Turn-by-turn measurements at the SLS

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Contents

- Refined Fourier analysis of turn by turn transverse position data measurements can determine several beam properties.e.g. transverse tunes, optics functions, phases, chromatic properties, coupling
- Most common used Fast Fourier Transform (FFT) algorithm is powerful however with a precision of $1/T$. To get fundamental frequencies of a system (tunes), very long integration, many data or turns are needed

- Jacques Laskar introduced a method of determining the fundamental frequencies of a Hamiltonian system e.g. the motion of a particle in phase space⟹Implemented by the Numerical Analysis of the Fundamental Frequency(NAFF) algorithm (see J.Laskar, Frequency analysis for multi-dimensional systems.Global dynamics and diffusion)
- Advantage of this method⟹If $f(t)$ is a quasiperiodic function and it is given numerically, it is possible to recover an approximation of $f(t)$ over a time span [-T T] with a precision of $\frac{1}{T^4}$

- In the case of a storage ring like SLS, f(t) can be a measured signal representing beam position data
- Expanding in Fourier series:

$$f(t) = \sum_{k=1}^{\infty} a_k e^{i\omega_k t} \tag{1}$$

where ω_k are the unknown frequencies of the system and a_k the amplitudes

- The NAFF algorithm provides an approximation $f(t)' = \sum_{k=1}^{N} a'_k e^{i\omega'_k t}$ and the amplitudes a'_k and fundamental frequencies ω'_k can be found with an iterative method

- ω_1' can be found by estimating the maximum value of the scalar product $\phi(\sigma) = < f(t), e^{i\sigma t} >$ where the scalar product is defined as,

$$< f(t), g(t) >= \frac{1}{2T} \int_{-T}^{T} f(t)g(t)dt \tag{2}$$

- After ω_1' is found, a_1' can be found by orthogonal projection on $e^{i\omega_1' t}$
- The process is repeated again for the function $f_1(t) = f(t) - a_1' e^{i\omega_1' t}$
- Its convergence is as fast as $\frac{1}{T^4}$ in comparison with standard FFT $\frac{1}{T}$

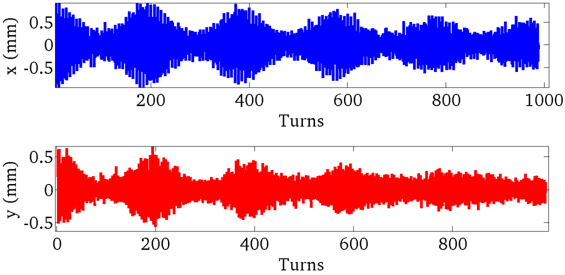
The SLS is a third generation light source which provides photon beams of high brilliance to 20 beam lines.

Table: Main Parameter of the SLS storage ring.

Parameter	Value
Circuference	288 m
Beam energy	2.4 GeV
Lattice	12 TBA
No. of BPMs	73
Betatron tunes (H/V)	20.44 / 8.74

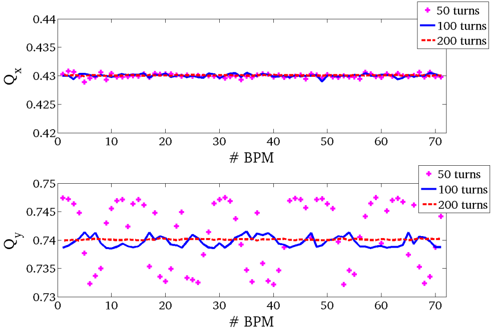
- In order to estimate the transverse tunes and other ring parameters, the bunches are kicked transversally by a kicker magnet to induce coherent betatron oscillations
- The resulting transverse position data are analysed with the NAFF method

- Some sample TBT data:



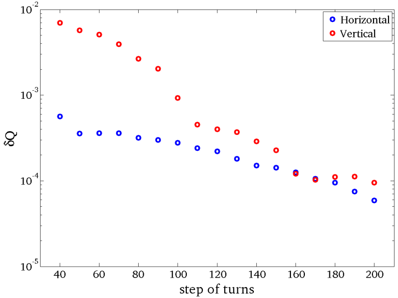
- BPMs 16 and 43 were noisy and they had to be ignored leaving a total of 71 BPMs for the analysis
- In the presence of chromaticity and amplitude dependent tune-shift, the BPM signal decoheres leaving a limited number of turns to be analysed

- Measurement of the fractional part of the tune⟹frequency analysis of the signal of each BPM indepedently



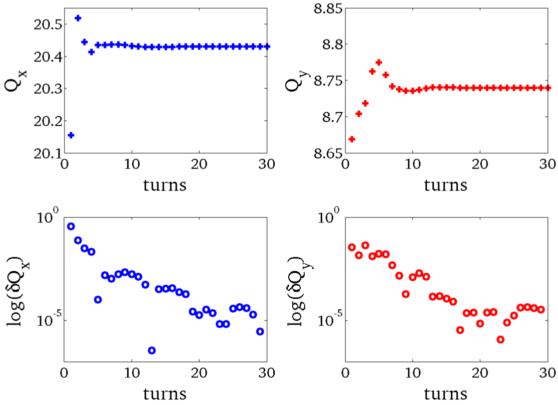
- The precision of the tune evaluation grows with the number of available turns
- 50 Turns⟹Large tune estimation fluctuation over the BPMs
- 200 Turns⟹Minimum tune estimation fluctuation especially in the vertical plane

Information about the accuracy of the tune estimation can be extracted from the standard deviation of the measured tune over all the BPMs



- Horizontal plane$\Longrightarrow$less than 10^{-3} from only 40 turns and gets even less towards 200 turns
- Vertical plane$\Longrightarrow$less than 10^{-3} from 100 turns.

- Mixing the BPM data simultaneously for every turn $\Longrightarrow$ Fast and accurate evaluation of the tune including its integer part provided that there are more BPMs than the tune integer units to prevent aliasing
- Plotting the tune estimation, and its error in logarithmic scale, will show how fast and how accurately the method works
- Reminder $\Longrightarrow$ SLS ring tune values: Q_x=20.44, Q_y=8.74



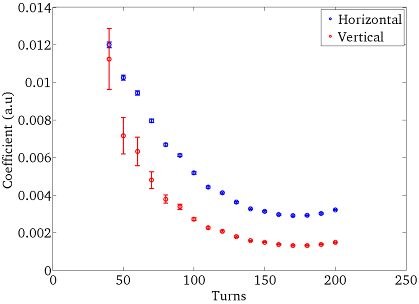
- Tune calculation almost from 10 turns for the horizontal plane. For the vertical 20 turns are needed$\Longrightarrow$Speed
- Difference of 10^{-5} and less between consecutive tune values from 20 turns$\Longrightarrow$Accuracy

- Fourier component amplitude associated to the main tunes is:

$$A_z^i = c_z \cdot S_i \sqrt{\beta_z^i} \tag{3}$$

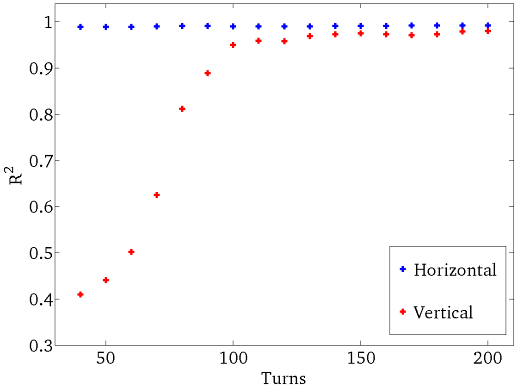
- Assuming well calibrated BPMs ($S_i = 1$), the beta function can be estimated by a linear fit of $(A_z^i)^2$ to the linear machine model

Representing the coefficient as a function of the time window used:



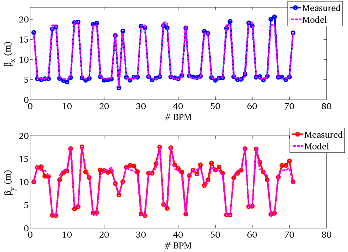
- Errorbars$\Longrightarrow$standard deviation of the least square fit$\Longrightarrow$Fit works even with 40 turns for Horizontal plane but for the Vertical plane at least 100 turns are needed to be less than 10^{-4}
- Increase of the step of turns converges to a single value of c_z with the least offset

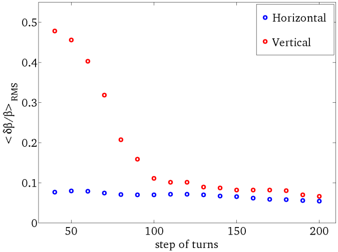
The precision of the linear fit can be evaluated from the correlation coefficient R^2



- Horizontal plane: R^2 is very close to 1 already for 40 turns
- Vertical Plane: At least 100 turns are needed

- The agreement between the measured beta functions and the ideal model values is excellent



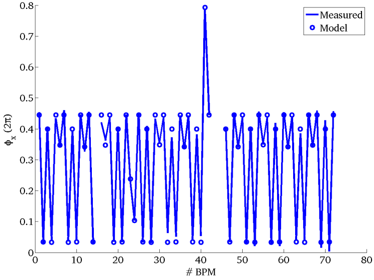


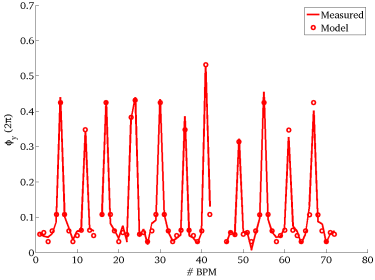
- From the plot of $< \frac{\delta\beta}{\beta} >_{RMS}$ $(Turns)$ $\Longrightarrow$Very good accuracy from less than 50 turns for the Horizontal plane, 150 turns are needed for the vertical plane$\Longrightarrow$ SLS operation is close to the ideal one

The phases of the main spectral lines for each BPM can also be used for evaluating the beta function difference between the machine and the model, using the measured phase advances between 3 concecutive BPMs(see P.Castro-Garcia Doctoral Thesis)

$$\begin{aligned}
\tilde{\beta}_1 &= \beta_1 \frac{\cot\tilde{\phi}_{12} - \cot\tilde{\phi}_{13}}{\cot\phi_{12} - \cot\phi_{13}} \\
\tilde{\beta}_2 &= \beta_2 \frac{\cot\tilde{\phi}_{12} - \cot\tilde{\phi}_{23}}{\cot\phi_{12} - \cot\phi_{23}} \\
\tilde{\beta}_3 &= \beta_3 \frac{\cot\tilde{\phi}_{23} - \cot\tilde{\phi}_{13}}{\cot\phi_{23} - \cot\phi_{13}}
\end{aligned} \tag{4}$$

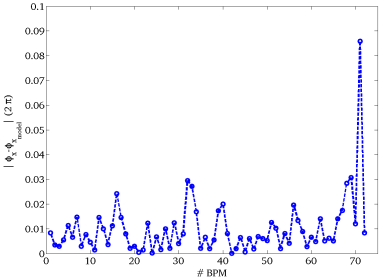
Before proceeding to the calculations⟹A comparison between the measured phase advances and the model ones would give a first impression of the accuracy of the method in the SLS case

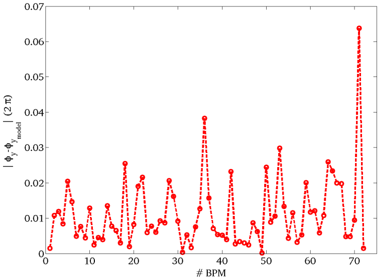




- Some discrepancies do exist in both planes

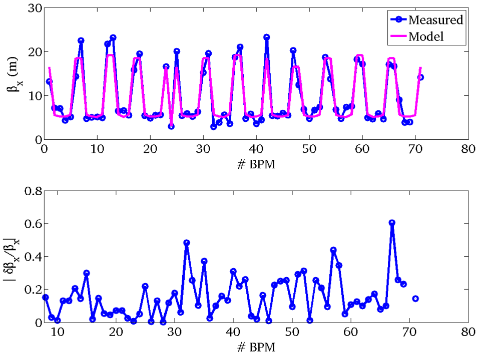
Plotting $|\phi - \phi_{model}|$ would give the size of these discrepancies





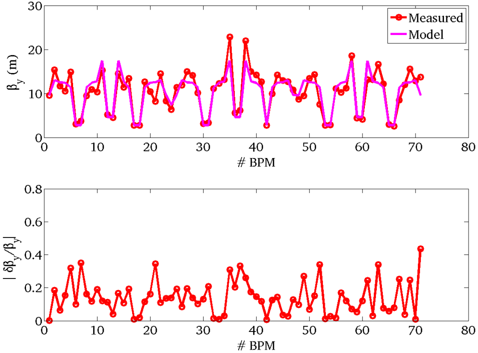
- The difference is below $4 \cdot 10^{-2}$ for most of BPMs in both planes$\Longrightarrow$however relatively large difference for $\phi_{71 \longrightarrow 1}$
- At first glance the method can be applied for the SLS case since the measured phase advances are relatively close to the model ones

For this method three beta function estimates per location can be obtained allowing for some statistics



- The relative beta difference between machine and model shown in the last plot implies that the accuracy is not as good as for the other method or some misinterpretation exists.

The same for the vertical plane:



- The small difference between measured and model phase advances in both planes cannot explain these discrepancies$\Longrightarrow$ Currently under investigation

- NAFF algorithm is a powerful refined Fourier analysis tool which reduces the computation time and reveals many details of the beam dynamics
- The measured fractional tune was measured quite accurately in around 200 turns using the traditional method
- Another method which uses combined BPM data in every turn was demonstrated. The measurement of the tune was possible in around 10 turns
- The amplitude and phase of the fundamental spectral line for each BPM were used to estimate beta functions revealing the accuracy of NAFF once again. The first method showed excellent accuracy but the second one did not and it must be further investigated for the SLS case

There are many open subjects under investigation using the capabilities of the NAFF algorithm:

- Chromaticity and energy deviation measurement from the Fourier spectrum, dispersion relation to the amplitude of the main spectral lines
- Signs of synchrobetatron coupling due to the presence of tune shift modulation as it was shown
- Possibility of estimating the beta functions by only using the measured phase advances

Thank You!

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