## • Neutrino Yukawa interaction:

$$\mathcal{L}_Y = h_{ij} \overline{L}_L^{ci} \cdot \Delta L_L^j$$

## • Higgs Potential:

$$V = m^2 \Phi^\dagger \Phi + M^2 \text{Tr}(\Delta^\dagger \Delta) + (\mu \Phi^T i \tau_2 \Delta^\dagger \Phi + \text{h.c.}) + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

Lepton number breaking parameter



*Cheng, Li (1980);  
Schechter, Valle, (1980);  
Magg, Wetterich, (1980);  
Mohapatra, Senjanovic, (1981).*

## • Neutrino mass matrix

$$(m_\nu)_{ij} = h_{ij} \frac{\mu \langle \phi^0 \rangle^2}{M_\Delta^2} = h_{ij} v_\Delta$$

$M_\Delta$  : Mass of triplet scalar boson.

$v_\Delta$  : VEV of the triplet Higgs

# The Higgs Triplet Model

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## • Neutrino Yukawa interaction:

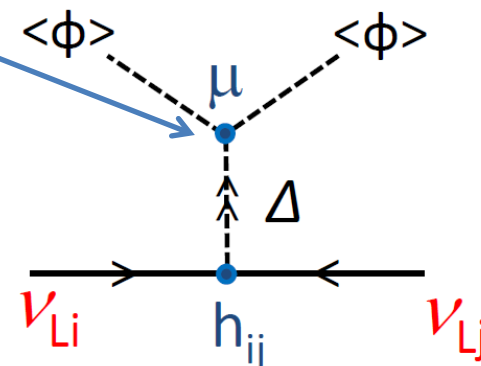
$$\mathcal{L}_Y = h_{ij} \overline{L}_L^{ci} \cdot \Delta L_L^j$$

## • Higgs Potential:

Lepton number breaking parameter

$$V = m^2 \Phi^\dagger \Phi + M^2 \text{Tr}(\Delta^\dagger \Delta) + (\mu \Phi^T i \tau_2 \Delta^\dagger \Phi + \text{h.c.}) \\ + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] \\ + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

*Cheng, Li (1980);  
Schechter, Valle, (1980);  
Magg, Wetterich, (1980);  
Mohapatra, Senjanovic, (1981).*



## • Neutrino mass matrix

$$(m_\nu)_{ij} = \frac{h_{ij} \mu \langle \phi^0 \rangle^2}{M_\Delta^2}$$

$O(0.1) \text{ eV}$  (circled in blue)     
  $O(0.1) \text{ eV}$  (circled in red)     
  $246 \text{ GeV}$  (circled in blue)     
  $O(100) \text{ GeV}$  (circled in green)

The HTM can be tested at colliders !!

# Important predictions

★ Rho parameter deviates from unity.

$$\rho_{\text{tree}} = \frac{1 + \frac{2v_{\Delta}^2}{v_{\Phi}^2}}{1 + \frac{4v_{\Delta}^2}{v_{\Phi}^2}} \simeq 1 - \frac{2v_{\Delta}^2}{v_{\Phi}^2}$$

★ Extra Higgs bosons

Doubly-charged  $H^{\pm\pm}$ , Singly-charged  $H^{\pm}$ , CP-odd  $A$  and CP-even Higgs boson  $H$

★ Characteristic mass relation is predicted.

$$m_{H^{++}}^2 - m_{H^+}^2 \simeq m_{H^+}^2 - m_A^2$$

Under  $v_{\Delta} \ll v_{\Phi}$  (From experimental data  $\rho_{\text{exp}} \sim 1$ )

$$m_h^2 \simeq 2\lambda_1 v^2$$

$$M_{\Delta}^2 \equiv \frac{v_{\Phi}^2 \mu}{\sqrt{2} v_{\Delta}}$$

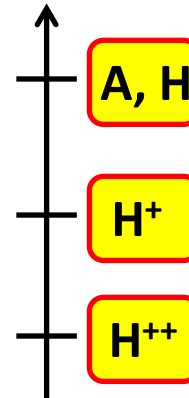
$$m_{H^{++}}^2 \simeq M_{\Delta}^2 - \frac{v^2}{2} \lambda_5$$

$$m_{H^+}^2 \simeq M_{\Delta}^2 - \frac{v^2}{4} \lambda_5$$

$$m_A^2 \simeq m_H^2 = M_{\Delta}^2$$

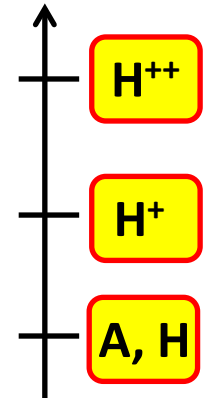
Case I ( $\lambda_5 > 0$ )

Mass



Case II ( $\lambda_5 < 0$ )

Mass



# Theoretical bounds

▶ Vacuum stability bound (Bounded from below) *Arhrib, et al., PRD84, (2011)*

$$\lim_{r \rightarrow \infty} V(rv_1, rv_2, \dots, rv_n) > 0$$

$$\lambda_2 = \lambda_3 = \lambda_\Delta$$

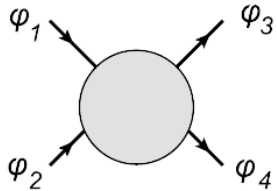
$$\lambda_1 > 0, \lambda_\Delta > 0,$$

$$2\sqrt{2\lambda_1\lambda_\Delta} + \lambda_4 + \text{MIN}[0, \lambda_5] > 0$$

*Lee, Quigg, Thacker, PRD16, (1977)*

*Aoki, Kanemura, PRD77, (2008); Arhrib, et al., PRD84, (2011)*

▶ Perturbative unitarity bound



$\varphi_i$ : Longitudinal modes of weak gauge bosons and physical Higgs bosons

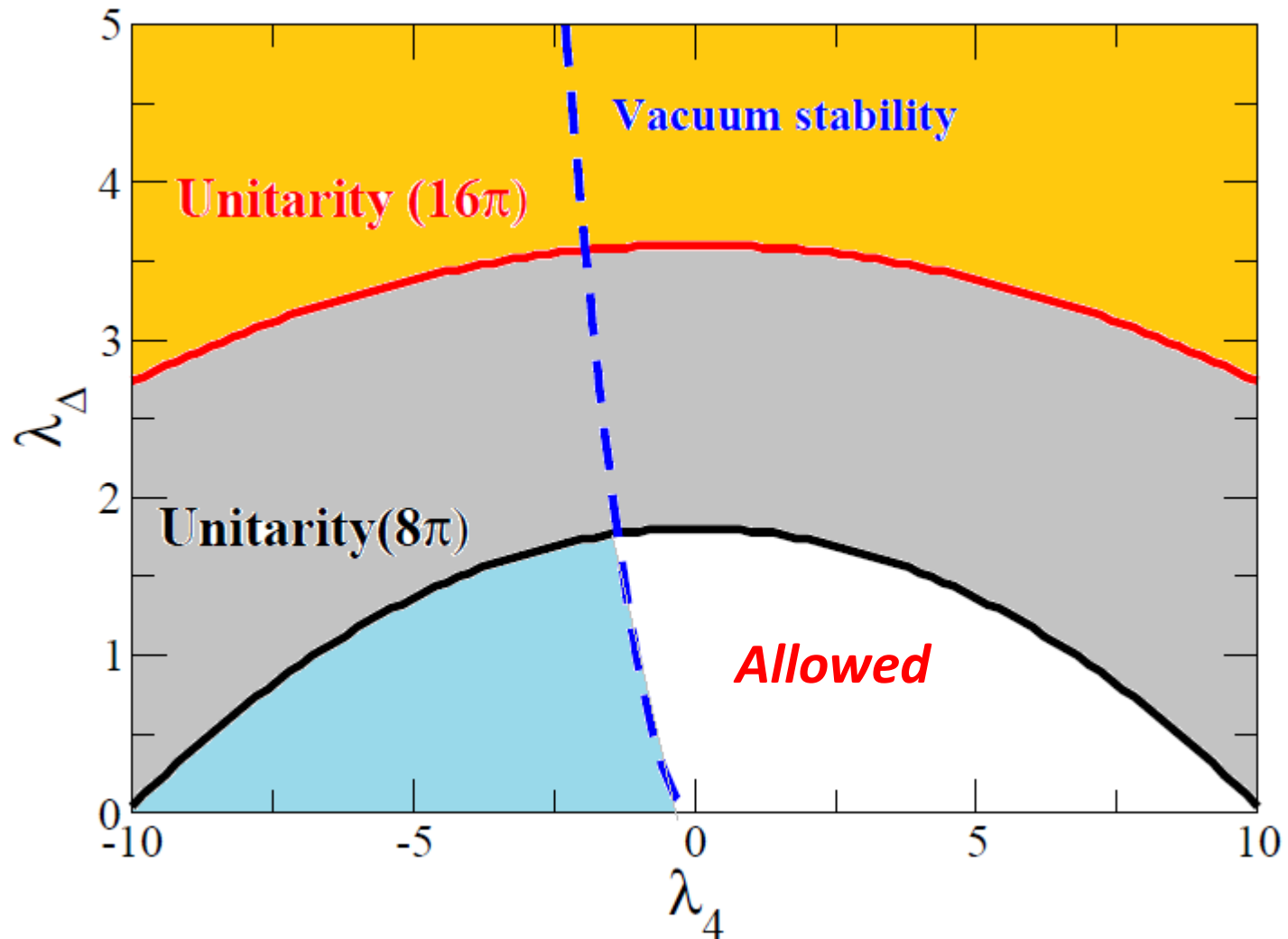
$$|\langle \varphi_3 \varphi_4 | a^0 | \varphi_1 \varphi_2 \rangle| < \frac{1}{2} \text{ or } 1$$

Eigenvalues  
of the matrix

$$\left\{ \begin{array}{l} x_1 = 3\lambda_1 + 7\lambda_\Delta + \sqrt{(3\lambda_1 - 7\lambda_\Delta)^2 + \frac{3}{2}(2\lambda_4 + \lambda_5)^2}, \\ x_2 = \frac{1}{2}(2\lambda_4 + 3\lambda_5), \\ x_3 = \frac{1}{2}(2\lambda_4 - \lambda_5) \end{array} \right. \quad |\mathbf{x}_i| < 8\pi \text{ or } 16\pi$$

# Theoretical bounds

Case for  $\lambda_5 = 0$  ( $\Delta m = 0$ )



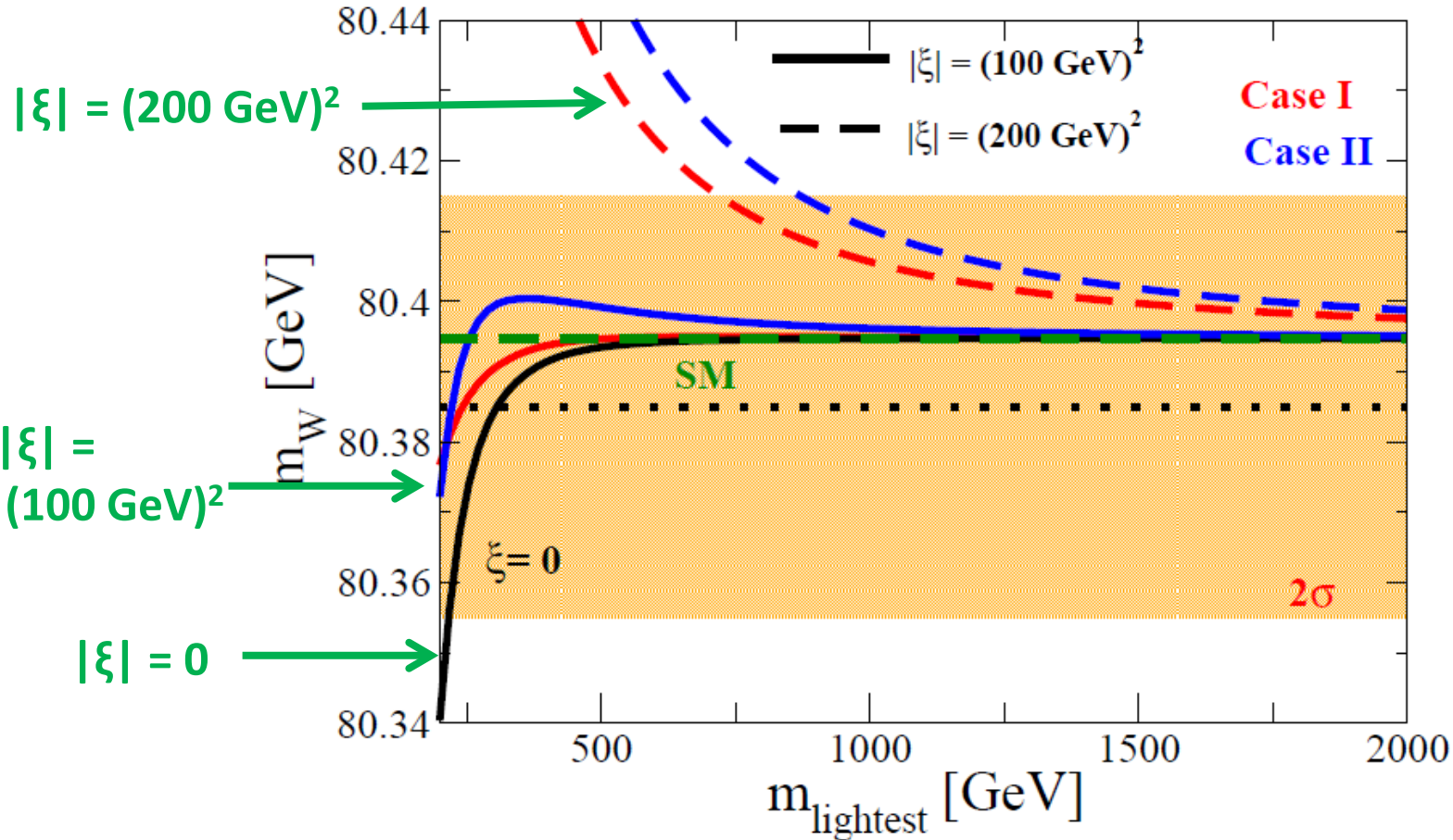
# 1-loop corrected W mass

$$\xi = m_{H^{++}}^2 - m_{H^+}^2$$

$m_h = 126 \text{ GeV}, m_t = 173 \text{ GeV}$

$$v_\Delta \simeq \mu \frac{v^2}{m_A^2}$$

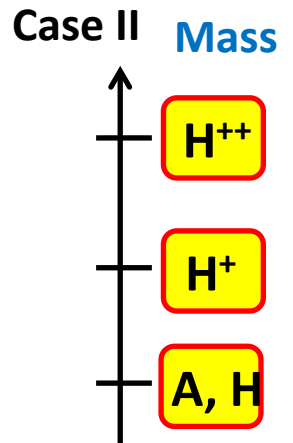
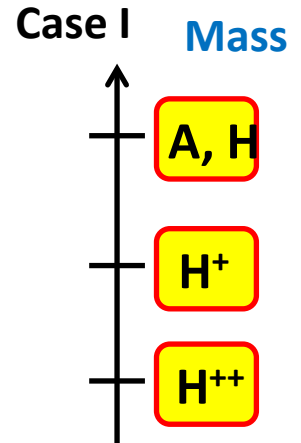
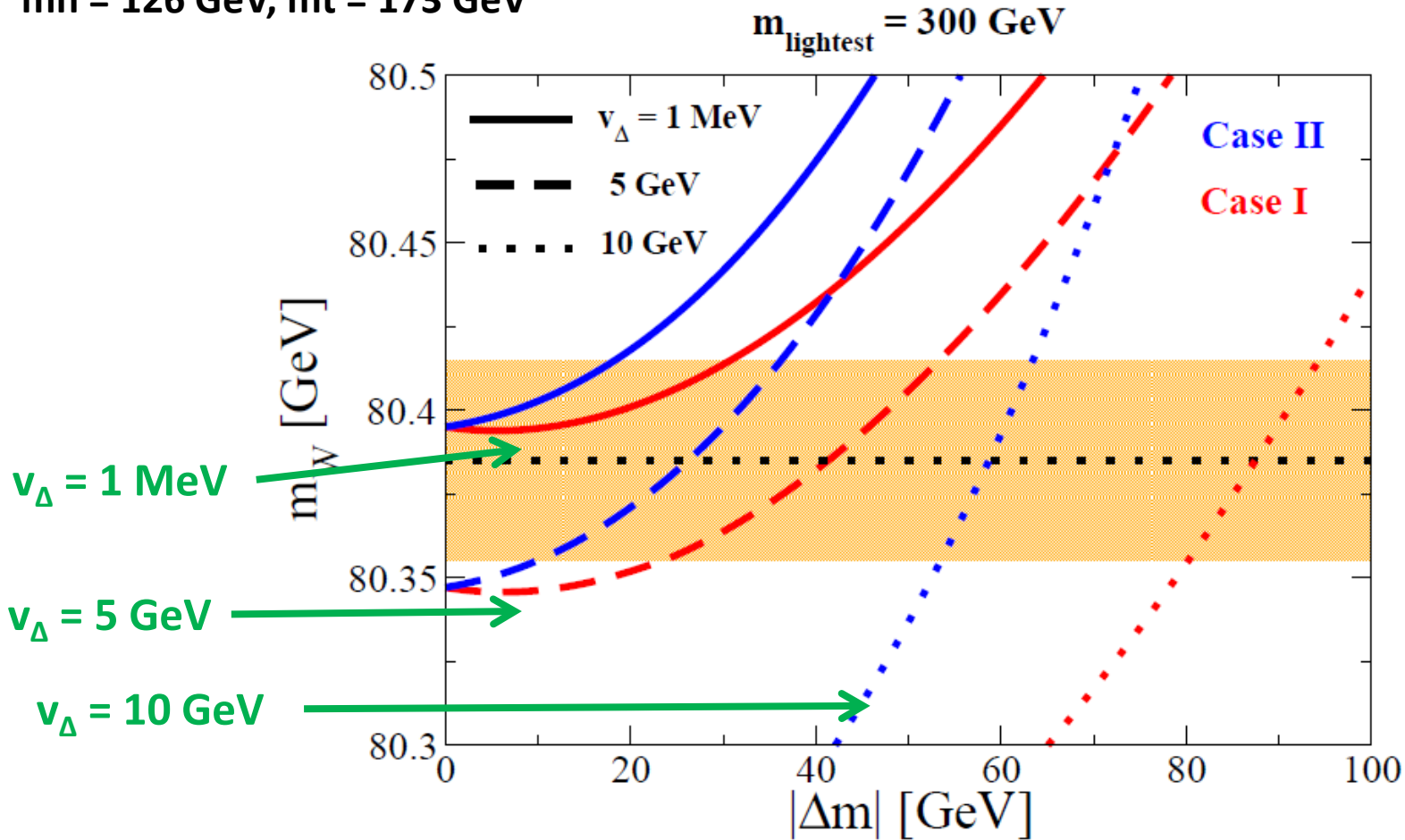
$\mu = 5 \text{ GeV}$



In the large mass limit, the HTM can be decoupled to the SM.

# 1-loop corrected W mass

$\Delta m = m_{H^{++}} - m_{H^+}$   
 $m_h = 126 \text{ GeV}, m_t = 173 \text{ GeV}$



For large triplet VEV case, large mass difference is favored.



# Testing the Higgs Triplet Model at colliders

- Indirect way (Decoupling case)

- Precise measurement for the Higgs couplings

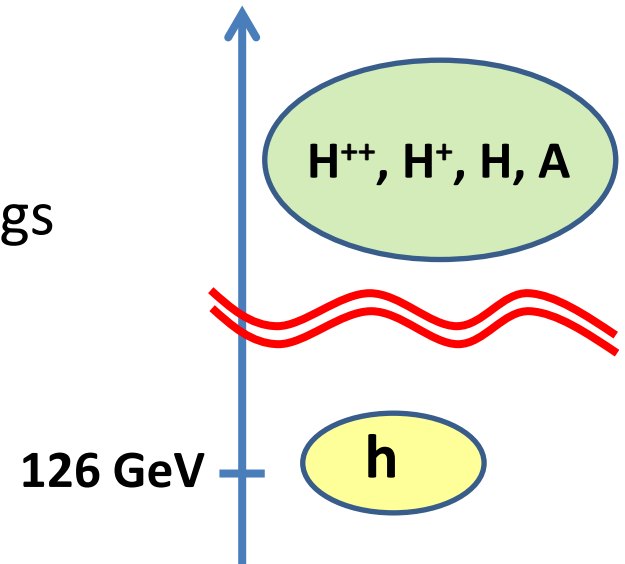
- Ex.  $h\gamma\gamma$ ,  $hhh$ ,  $hWW$ ,  $hZZ$ , ...

- Direct way

- Discovery of extra Higgs bosons

- Ex. Doubly-charged Higgs boson, Singly-charged Higgs boson, ...

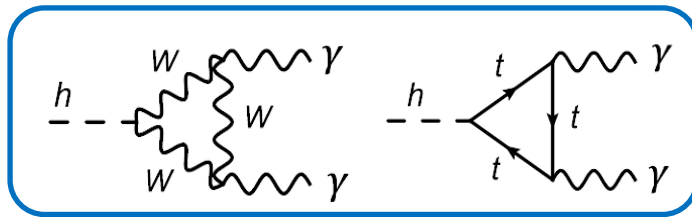
- Testing the mass spectrum among the triplet like Higgs bosons.



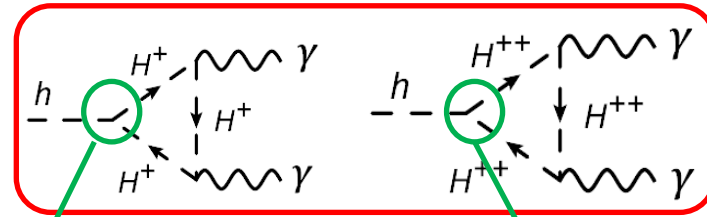
# Higgs $\rightarrow \gamma\gamma$

Arhrib, et al. JHEP04 (2012);

Kanemura, KY, PRD85 (2012); Akeroyd, Moretti PRD86 (2012)



SM contribution



Triplet-like Higgs loop contribution

$$\lambda_{hH^+H^-} \approx -(\lambda_4 + \lambda_5/2)v$$

$$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$$

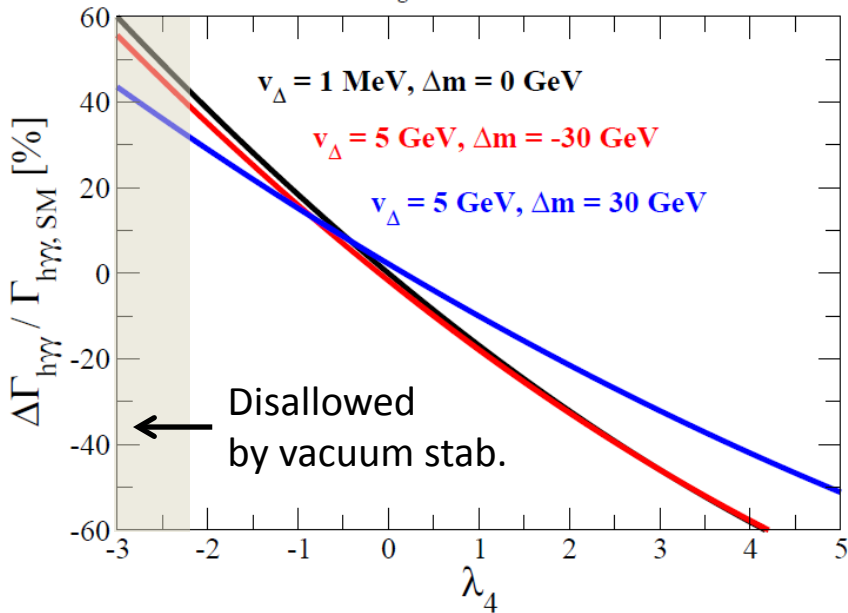
Sign of  $\lambda_4$  is quite important!



If  $\lambda_4 < 0 \rightarrow$  **Constructive** contribution

If  $\lambda_4 > 0 \rightarrow$  **Destructive** contribution

$m_{\text{lightest}} = 300 \text{ GeV}$

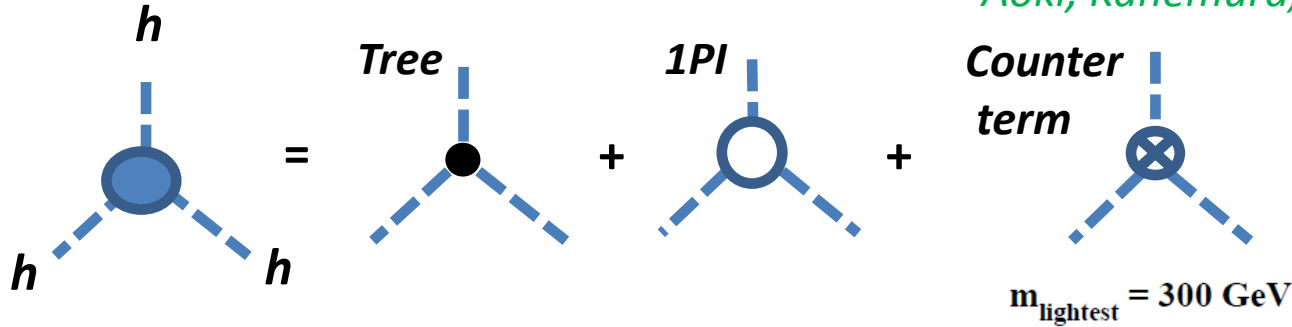


Signal strength ( $\sigma_{\text{obs}}/\sigma_{\text{SM}}$ ) is  
 $1.56 \pm 0.43$  (CMS) and  $1.9 \pm 0.5$  (ATLAS).

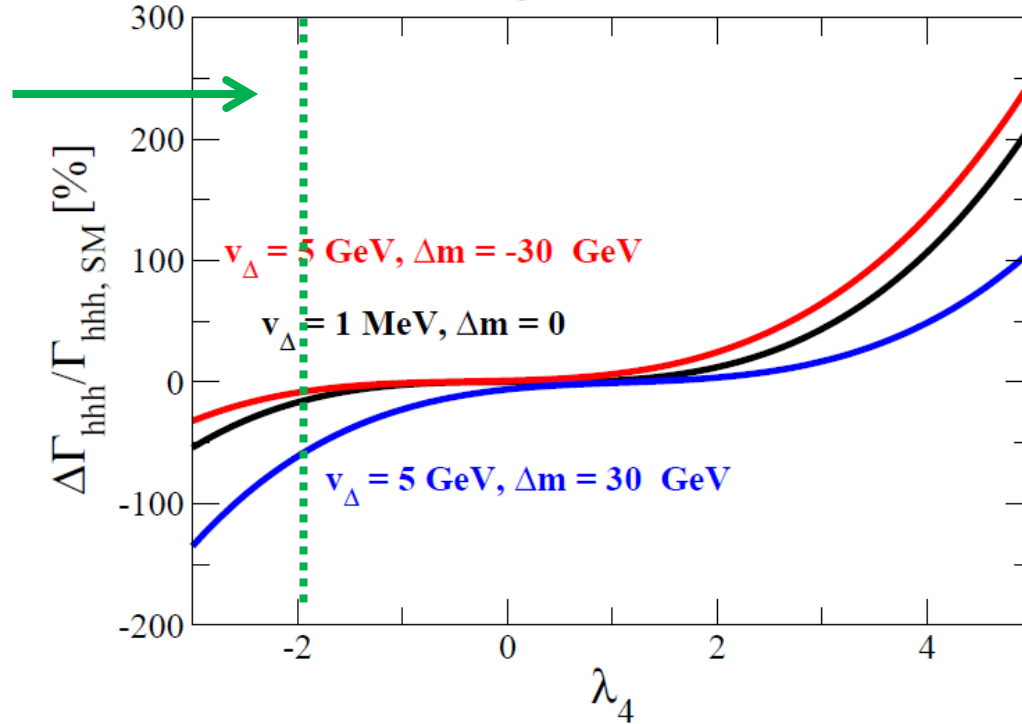
When triplet Higgs mass  $\sim 300 \text{ GeV}$ ,  
 $h\gamma\gamma$  can be enhanced by  $\sim +40\% \sim +50\%$ .

# Renormalized hhh coupling

*Aoki, Kanemura, Kikuchi, KY, PLB714 (2012)*



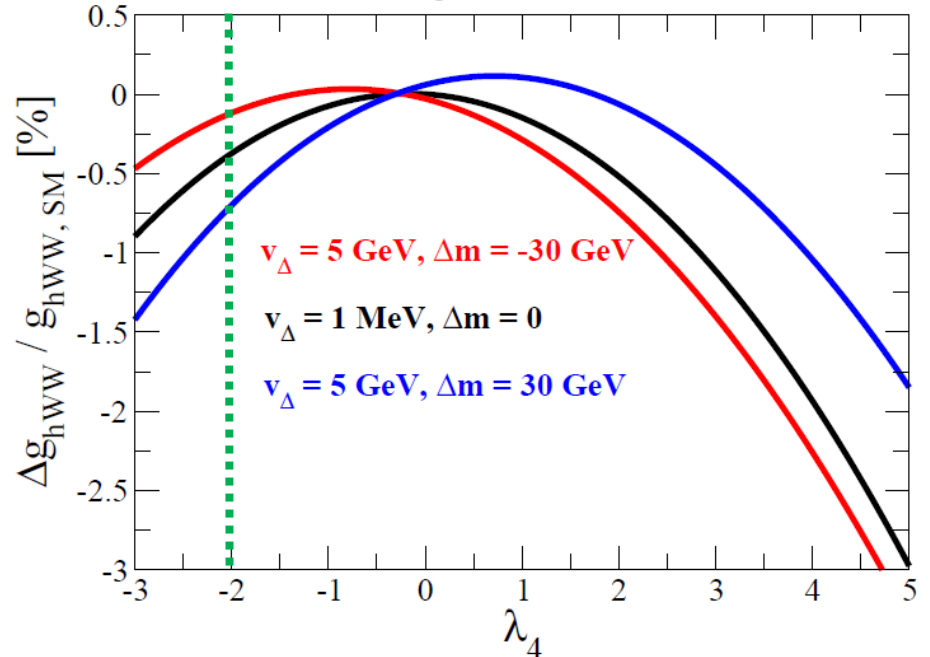
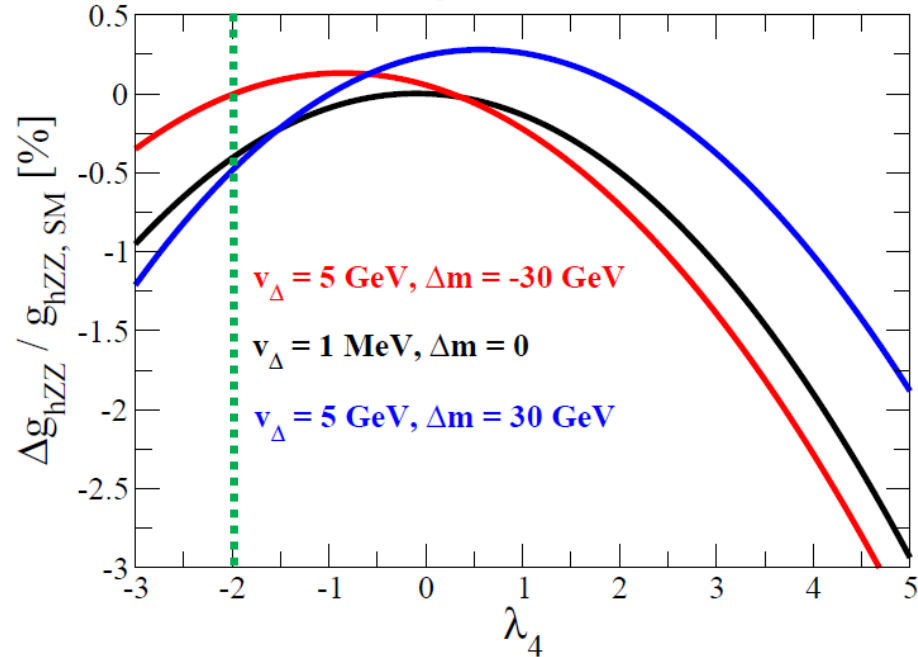
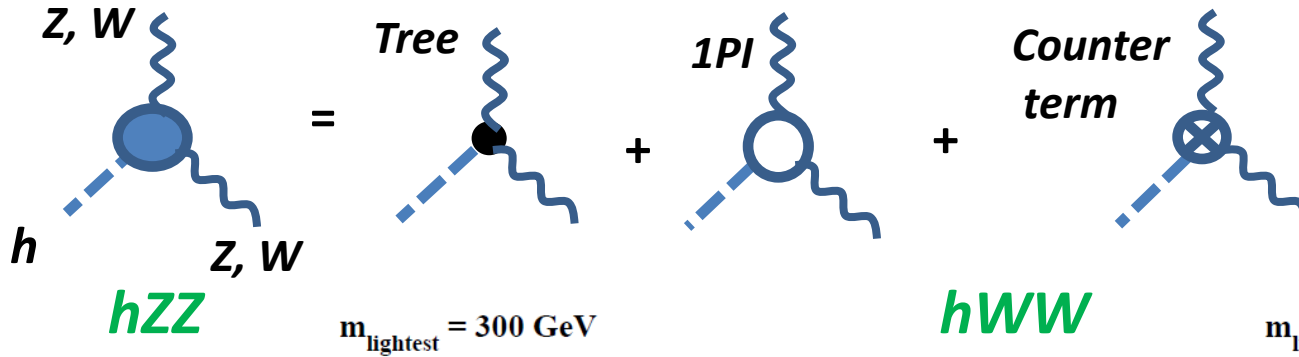
Corresponds to  $\Delta h\gamma\gamma \sim +50\%$



When  $h\gamma\gamma$  (HTM)  $>$   $h\gamma\gamma$  (SM) [ $\lambda_4 < 0$ ],  $hhh$  (HTM)  $<$   $hhh$  (SM).

In the case of  $\lambda_4 \sim -2$ , deviation of the  $hhh$  coupling is  $-10 \sim -60\%$ .

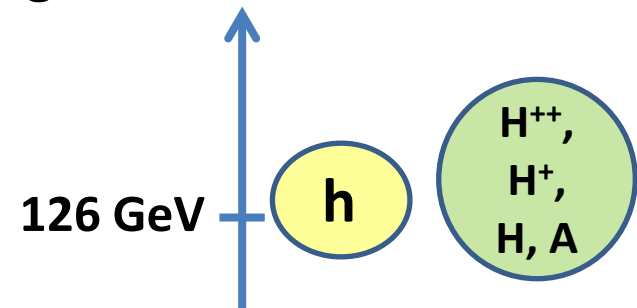
# Renormalized hZZ and hWW coupling



In the case of  $\lambda_4 \sim -2$ ,  
 deviation of the hZZ (hWW) coupling is  $0 \sim -0.5\%$  ( $-0.1 \sim -0.7\%$ ).

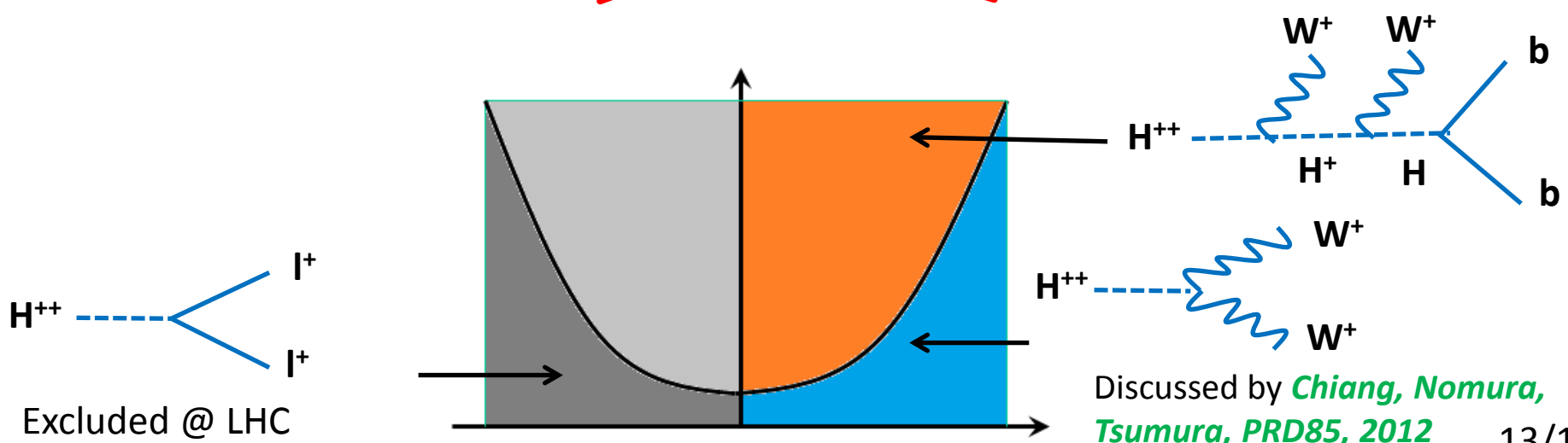
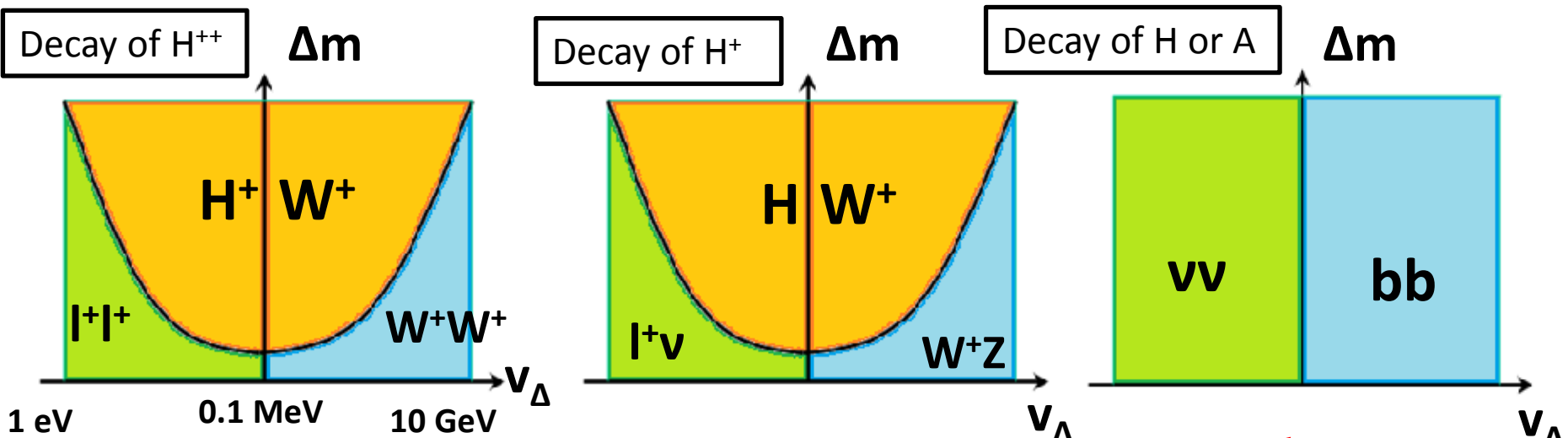
# Testing the Higgs Triplet Model at colliders

- Indirect way (Decoupling case)
  - Precise measurement for the Higgs couplings
  - Ex.  $h\gamma\gamma$ ,  $hWW$ ,  $hZZ$ ,  $hhh$ , ...



- **Direct way**
  - Discovery of extra Higgs bosons
  - Ex. Doubly-charged Higgs boson, Singly-charged Higgs boson, ...
  - Testing the mass spectrum among the triplet like Higgs bosons.

# Decay property of the triplet-like Higgs bosons [O(100) GeV case]



# Mass reconstruction at LHC

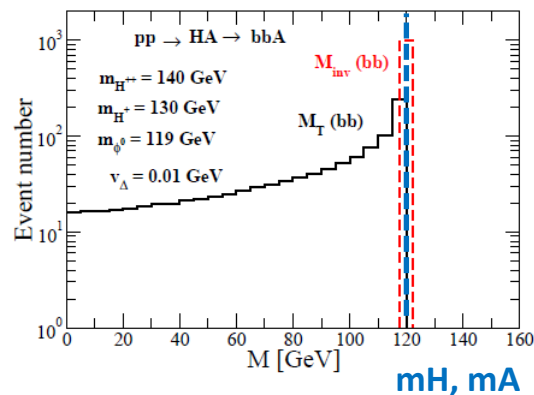
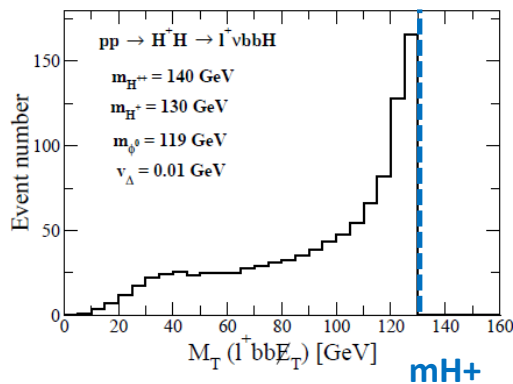
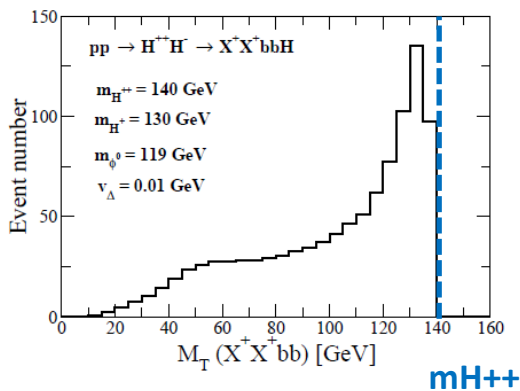
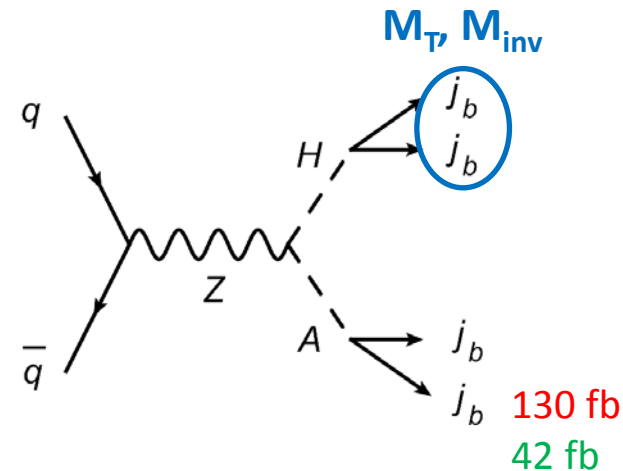
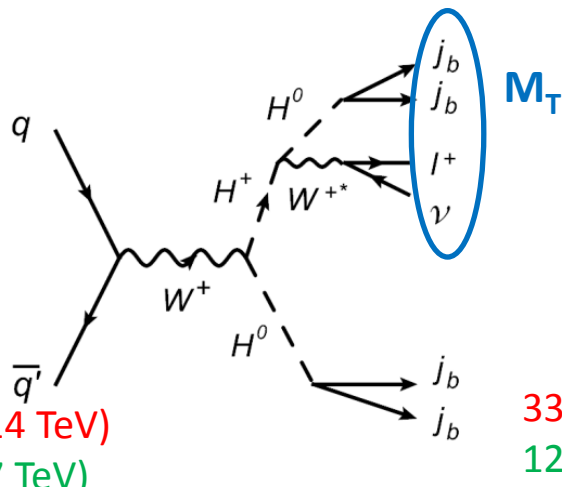
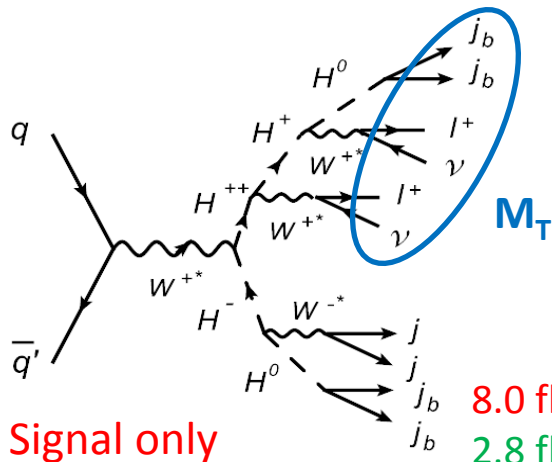
Aoki, Kanemura, KY, PRD85(2012)

|          |   |         |
|----------|---|---------|
| $H^{++}$ | ↑ | 140 GeV |
| $H^+$    | — | 130 GeV |
| $H, A$   | — | 119 GeV |
| $h$      | — | 114 GeV |

$$qq' \rightarrow H^{++}H^- \rightarrow (l^+l^+vvbb)(jjbb)$$

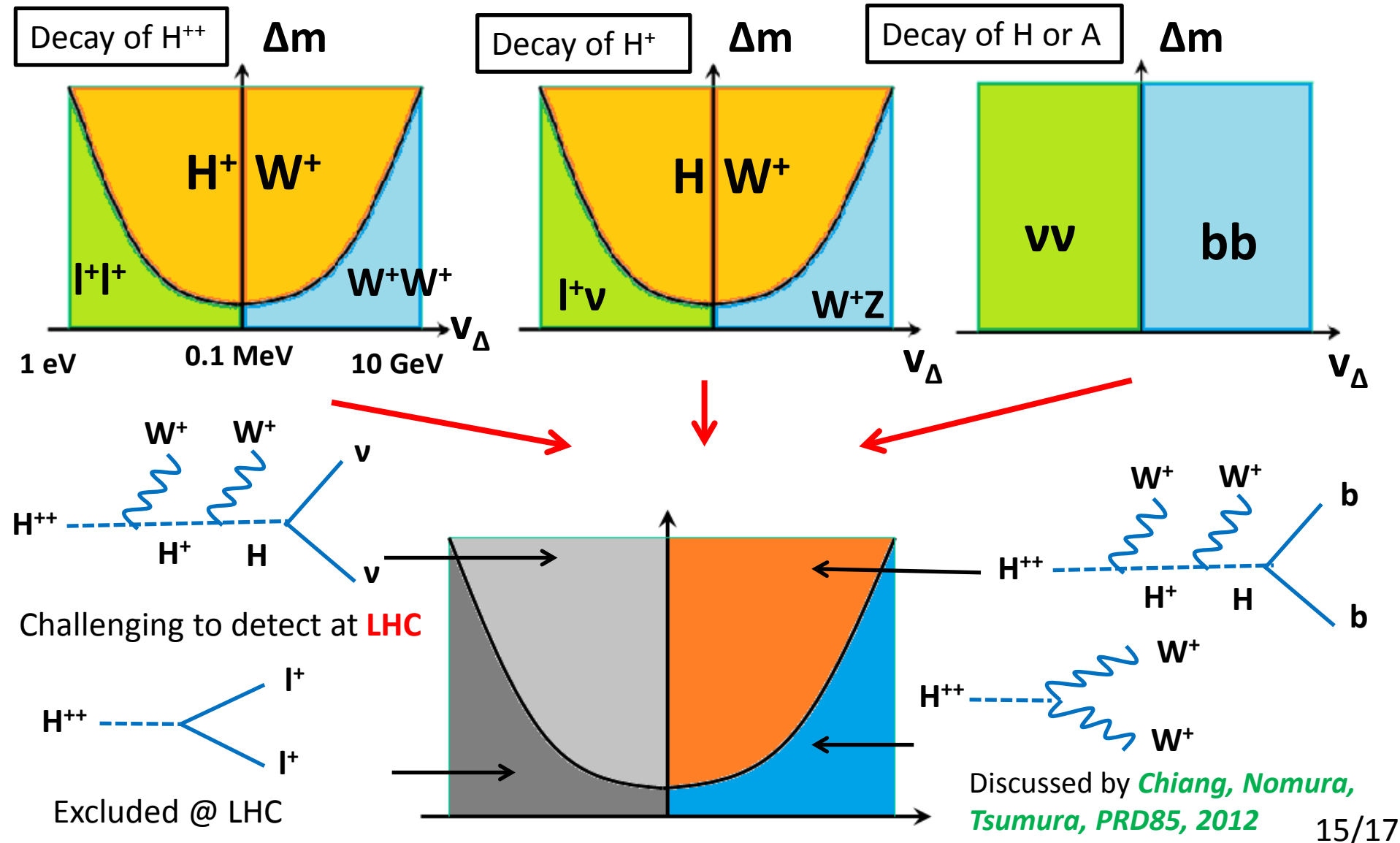
$$qq' \rightarrow H^+H \rightarrow (l^+vbb)(bb)$$

$$qq \rightarrow HA \rightarrow (bb)(bb)$$



All the masses of the  $\Delta$ -like scalar bosons may be reconstructed.

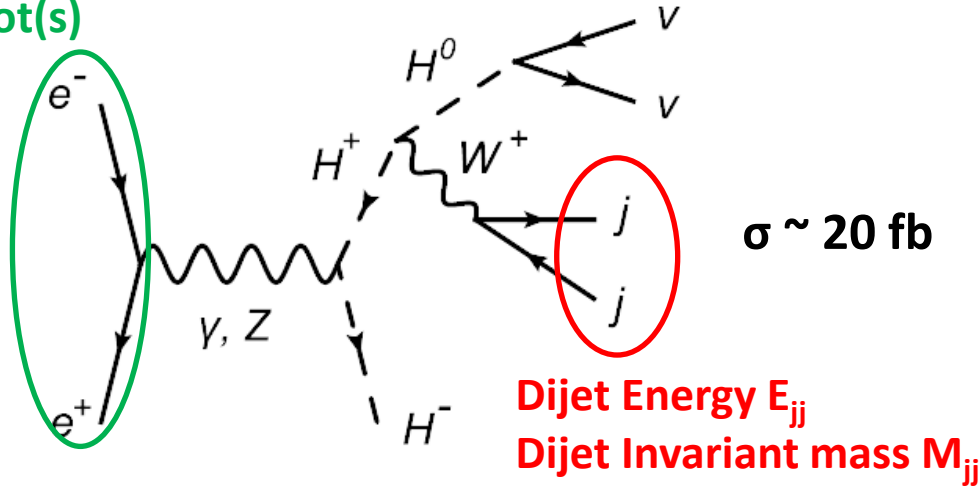
# Decay property of the triplet-like Higgs bosons [O(100) GeV case]



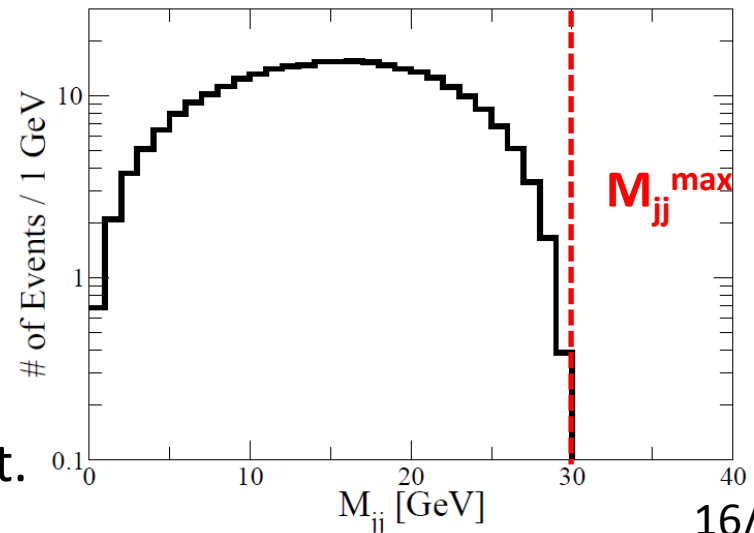
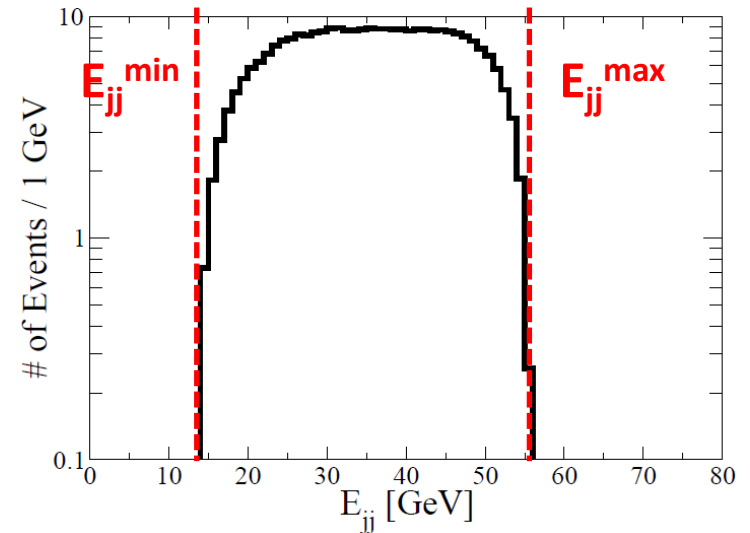


# Mass reconstruction at ILC

Root(s)



$m_{H^+} = 200 \text{ GeV}$ ,  $m_H = 170 \text{ GeV}$ ,  
 $\text{Root}(s) = 500 \text{ GeV}$ ,  $100 \text{ fb}^{-1}$



$$m_{H^+}^2 \simeq \frac{s}{4} \left[ 1 - \left( \frac{E_{jj}^{\max} - E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}} \right)^2 \right]$$

$$m_{H^0}^2 \simeq \frac{s}{4} \left[ 1 - \left( \frac{E_{jj}^{\max} - E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}} \right)^2 \right] - \frac{2\sqrt{s}E_{jj}^{\max}E_{jj}^{\min}}{E_{jj}^{\max} + E_{jj}^{\min}}$$

$$M_{jj}^{\max} \simeq m_{H^+} - m_H$$

$H^{++}$  can be measured by looking at the excess of the SS dilepton + jets + missing event.

# Summary

## ▶ **The Higgs Triplet Model (HTM) :**

Tiny neutrino masses can be explained.

## ▶ **Indirect way to test the HTM at colliders (Decoupling case):**

Measuring the deviation of the Higgs coupling from the SM prediction.

Ex) Triplet Higgs mass = 300 GeV case

- $h\gamma\gamma \rightarrow +50\%$       Current LHC data can be reproduced.
- $hhh \rightarrow -60\% \sim +100\%$       Direction of the correction is opposite to  $h\gamma\gamma$ .
- $hZZ, hWW \rightarrow \sim -1\%$        $O(1\%)$  deviation of  $hVV$  can be measured at the ILC.

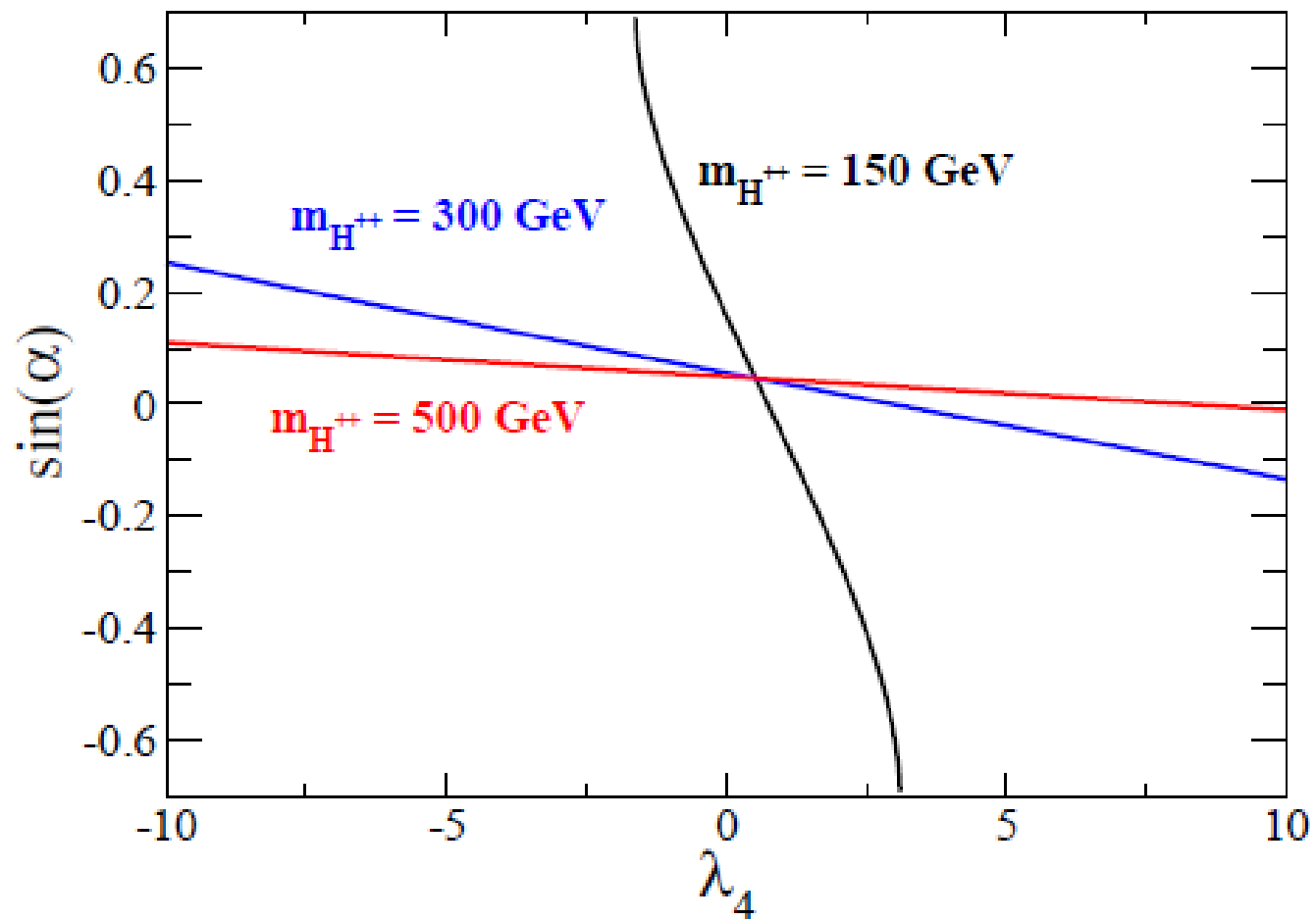
## ▶ **Direct way to test the HTM at colliders (Light triplet Higgs case):**

At the LHC,  $m_H > m_{H^+} > m_{H^{++}}$  with small  $v\Delta$  case is challenging to test the model.

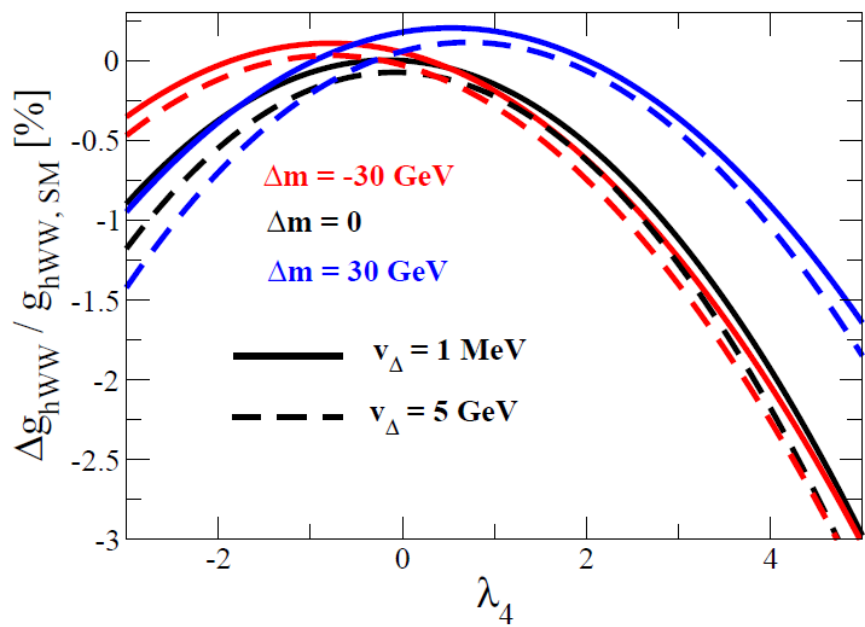
At the ILC, even if this scenario is realized, triplet-like Higgs bosons may be detectable by using the dijet energy and invariant mass distribution.

**ILC is necessary to test the HTM in both indirect way and direct way!!**

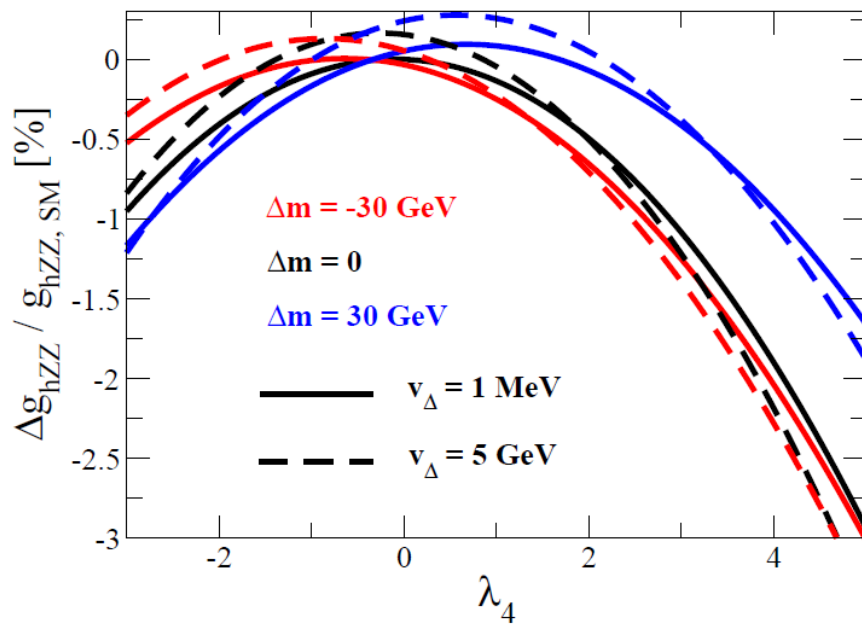
$v_\Delta = 5.69 \text{ GeV}, \Delta m = 0$



$m_{\text{lightest}} = 300 \text{ GeV}$

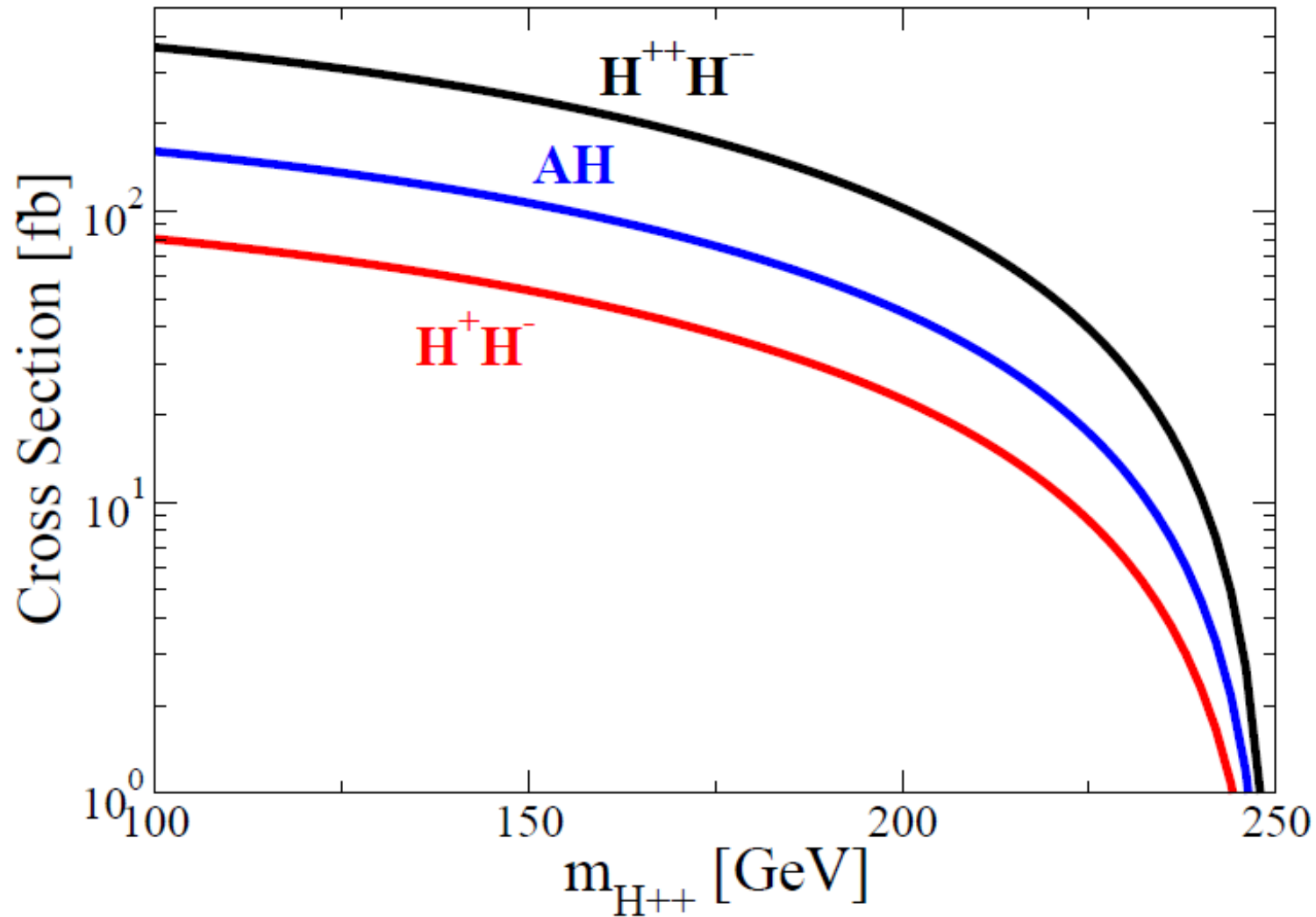


$m_{\text{lightest}} = 300 \text{ GeV}$



# Cross section

Root(s) = 500 GeV,  $\Delta m = 0$

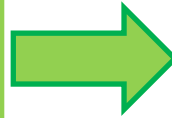


# Renormalization of the Higgs potential

Aoki, Kanemura, Kikuchi, KY, PLB714

8 parameters in the potential

$$\mu, m, M, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$$



8 physical parameters

$$v, m_{H^{++}}, m_{H^+}, m_A, m_h, m_H, \alpha, \beta' (v\Delta)$$

Counter terms

$$\delta v, \delta m_{H^{++}}^2, \delta m_{H^+}^2, \delta m_A^2, \delta m_h^2, \delta m_H^2, \delta\alpha, \delta\beta'$$

$$\text{Tadpole: } \delta T_\varphi, \delta T_\Delta,$$

$$\text{Wave function renormalization: } \delta Z_{H^{++}}, \delta Z_{H^+}, \delta Z_A, \delta Z_H, \delta Z_h$$

Reno. of EW parameters



$$\delta v$$

Vanishing 1-point function

$$\text{Diagram: } \text{circle} \text{---} = \text{circle with cross} \text{---} + \text{circle with 1PI} \text{---} = 0 \quad \Rightarrow \quad \delta T_\varphi, \delta T_\Delta$$

On-shell condition

$$\left. \text{Diagram: } \phi \text{---} \text{circle} \text{---} \phi \right|_{p^2 = \phi^2} = 0 \quad \Rightarrow \quad \delta m_{H^{++}}^2, \delta m_{H^+}^2, \delta m_A^2, \delta m_h^2, \delta m_H^2$$

$$\frac{d}{dp^2} \left. \text{Diagram: } \phi \text{---} \text{circle} \text{---} \phi \right|_{p^2 = \phi^2} = 0 \quad \Rightarrow \quad \delta Z_{H^{++}}, \delta Z_{H^+}, \delta Z_A, \delta Z_H, \delta Z_h$$

No-mixing condition

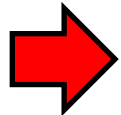
$$\left. \text{Diagram: } \phi \text{---} \text{circle} \text{---} \phi' \right|_{p^2 = \phi^2, \phi'^2} = 0 \quad \Rightarrow \quad \delta\alpha, \delta\beta'$$

where 2-point function is defined by

$$\text{Diagram: } \text{circle} \text{---} = \text{circle with cross} \text{---} + \text{circle with 1PI} \text{---}$$

# Constraints from EW precision data

There are 3 parameters:  $g$ ,  $g'$  and  $v$  in the kinetic term of Higgs fields.

 The electroweak observables are described by the 3 input parameters.

**We can choose  $\alpha_{em}$ ,  $m_W$  and  $m_Z$  as the 3 input parameters.**

The weak angle  $\sin^2\theta_W = s_W^2$  can be described in terms of the gauge boson masses.

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$$

The counter term of  $\delta s_W^2$  is derived as

$$\sim \delta\rho = \rho - 1 = \alpha_{em} T$$

$$\frac{\delta s_W^2}{s_W^2} = \frac{s_W^2}{c_W^2} \left[ \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right] = \frac{s_W^2}{c_W^2} \left[ \frac{\Pi^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi^{WW}(m_W^2)}{m_W^2} \right]$$

**The quantity  $\delta\rho$  (or  $T$ ) measures the violation of the custodial symmetry.**

# Constraints from EW precision data

There are **4** parameters (instead of 3 in the SM):  
 $g$ ,  $g'$ ,  $v$  and  $v_\Delta$  in the kinetic term of Higgs fields.

 The electroweak observables are described by the **4** input parameters.

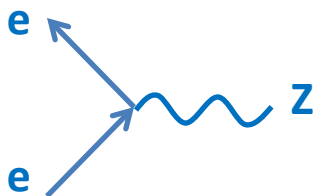
The weak angle  $\sin^2\theta_W = s_W^2$  cannot be given in terms of the gauge boson masses.

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$$

Scheme 1 *Blank, Hollik (1997)*

Input parameters:

$m_W$ ,  $m_Z$ ,  $\alpha_{em}$ ,  $s_W^2$



$$1 - 4\hat{s}_W^2(m_Z) = \frac{\text{Re}(v_e)}{\text{Re}(a_e)}$$

Scheme 2

Input parameters:

$m_W$ ,  $m_Z$ ,  $\alpha_{em}$ ,  $v_\Delta$

*Chankowski, Pokorski,  
Wagner (2007);  
Chen, Dawson, Jackson  
(2008)*

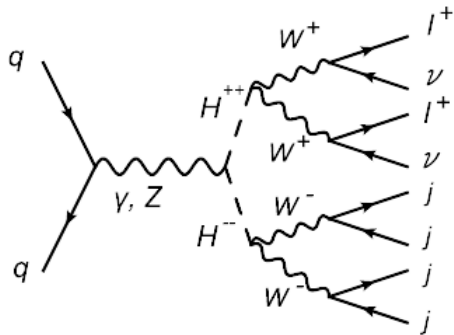




# Diboson decay scenario

Realizing  $v_{\Delta} > 0.1$  MeV with Case I

► Signal: Same-sign dilepton + Jets + Missing



When  $m_{H^{++}} = 100$  GeV,  $\sqrt{s} = 7$  TeV  
the signal cross section is  $\sim 3$  fb.

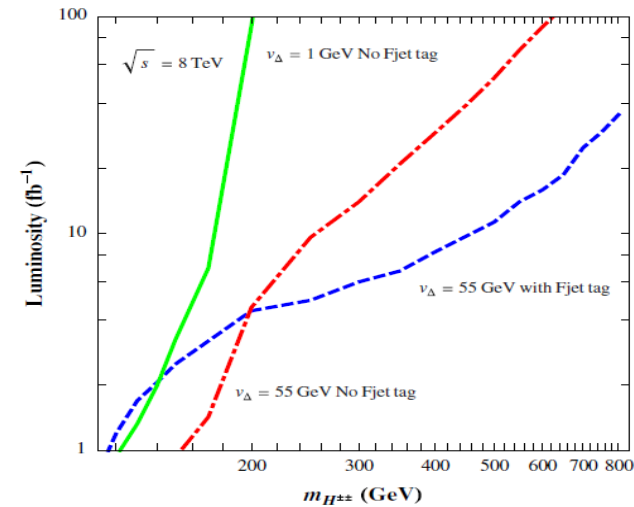
► Chiang, Nomura and Tsumura studied discovery potential for this scenario at the LHC.

► Data:  $L \sim 2 \text{ fb}^{-1}$ ,  $\sqrt{s} = 7$  TeV at ATLAS

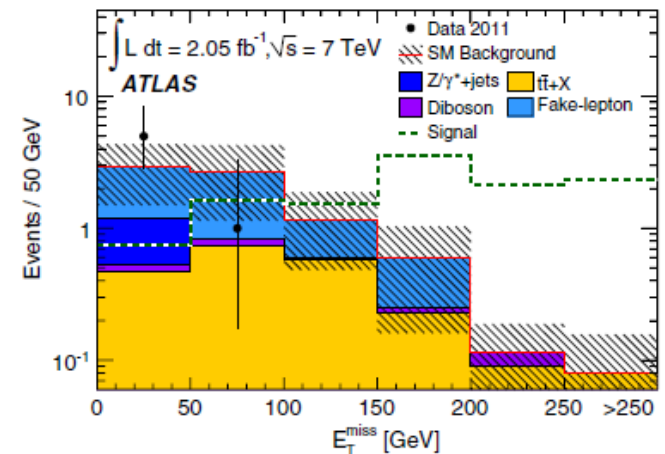
6 events have been discovered, which is consistent with the SM prediction.

Can we set a lower bound for the mass of  $H^{++}$   
in the case of  $H^{++}$  decays into diboson?

Chiang, Nomura, Tsumura, 2012

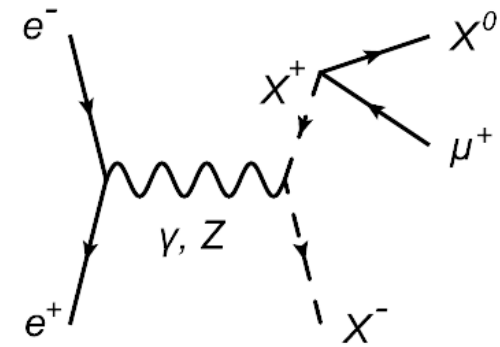
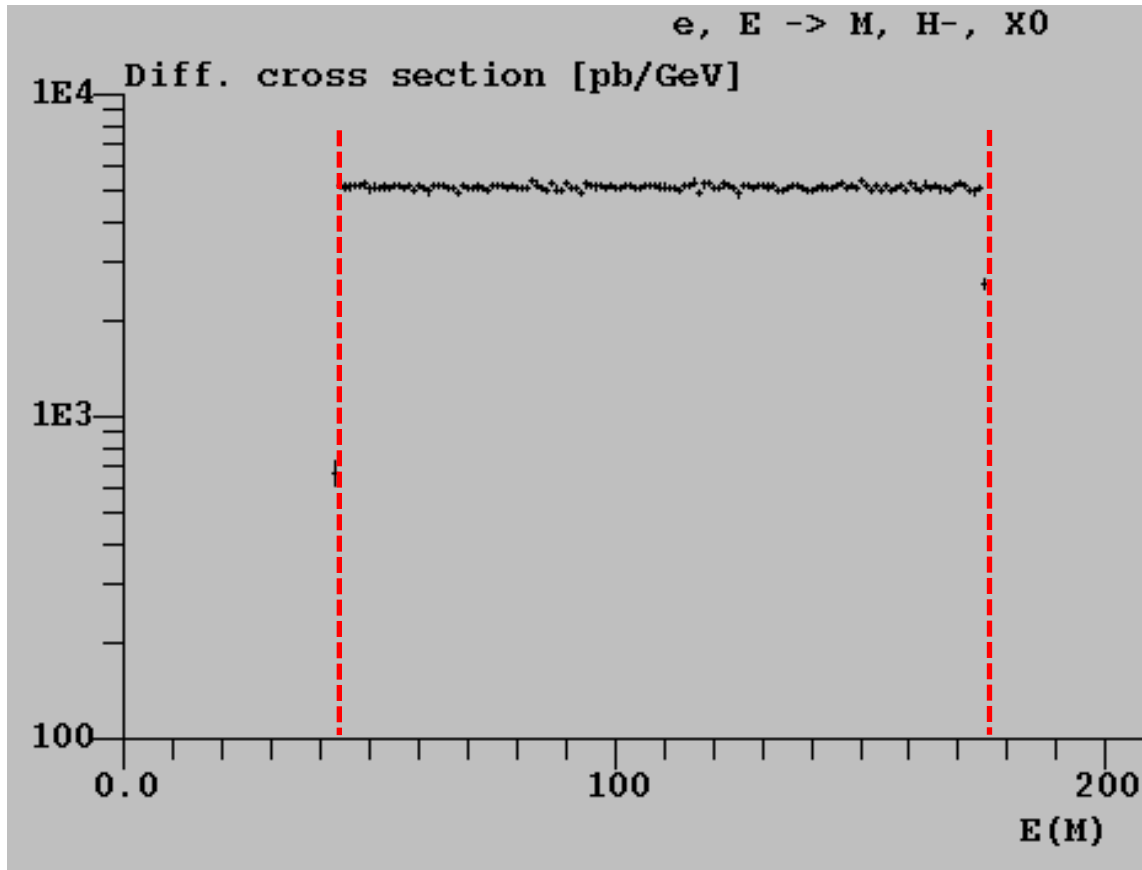


ATLAS, data 2011



# Mass reconstruction at ILC

Ex.  $m_{X^+} = 200 \text{ GeV}$ ,  $m_{X^0} = 70 \text{ GeV}$

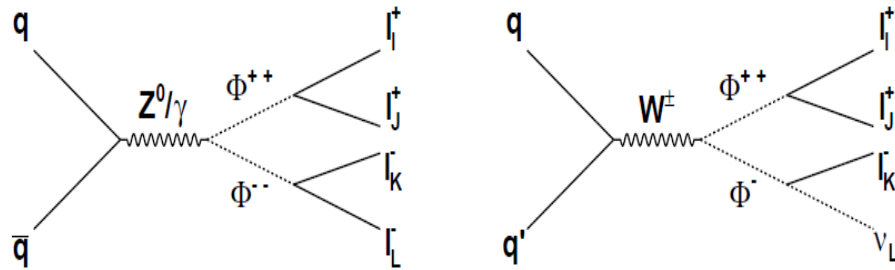


$$m_{X^0}^2 = \frac{s}{4} \left[ 1 - \left( \frac{E_{\mu}^{\max} - E_{\mu}^{\min}}{E_{\mu}^{\max} + E_{\mu}^{\min}} \right)^2 \right] - \frac{2\sqrt{s}E_{\mu}^{\max}E_{\mu}^{\min}}{E_{\mu}^{\max} + E_{\mu}^{\min}}$$

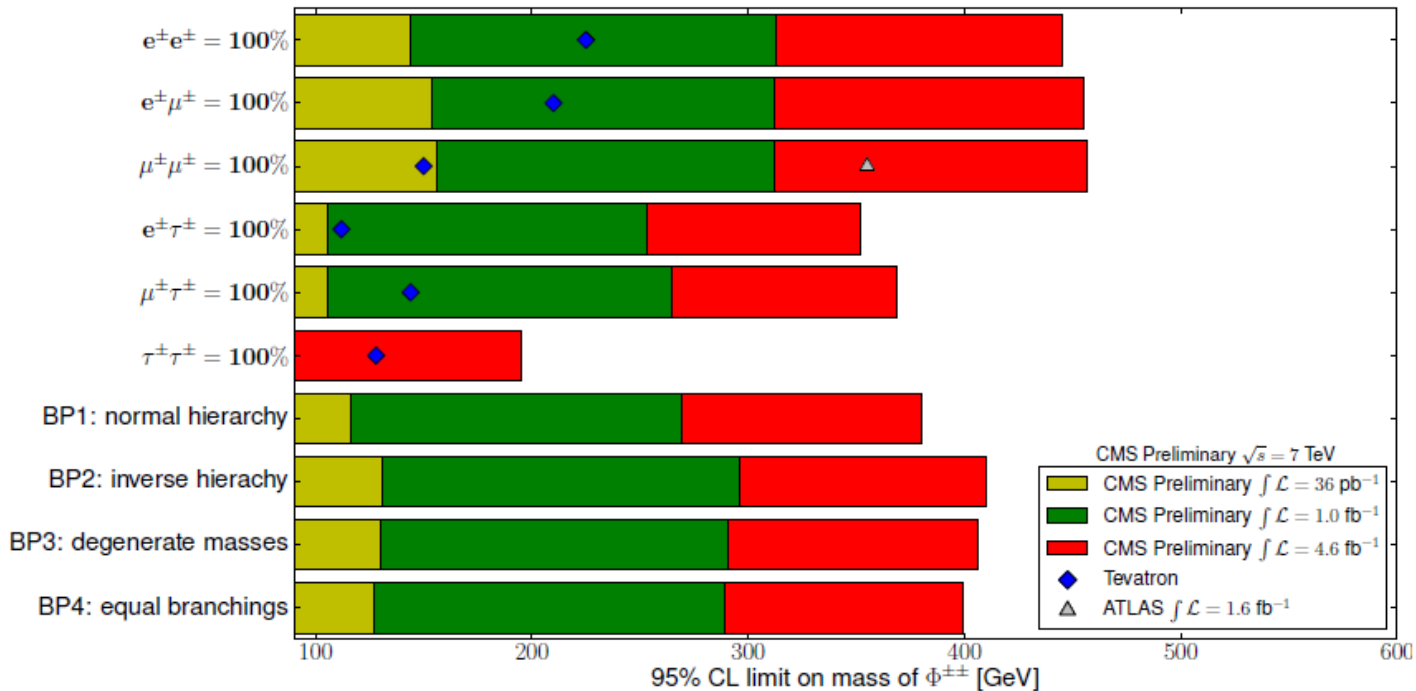
$$m_{X^+}^2 = \frac{s}{4} \left[ 1 - \left( \frac{E_{\mu}^{\max} - E_{\mu}^{\min}}{E_{\mu}^{\max} + E_{\mu}^{\min}} \right)^2 \right]$$

# Experimental bounds (Direct)

Assuming 100% same-sign leptonic decay of the doubly-charged Higgs boson



*CMS PAS HIG-12-005, 7TeV, 4.6 fb<sup>-1</sup>*

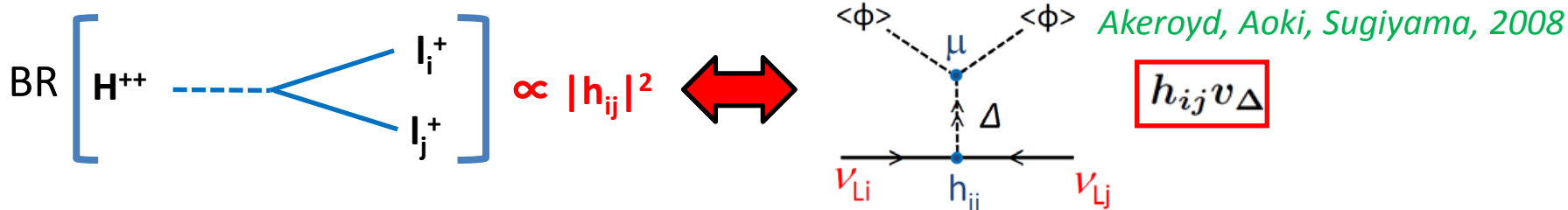


**$m_{H^{++}} \gtrsim 400 \text{ GeV}$**

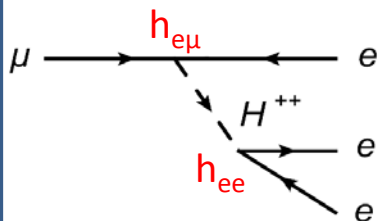
# Dilepton decay scenario

Realizing  $v_\Delta < 0.1$  MeV with Case I.

By measuring the pattern of leptonic decay, we can directly test the neutrino mass matrix.



Lepton flavor violation ( $\mu \rightarrow 3e$ ,  $\mu$ -e conversion, etc)



*Chun, Lee, Park, 2003; Kakizaki, Ogura, Shima, 2003; Abada, Biggio, Bonnet, Gavela, Hambey, 2007, ...*

$$BR(\mu \rightarrow 3e) \simeq \frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} \simeq \frac{|h_{ee}h_{e\mu}|^2}{4m_{H^{++}}^4 G_F^2} < 10^{-12} \Rightarrow |h_{ee}h_{e\mu}| \lesssim 10^{-5} \times \left(\frac{m_{H^{++}}}{1 \text{ TeV}}\right)^2$$

LHC phenomenology

-4 lepton, 3 lepton signature *Perez, Han, Huang, Li, 2008; Akeroyd, Chiang, Gaur, 2010, ...*

-Using tau polarization *Sugiyama, Tsumura, Yokoya, arXiv:1207.0179*

Discrimination of chiral structure of the Yukawa coupling

-Same sign tetra-lepton signature *Chun, Sharma, arXiv:1206.6278*

# Experimental bounds (Indirect)

Gauge boson self-energies

6 nondec. d.o.f.

Model w/  $\rho_{\text{tree}} = 1$

$$\text{W self-energy diagram} = M_{\text{New}}^2 + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots$$

$$\text{Z self-energy diagram} = M_{\text{New}}^2 + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots$$

$$\text{Y-Z mixing self-energy diagram} = p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots$$

$$\text{Y-Y self-energy diagram} = p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots$$

3 input parameters

→ 6-3 (Ren. conditions) = 3 nondec. d.o.f.

= S, T, U *Peskin, Takeuchi, PRL65, (1990)*

$$\delta\rho \simeq \frac{1}{16\pi^2} \frac{(m_t - m_b)^2}{m_W^2} + \dots$$

Model w/o  $\rho_{\text{tree}} = 1$

4 input parameters *Blank, Hollik, NPB514, (1998)*

→ 6-4 (Ren. conditions) = 2 nondec. d.o.f.

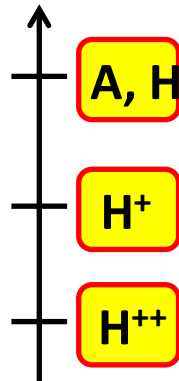
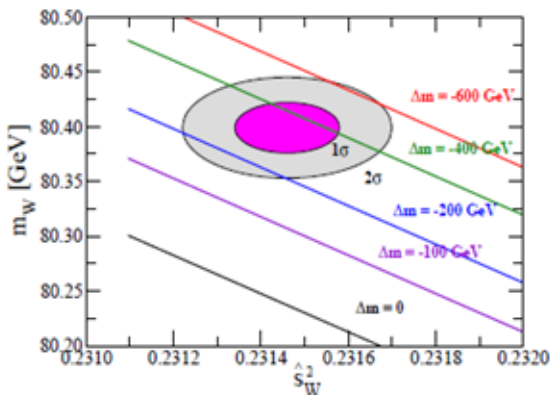
$$\delta\rho \simeq \frac{1}{16\pi^2} \ln \frac{m_t}{m_b} + \dots$$

1-loop corrected W mass

*Kanemura, KY, PRD85 (2012)*

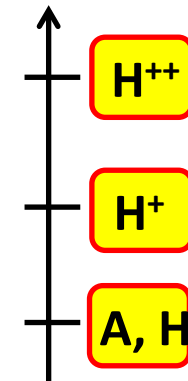
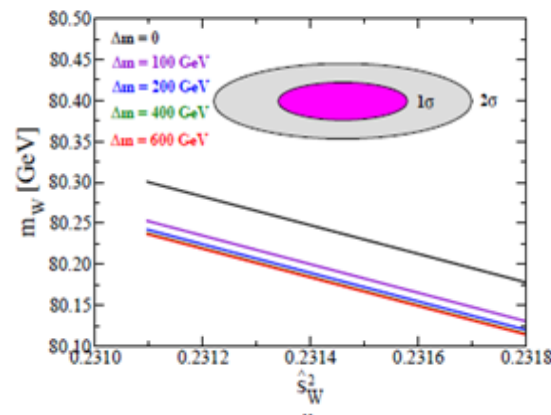
Case I:  $m_{H^{\pm\pm}} = 150 \text{ GeV}$ ,  $m_h = 125 \text{ GeV}$ ,  $\tan\alpha = 0$

Case I



Case II:  $m_A = 150 \text{ GeV}$ ,  $m_h = 125 \text{ GeV}$ ,  $\tan\alpha = 0$

Case II



# Radiative corrections to the mass spectrum

Aoki, Kanemura, Kikuchi, KY, arXiv: 1204.1951

Ratio of the squared mass difference  $R$

$$R \equiv \frac{m_{H^{++}}^2 - m_{H^+}^2}{m_{H^+}^2 - m_A^2}$$

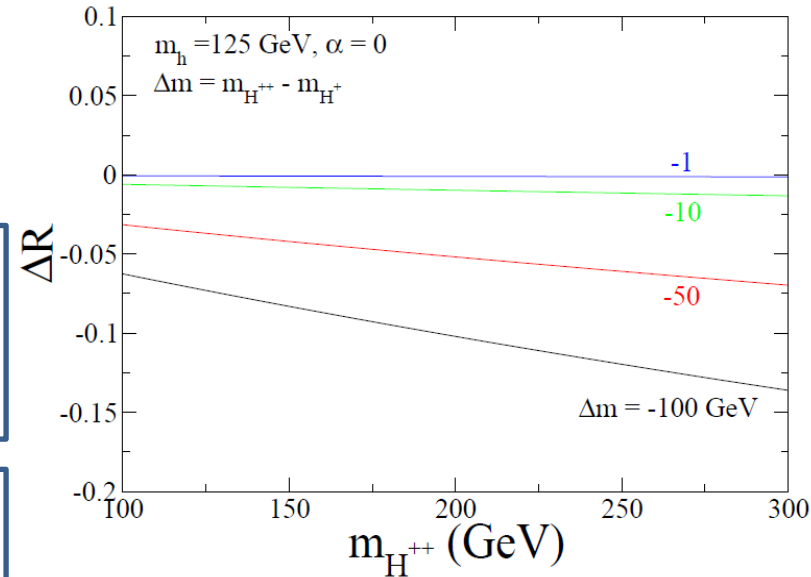
Tree level:  $R^{\text{tree}} = 1 + \left( \frac{v_\Delta^2}{v^2} \right) \simeq 1$   
Less than  $10^{-3}$

Loop level:  $R^{\text{loop}} = 1 + \underbrace{\Delta R}_{\text{Loop correction}} + \left( \frac{v_\Delta^2}{v^2} \right)$

$$\Delta R = \frac{\Pi_{H^{++}H^{--}}^{1\text{PI}}[m_{H^{++}}^2] - 2\Pi_{H^+H^-}^{1\text{PI}}[m_{H^+}^2] + \Pi_{AA}^{1\text{PI}}[(m_A^2)_{\text{tree}}]}{m_{H^{++}}^2 - m_{H^+}^2}$$

$(m_A^2)_{\text{tree}}$  is determined by  $m_{H^{++}}^2$  and  $m_{H^+}^2$ :  $(m_A^2)_{\text{tree}} = 2m_{H^+}^2 - m_{H^{++}}^2$

**Case I**



In favored parameter sets by EW precision data:  $m_{H^{++}} = O(100)\text{GeV}$ ,  
 $|\Delta m| \sim 100\text{GeV}$ ,  $\Delta R$  can be as large as  **$O(10)\%$** .

# Custodial Symmetry

The SM Lagrangian can be written by the  $2 \times 2$  matrix form of the Higgs doublet:

$$\Sigma = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

## ★ Kinetic term

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \text{Tr} \left[ (\tilde{D}_\mu \Sigma)^\dagger (\tilde{D}^\mu \Sigma) \right]$$

$$\tilde{D}_\mu \Sigma = \partial_\mu \Sigma + i \frac{g}{2} \tau \cdot W_\mu \Sigma - i \frac{g'}{2} B_\mu \Sigma \tau_3$$

## ★ Higgs potential

$$V = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$= -\frac{\mu^2}{2} \text{Tr}(\Sigma^\dagger \Sigma) + \frac{\lambda}{4} \text{Tr}(\Sigma^\dagger \Sigma)^2$$

## ★ Yukawa interaction (top-bottom sector)

$$\mathcal{L}_Y = y_t \bar{Q}_L \tilde{\Phi} t_R + y_b \bar{Q}_L \Phi b_R + \text{h.c.}$$

$$= y_V \bar{Q}_L \Sigma Q_R + y_A \bar{Q}_L \Sigma \tau_3 Q_R + \text{h.c.}$$

When we take  $g'$  and  $y_A \rightarrow 0$ ,

Lagrangian is invariant under  $SU(2)_L \times SU(2)_R$

$$\Sigma \rightarrow U_L \Sigma U_R^\dagger, \quad Q_{L,R} \rightarrow U_{L,R} Q_{L,R}$$

After the Higgs field gets the VEV:

$$\Sigma \rightarrow \langle \Sigma \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \langle \Delta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

this symmetry is reduced to

$SU(2)_L = SU(2)_R = SU(2)_V$  (custodial symmetry).

**$SU(2)_V$  breaking by  $g'$  is included in the definition of the rho parameter, while that by  $y_A$  is not.**

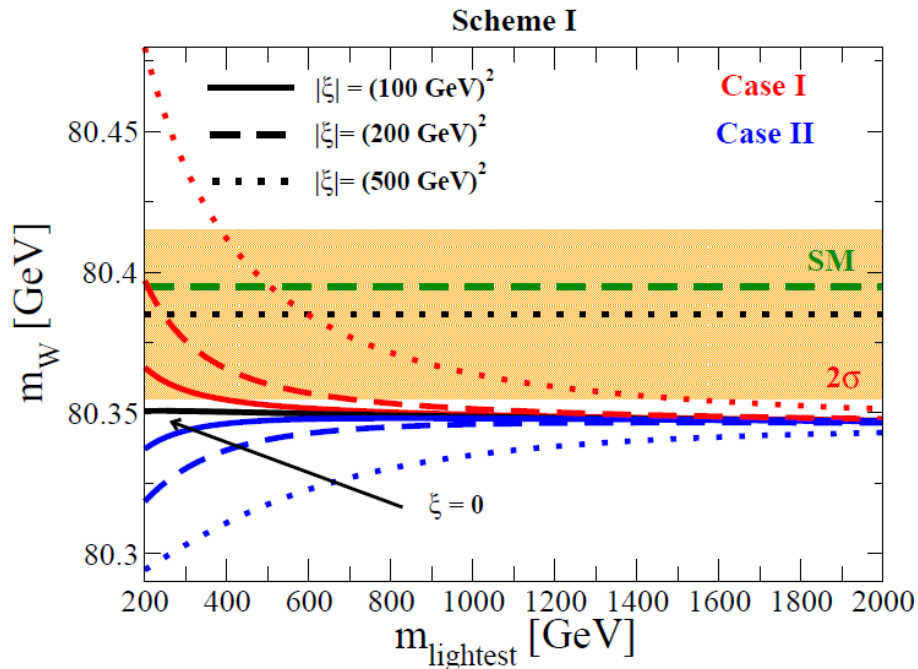
**There is a significant contribution to the deviation of rho = 1 from the top-bottom sector by the loop effect.**



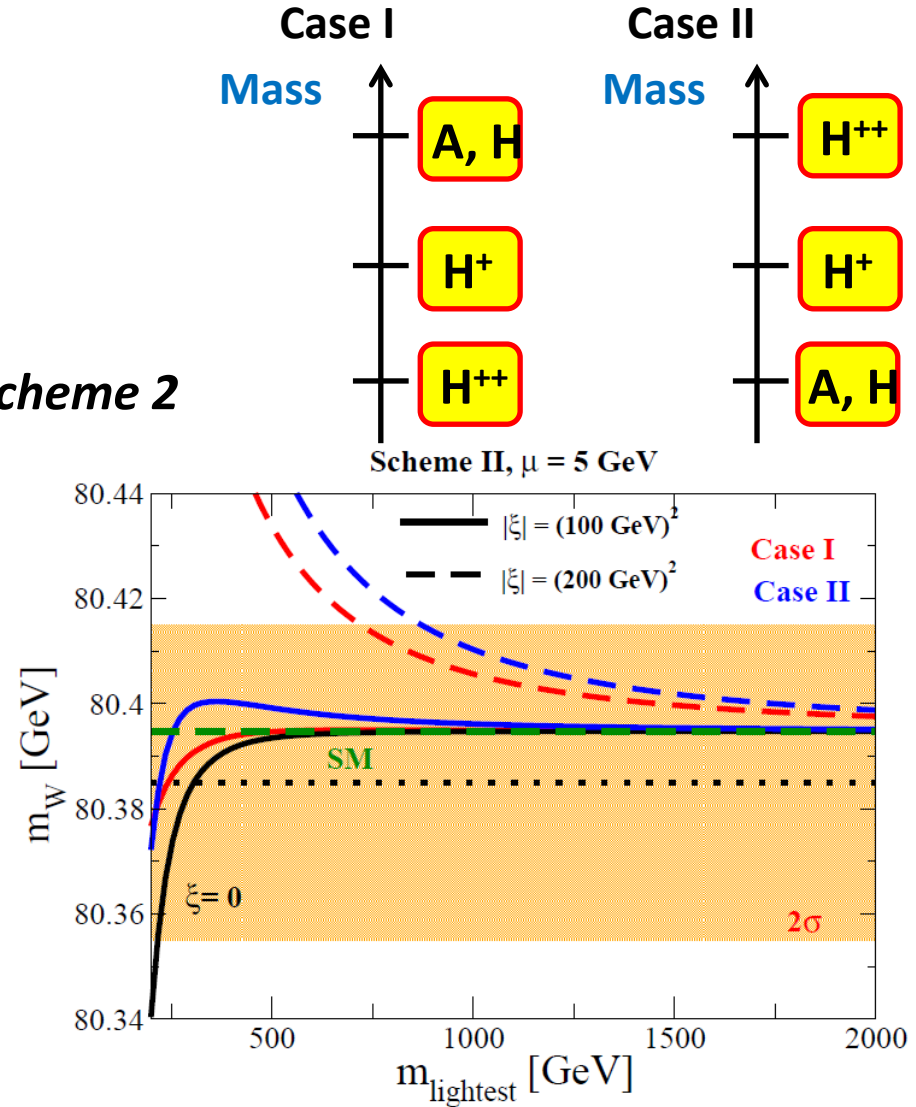
# 1-loop corrected W mass

$$\xi = m_{H^{++}}^2 - m_{H^+}^2$$

*Scheme 1*



*Scheme 2*



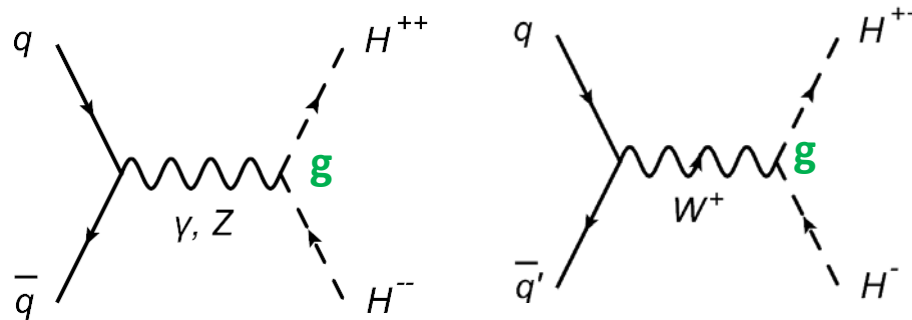
In Scheme II, decoupling limit can be taken in the heavy mass limit.

# Production mechanisms at LHC

## Main production process

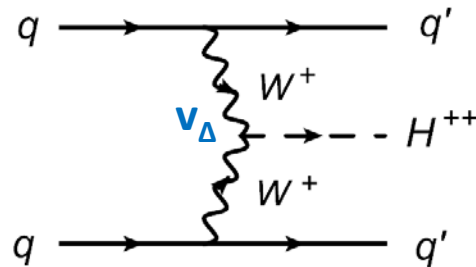
### Drell-Yan

- depends on the gauge coupling



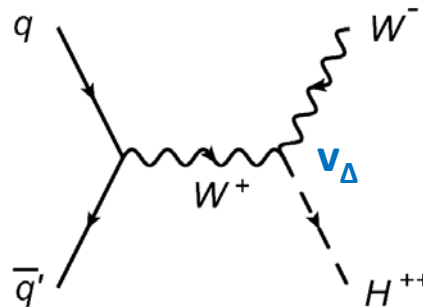
### Vector Boson Fusion

- depends on  $v_\Delta \rightarrow$  **Suppressed**

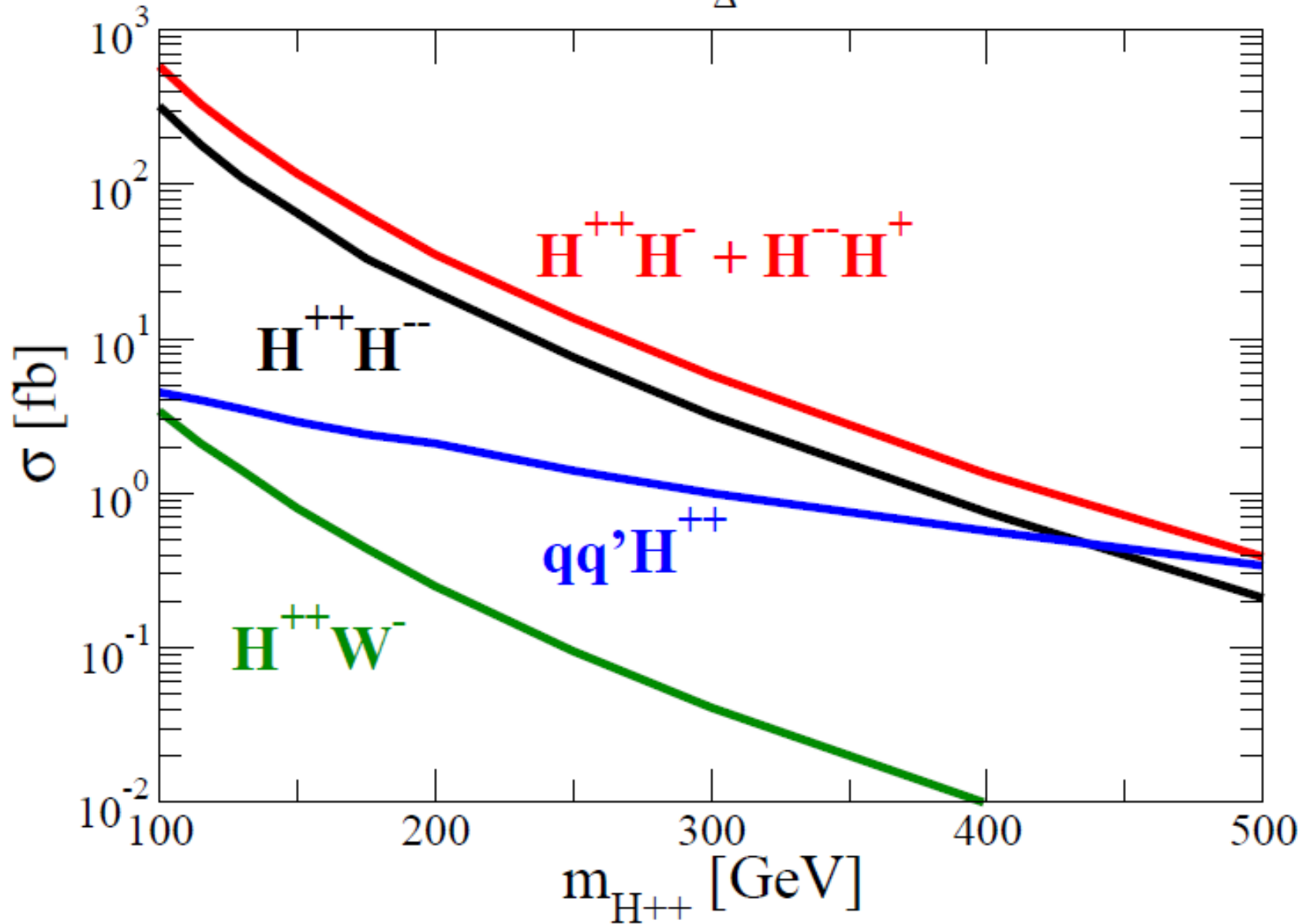


### W associate

- depends on  $v_\Delta \rightarrow$  **Suppressed**



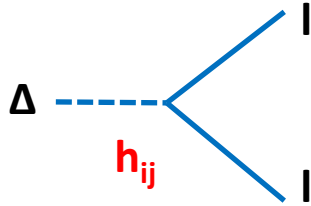
Root(s) = 7 TeV,  $v_{\Delta} = 8$  GeV,  $\Delta m = 0$



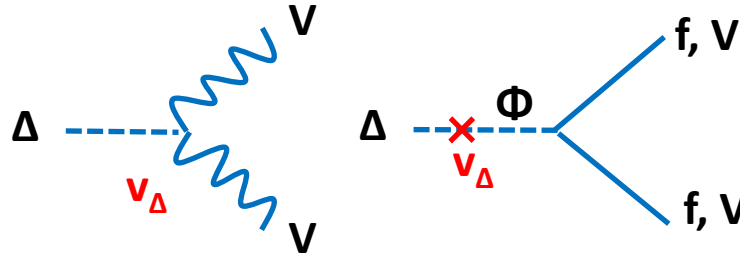
# Decay

The decay of  $\Delta$ -like Higgs bosons can be classified into 3 modes.

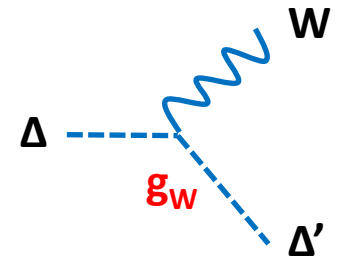
1. Decay via  $h_{ij}$



2. Decay via  $v_\Delta$

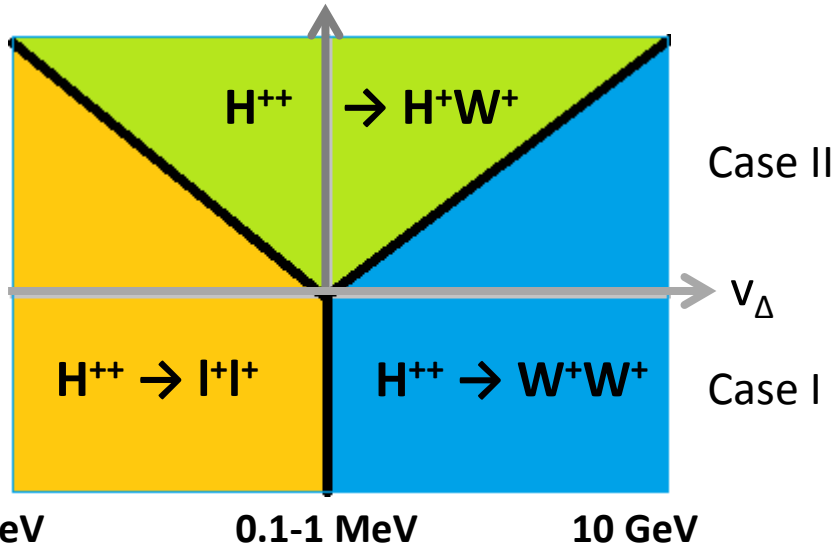


3. Decay via  $g$



Decay of  $H^{++}$

$\Delta m (= m_{H^{++}} - m_{H^+})$



Decay modes of 1 and 2 are related to each other by the relation:

$$(m_\nu)_{ij} = h_{ij} v_\Delta$$

Decay of the triplet like scalar bosons strongly depend on  $v_\Delta$  and  $\Delta m (\equiv m_{H^{++}} - m_{H^+})$ .