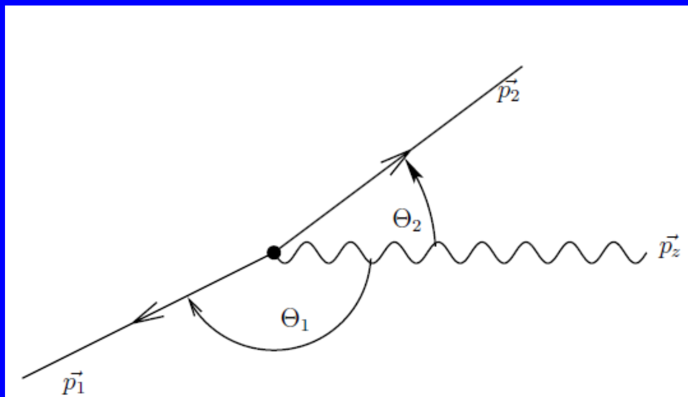
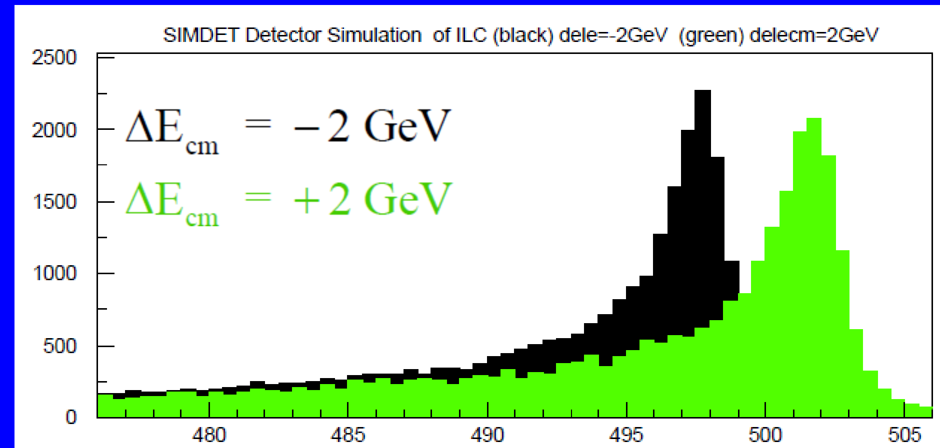
Investigating In-Situ $\sqrt{s}$ Determination with $\mu\mu(\gamma)$

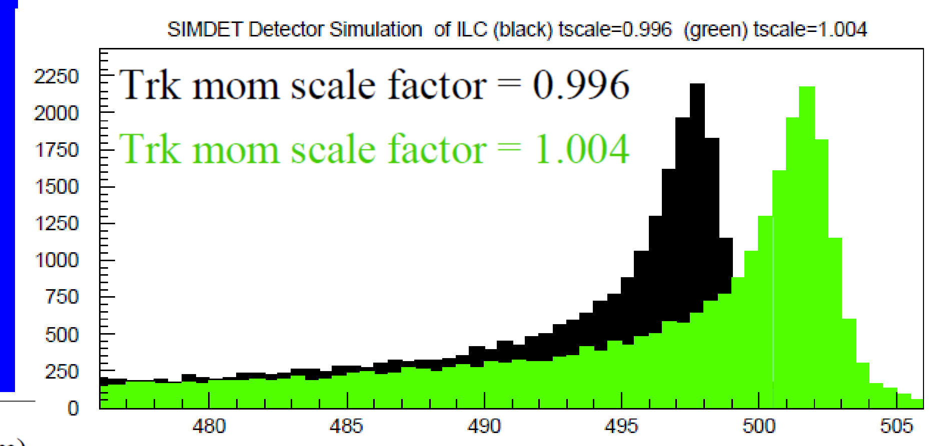
- ILC physics capabilities will benefit from a well understood centre-of-mass energy
  - Preferably determined from collision events.
- Measure precisely W, top, Higgs masses. (and Z ?)
- Two methods using $\mu\ \mu\ (\gamma)$ events have been discussed:
  - Method A: Angle-Based Measurement
  - Method P: Momentum-Based Measurement

Graham W. Wilson, Univ of Kansas,
ECFA LC2013, Hamburg, May 28th 2013.

Using $Z\gamma \to \mu\mu\gamma$ for $\sqrt{s}$ determination







Two methods:

A) Use angles only, measure $m_{12}/\sqrt{s}$.
Use known m_Z to reconstruct $\sqrt{s}$.

P) Use muon momenta.
Measure $E_1 + E_2 + p_{12}$.

Tim Barklow study. (assume dL/dx_1dx_2 known)

ECM = 350 GeV 100 fb^{-1}

measured var	fit var	ΔE_{cm} (GeV)	$\frac{\Delta E_{cm}}{E_{cm}}$ (ppm)
$E_{Z\gamma}$ using angles only	E_{cm}	0.0425	121
$E_{Z\gamma}$ using momenta & angles	E_{cm}	0.0035	10
$E_{Z\gamma}$, M_Z using momenta & angles	E_{cm} & t	0.0045	13
$E_{Z\gamma}$ using momenta & angles	E_{cm} & t	0.0048	14

With detectors designed for 0.14% $\Delta p_T/p_T$ at 45 GeV, it is feasible to improve by an order of magnitude over the Γ_Z dominated method.
May also scale better with $\sqrt{s}$?

Method A: Angles

Hinze & Moenig

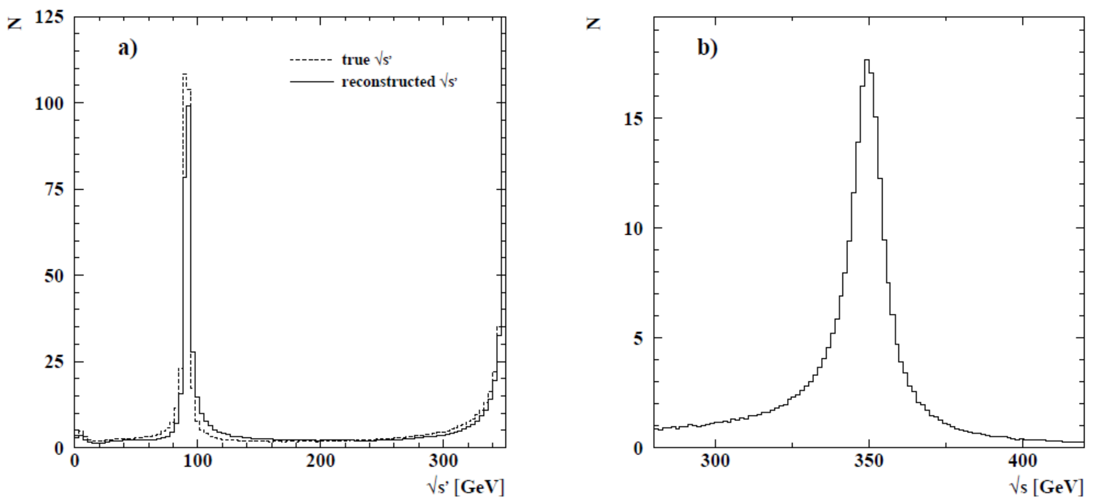


Figure 2: True and reconstructed $\sqrt{s'}$ (a) and reconstructed $\sqrt{s}$ for $e^+e^- \to Z\gamma \to \mu^+\mu^-\gamma$ at $\sqrt{s} = 350\,\mathrm{GeV}$

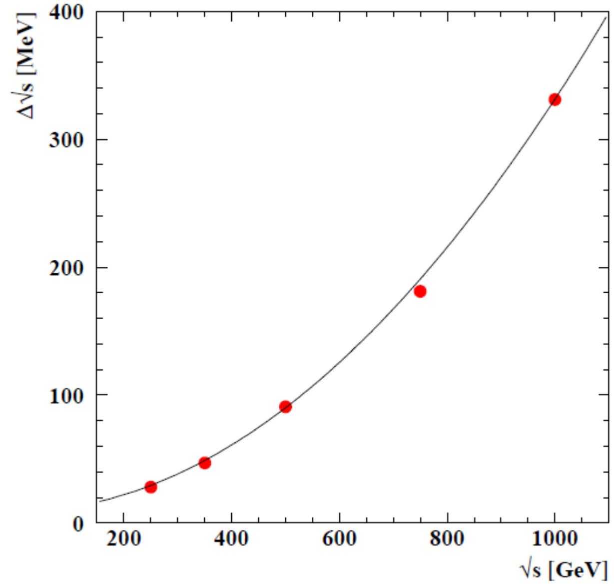


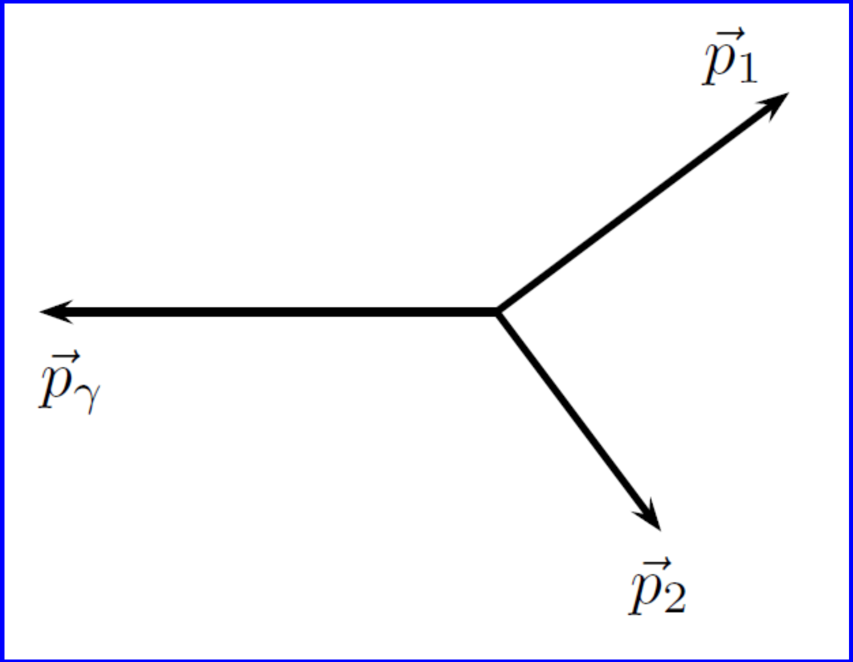
Figure 3: Energy dependence of $\Delta\sqrt{s}$ for $\mathcal{L} = 100\ \mathrm{fb}^{-1}$.

$$\sqrt{s} = m_Z \sqrt{\frac{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1+\theta_2)}{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1+\theta_2)}}$$

1. Statistical error per event of order $\Gamma/M = 2.7\%$
2. Error degrades fast with $\sqrt{s}$.

(Note. At 161 GeV my error estimate (ee,μμ) on $\sqrt{s}$ is 5 MeV: 31 ppm)

Method P: Muon Momenta



Under the assumption of a massless photonic system balancing the measured di-muon, the momentum (and energy) of this photonic system is given simply by the momentum of the di-muon system.

So the center-of-mass energy can be estimated from the sum of the energies of the two muons and the inferred photonic energy.

$(\sqrt{s})_P = E_1 + E_2 + |\, \mathbf{p_1} + \mathbf{p}_2 \,|$

In the specific case, where the photonic system has zero p_T, the expression is particularly straightforward. It is well approximated by

$$\sqrt{s}_P = p_T \left(\frac{1+\cos\theta_1}{\sin\theta_1} + \frac{1+\cos\theta_2}{\sin\theta_2} \right)$$

where p_T is the p_T of each muon. Assuming excellent resolution on angles, the resolution on $(\sqrt{s})_P$ is determined by the θ dependent p_T resolution.

Method can also use non radiative return events with $m_{12} \gg m_Z$

Very simplified 3-body MC with $m_{12} \approx m_Z$ to show the potential)

Method A (Angles)

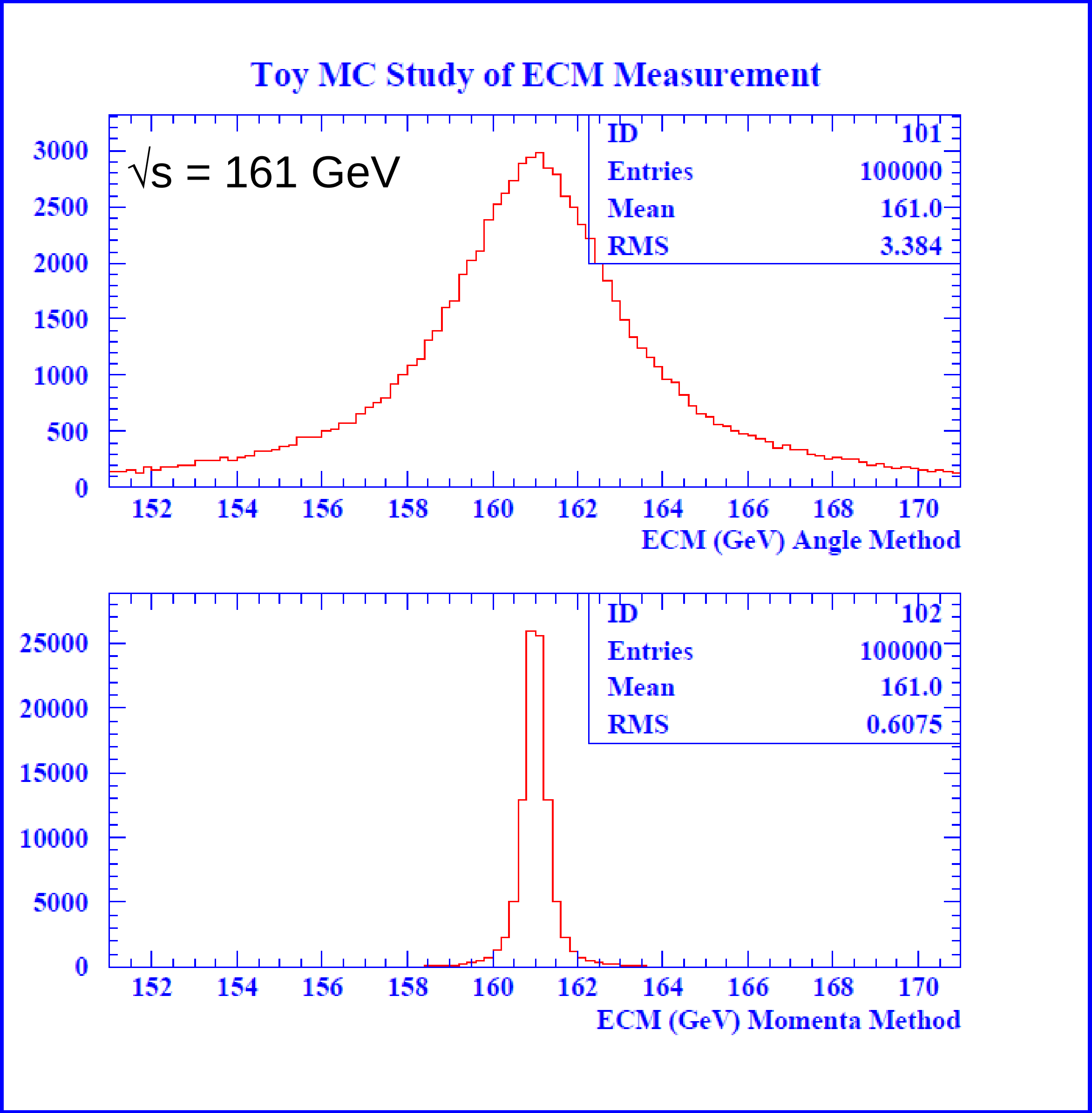
(Absolute scale driven by m_Z – known very well)

Method P (Momenta)

(Absolute scale driven by tracker momentum scale).

Momenta smeared.

Resolution is effectively 10 times better !



Momentum Resolution

Use the standard parametrization fitted to single muons from the ILD DBD.

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin\theta)$$

Where typically

$a = 2 \times 10^{-5}\,\mathrm{GeV}^{-1}$ and $b = 1 \times 10^{-3}$

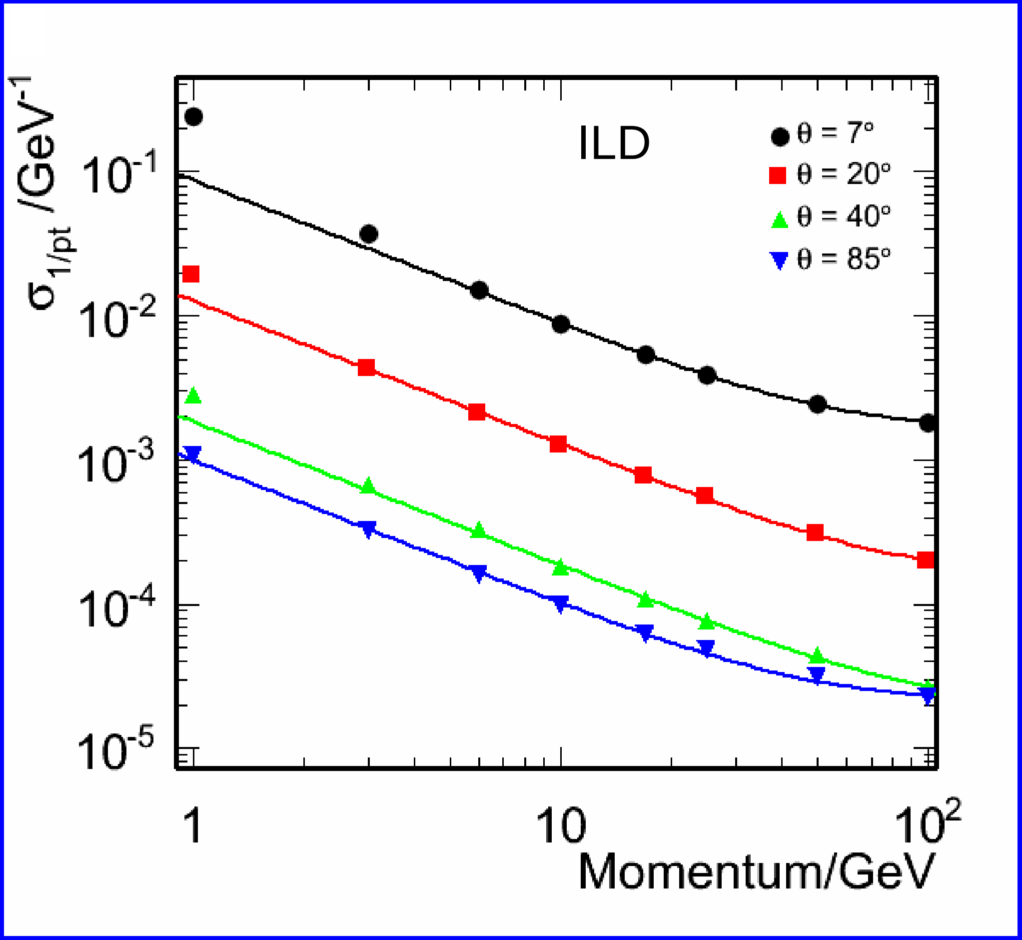
for the full TPC coverage ($\theta > 37°$)

Fit momentum resolution in the p≥10 GeV range.

Superimposed curves are fits for the a,b parameters at 4 polar angles.

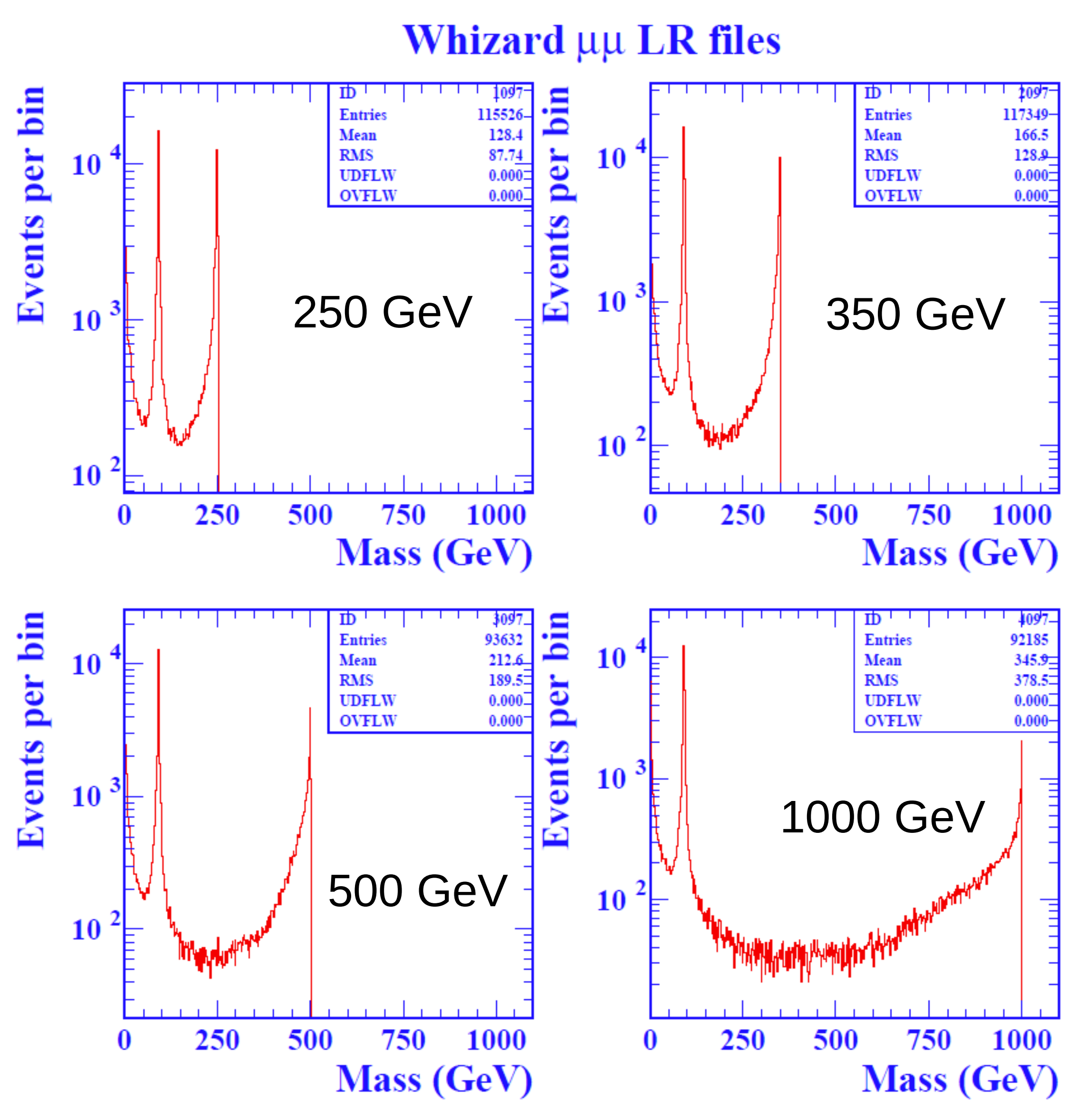
Maximum deviation from fit with this simple parametric form is 6%.

Interpolate between polar angles in endcap (use R^2 scaling for the a term).

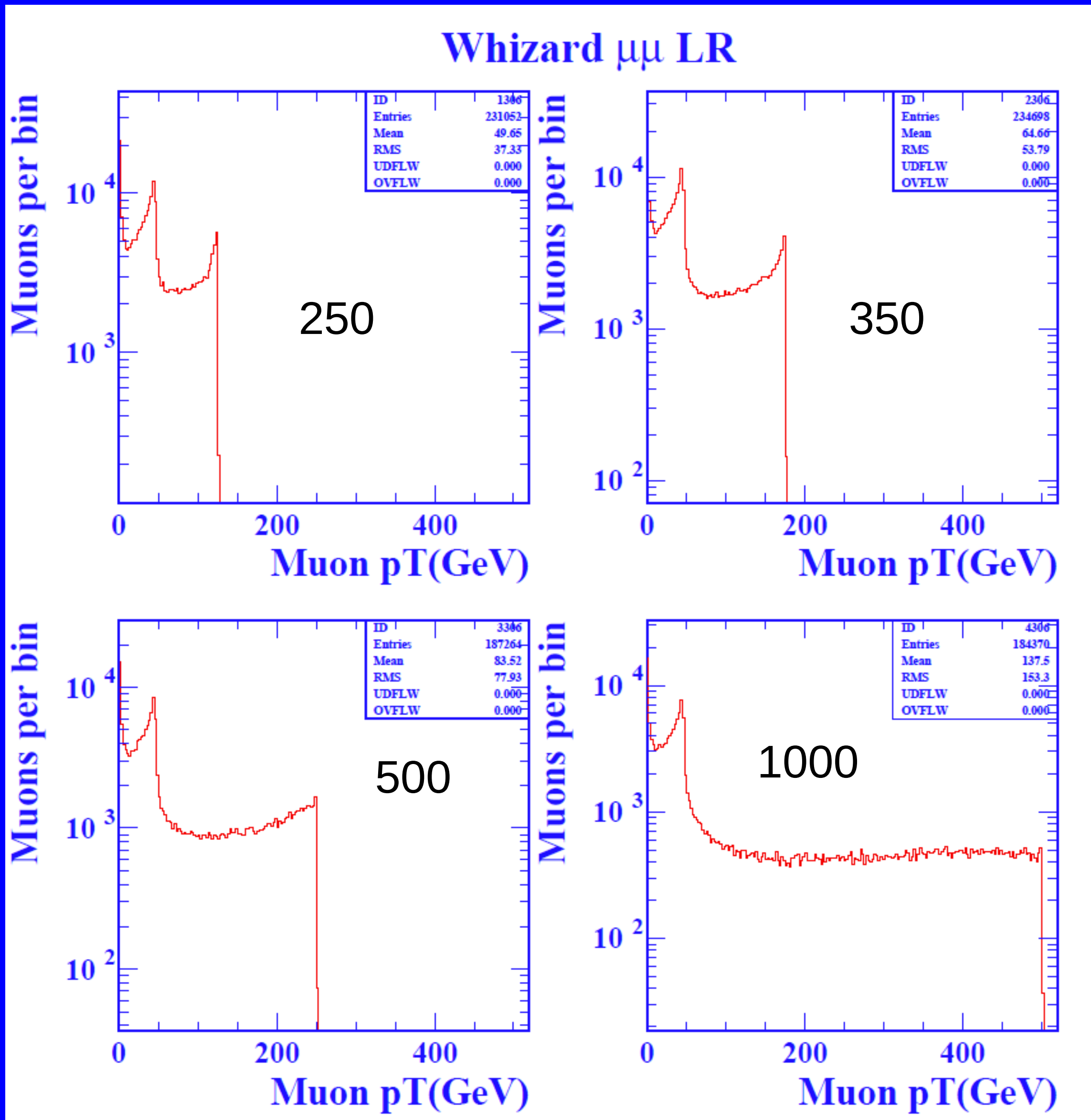


Generator Data-sets

- Use DBD Whizard 4-vector files.
- At ECM=250, 350, 500, 1000 GeV.
- Use 1 stdhep file per energy. (e^-_L, e^+_R).
- Lumis are 10.4, 20.1, 32.2, 109 fb^{-1}.
- Events of interest have a wide range of di-muon mass values.

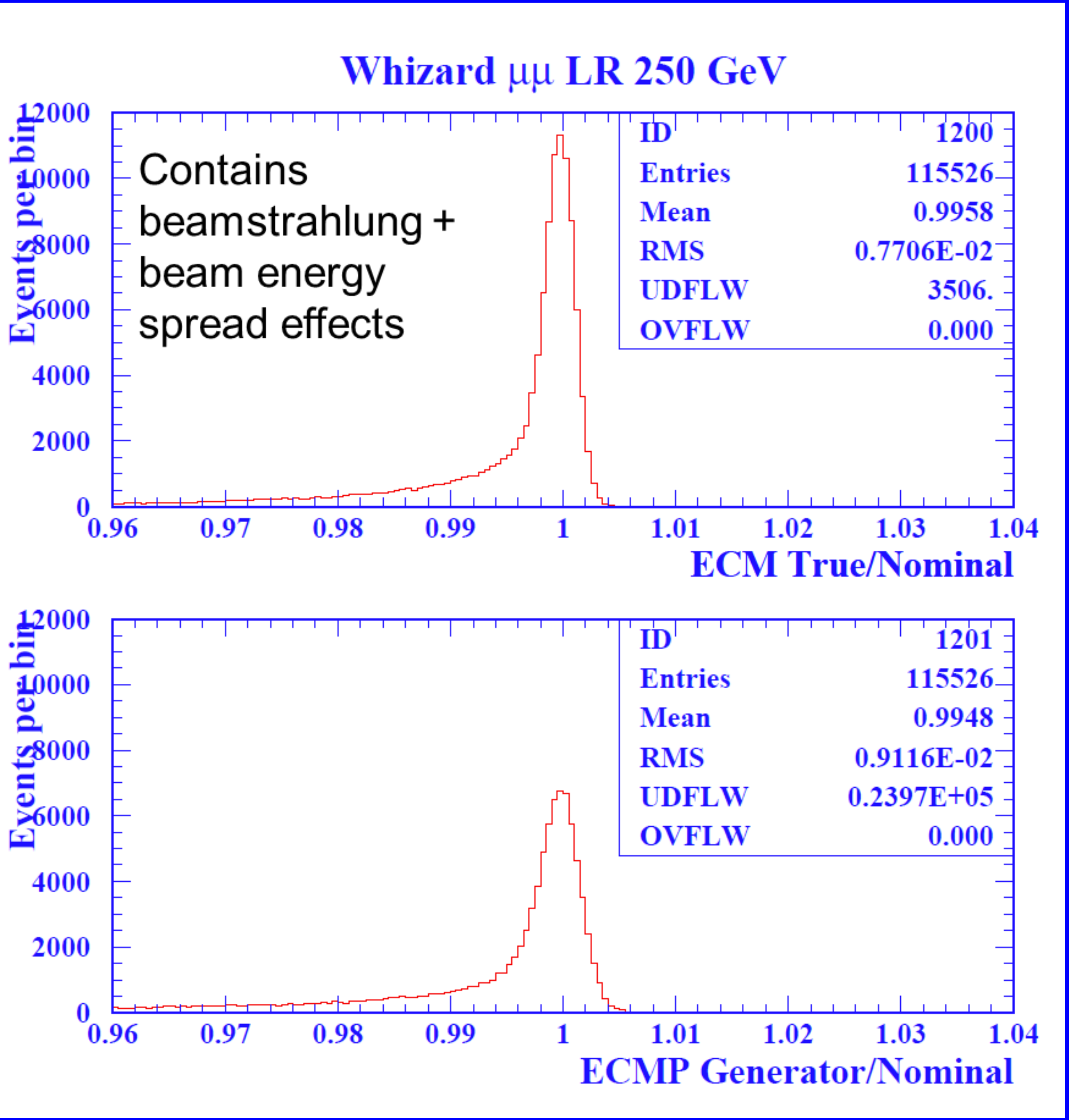


Muon pT distributions

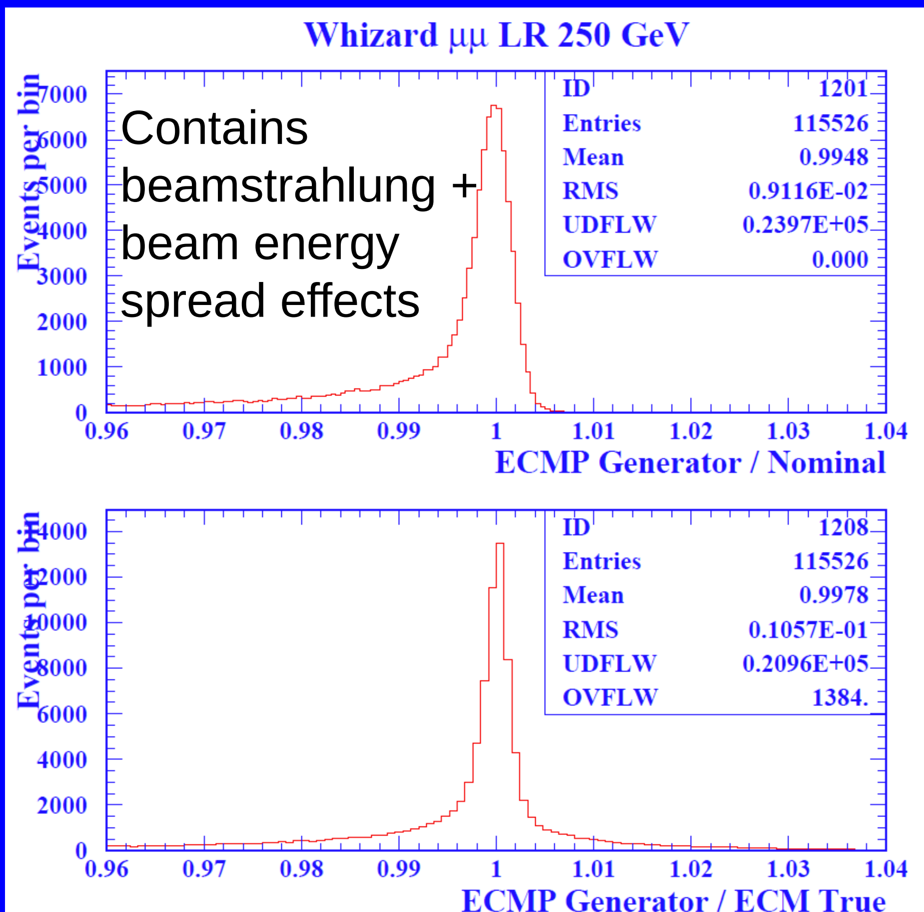


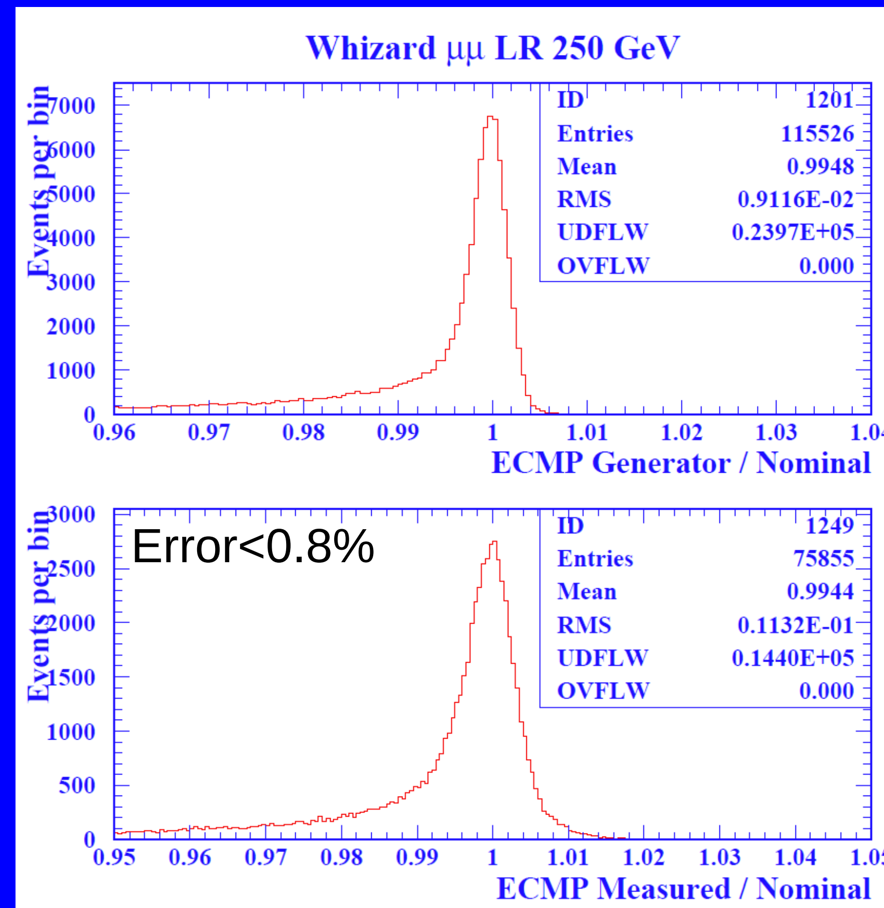
Note that ILD DBD momentum resolution numbers only verified up to p =100 GeV. But expected to be reliable.

ECMP as an estimator of ECM



ECMP as an estimator of ECM





ECMP often is very well correlated with ECM. But long tails : eg hard ISR from BOTH beams

ECMP measured has additional effects from momentum resolution

Calculating error on $\sqrt{s_P}$

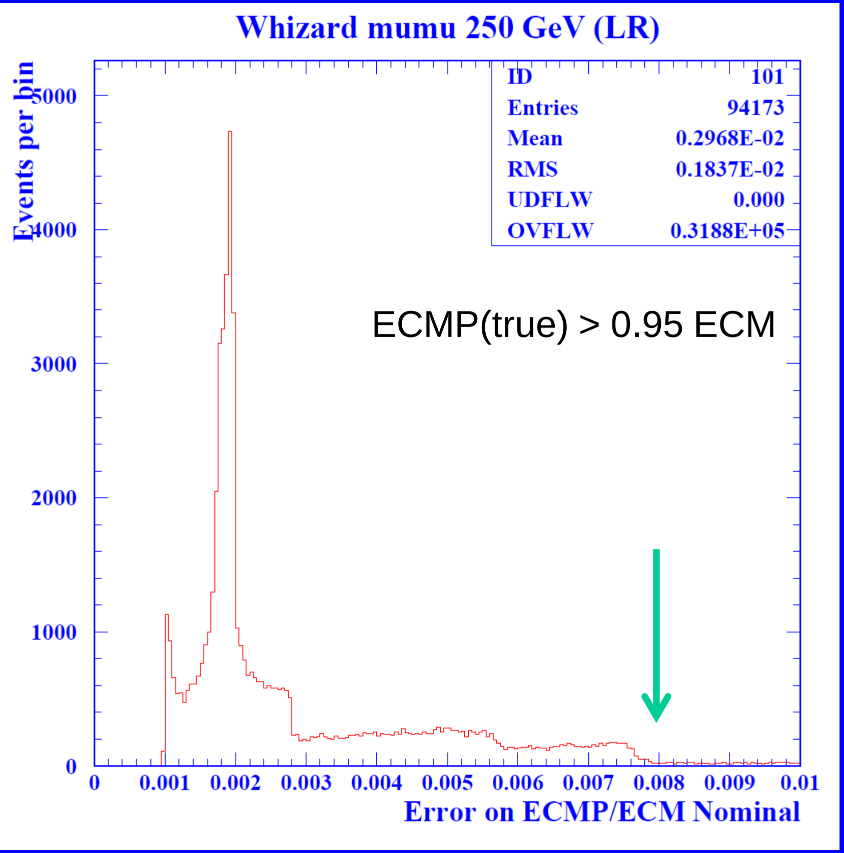
- Can write

$$\sqrt{s_P} = E_1 + E_2 + |p_{12}|$$
$$= \sqrt{(p_1^2 + m^2)} + \sqrt{(p_2^2 + m^2)}$$
$$+ \sqrt{(p_1^2 + p_2^2 + 2p_1p_2\cos\psi_{12})}$$

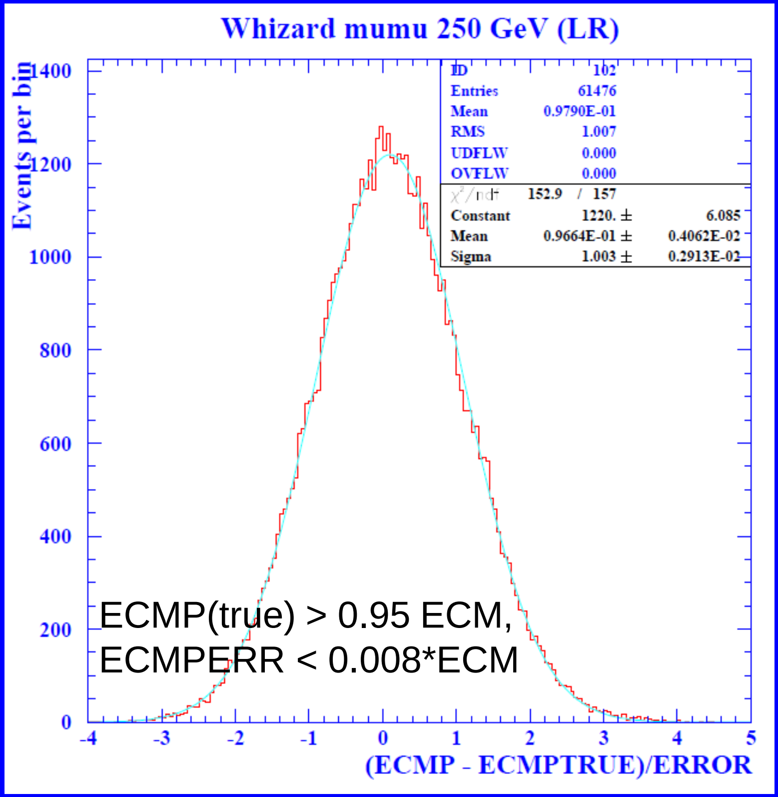
- Then write $p_1 = \csc\theta_1/\kappa_1$ with $\kappa_1 = 1/pT_1$ and similarly for p_2. Use errors on κ from DBD.
- Then do error propagation (neglecting angle errors).

Error on $\sqrt{s_P}$ estimator from momentum resolution

- Using general expression with error propagation. Does not use zero pT approximation. Assumes angle errors negligible.

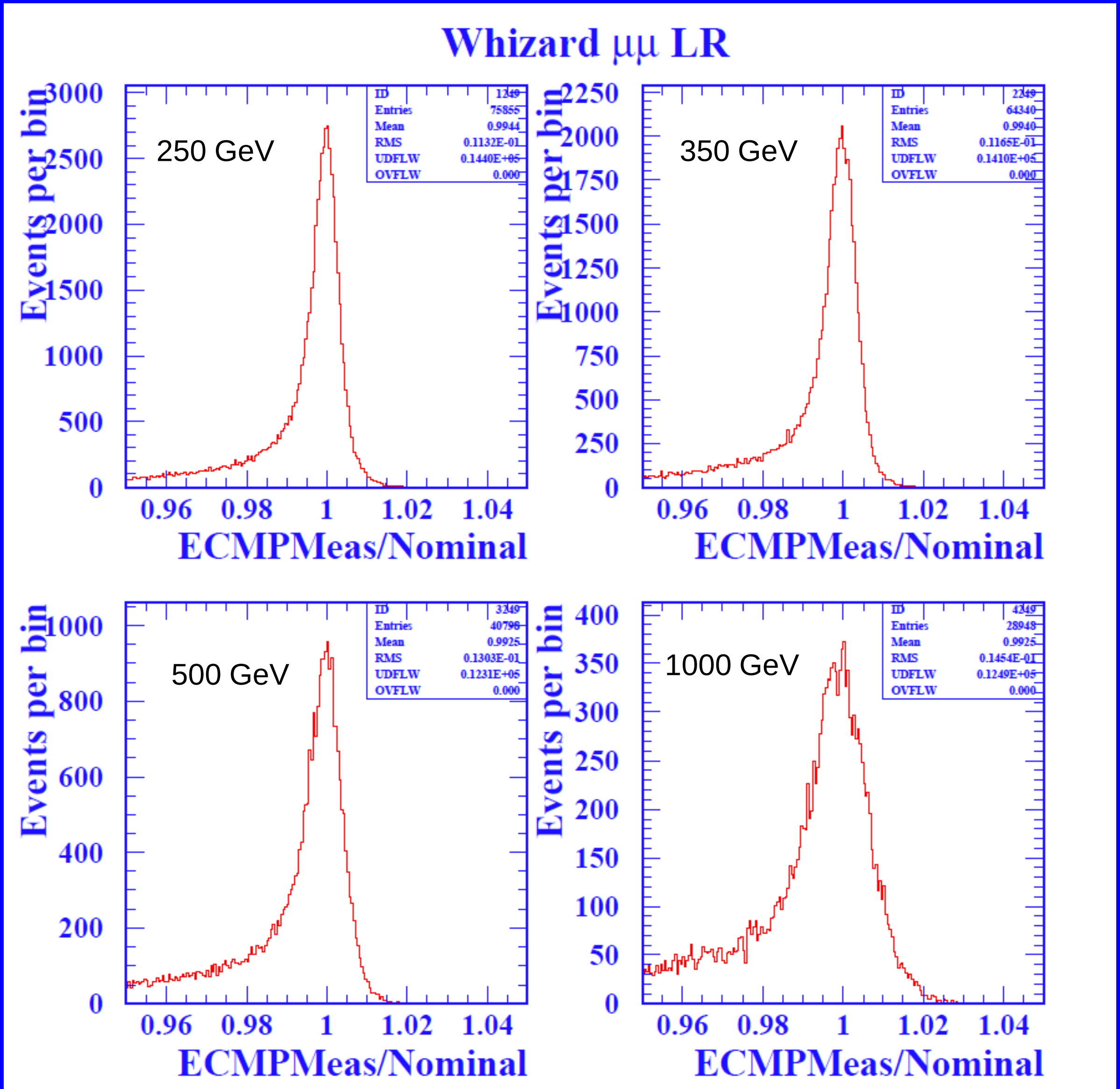


Error distribution is complicated. Reflects the kinematics, beamstrahlung, ISR, FSR, polar angles and p resolution.



Pull distribution has correct width. 10% +ve bias presumably due to errors being Gaussian in curvature (1/pT) not in p.

ECMP Distributions (error<0.8%)

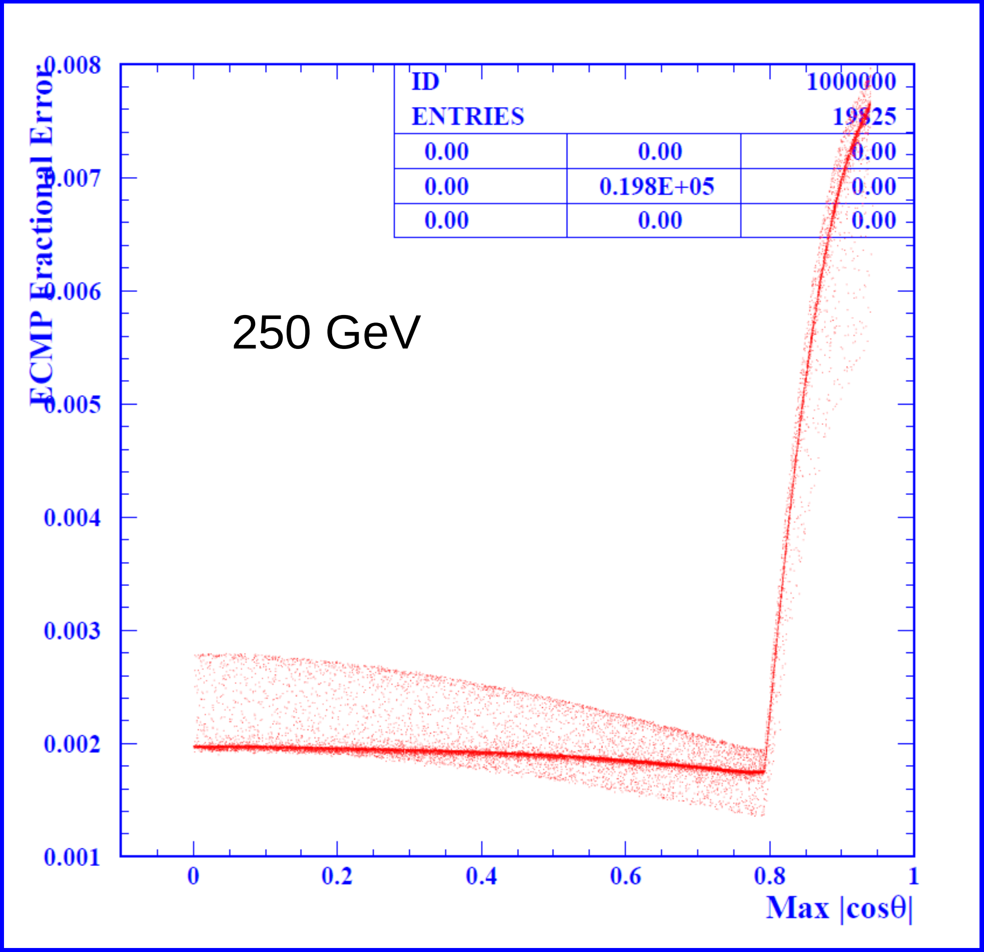


M > 245 GeV

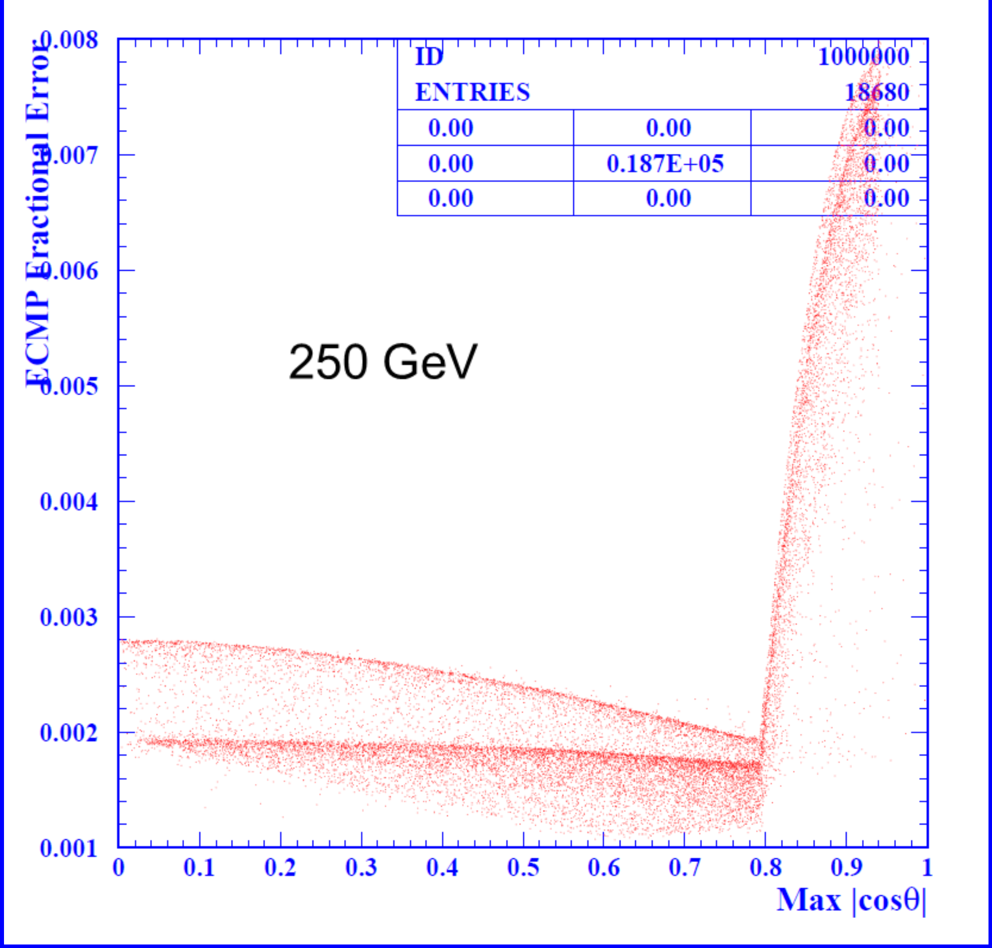
Why is the error distribution so complicated ??

I don't fully understand but is a complicated mix of p1, p2, cosθ1, cosθ2 and the x1, x2 distributions.

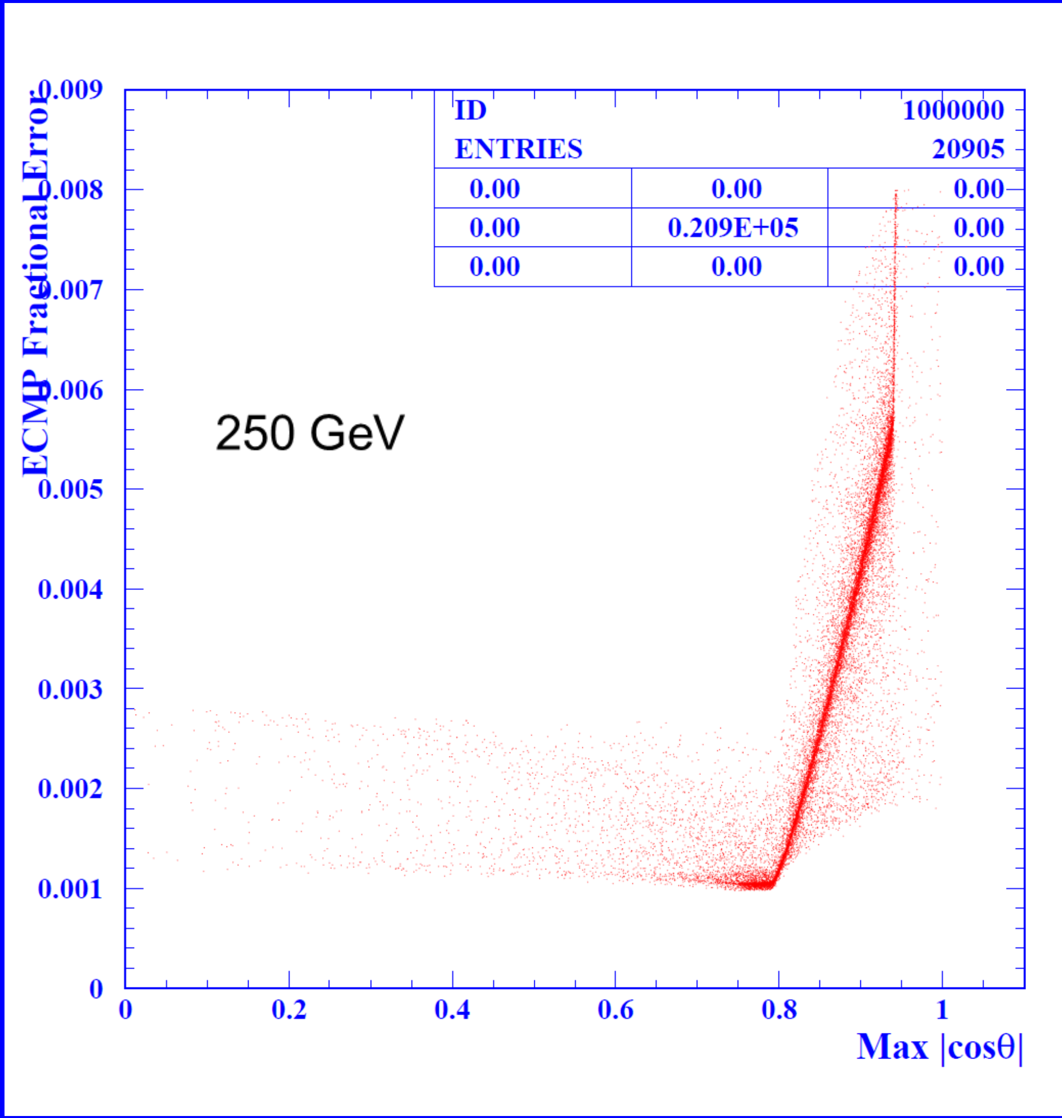
This slide and next ones show error vs cosθ of most forward muon for various di-muon mass bins.



120 < M < 245 GeV


ID 1000000
ENTRIES 18680
0.00 0.00 0.00
0.00 0.187E+05 0.00
0.00 0.00 0.00
250 GeV
ECMP Fractional Error
Max |cosθ|
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

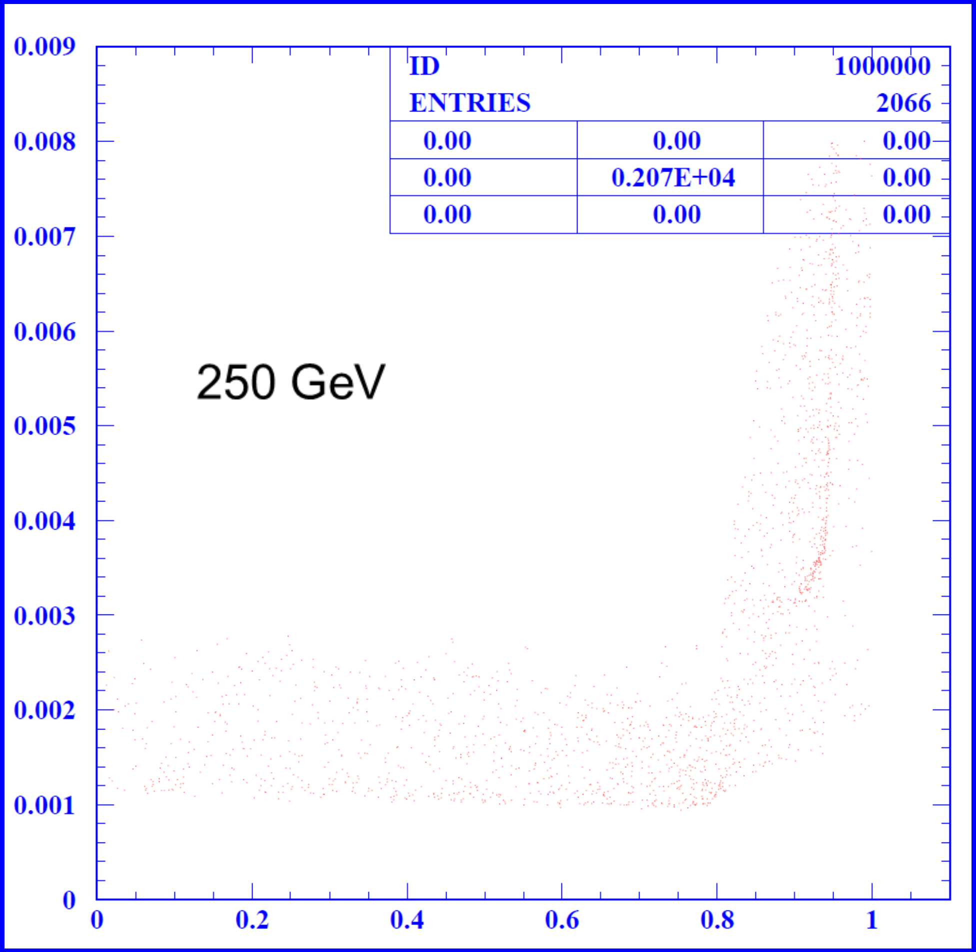
Z Events (60 < M < 120 GeV)



M < 60 GeV

Error on ECMP divided by nominal ECM.

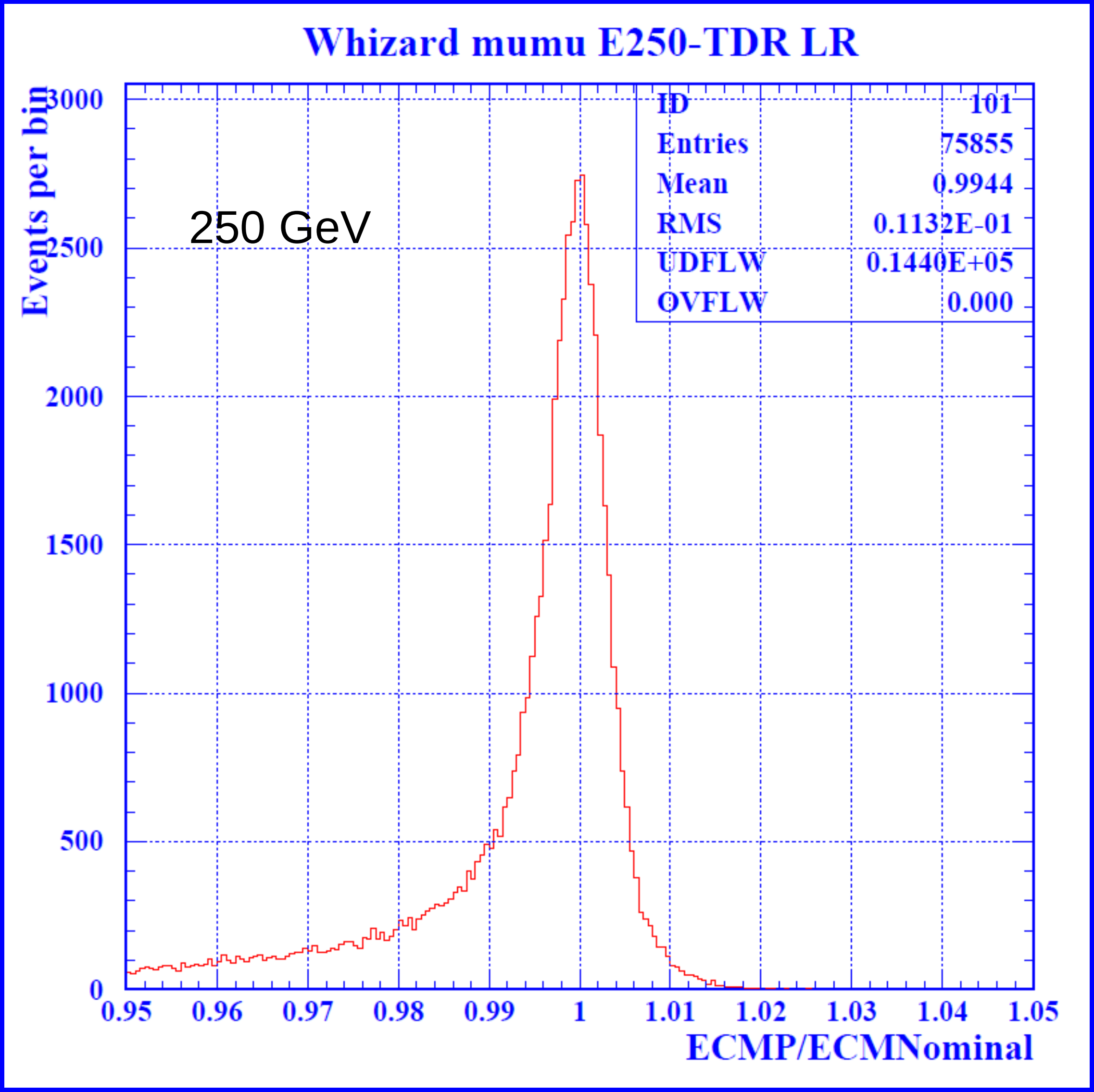
Not many events in this region with small error.



Max |cosθ|

Basic selection at 250 GeV: require error < 0.8%

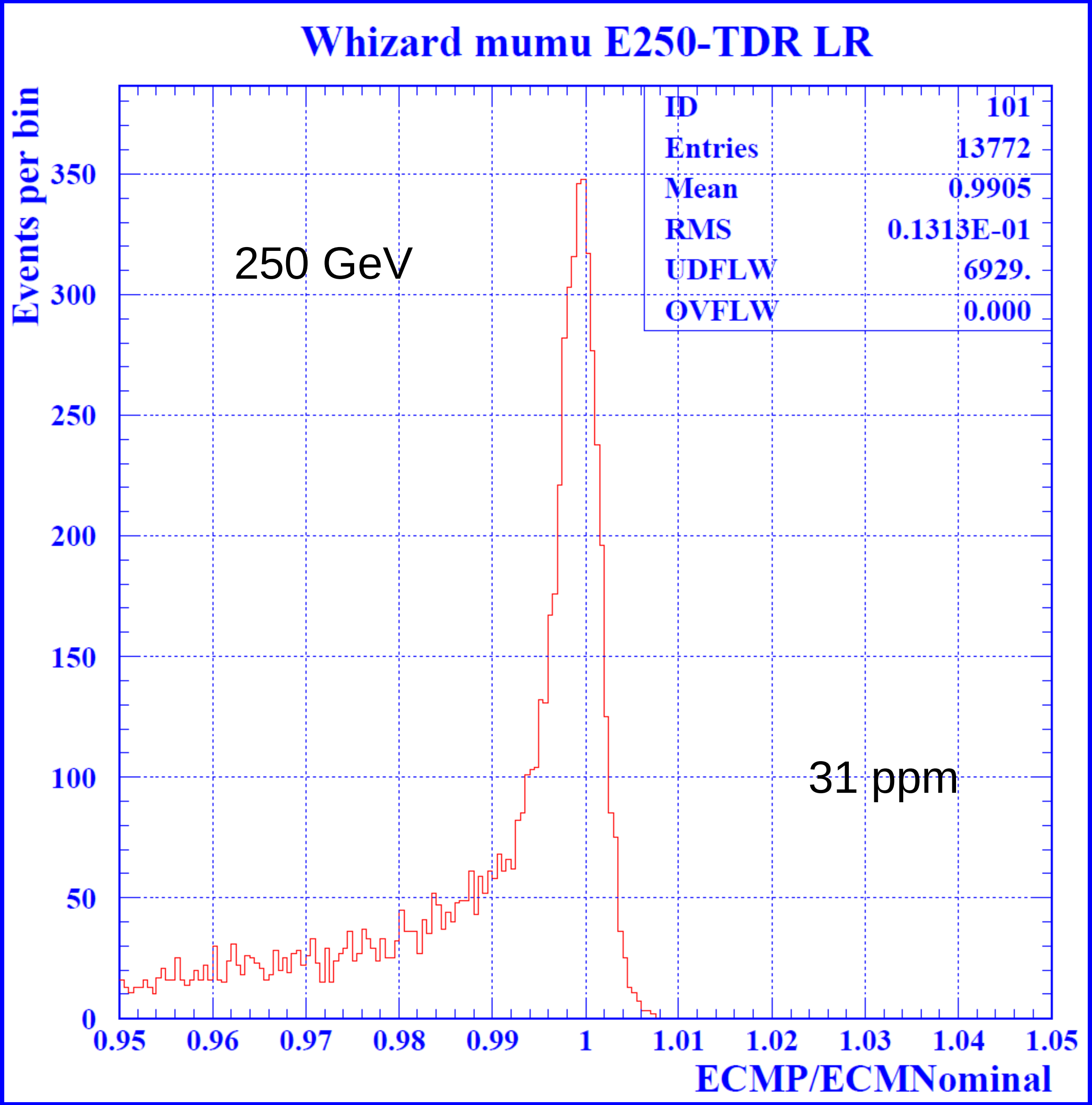
- Beam energy spread contributes 0.122% at 250 GeV.
- ECMP is well measured experimentally when the muons are in the acceptance.



Error < 0.15%

RMS width of peak is less than 0.20%.
As expected from convolving 0.12% with something like 0.13%.

Estimate error of 31 ppm for this sample based on 0.20% error and 60% of these events contributing to a measurement of the peak position.

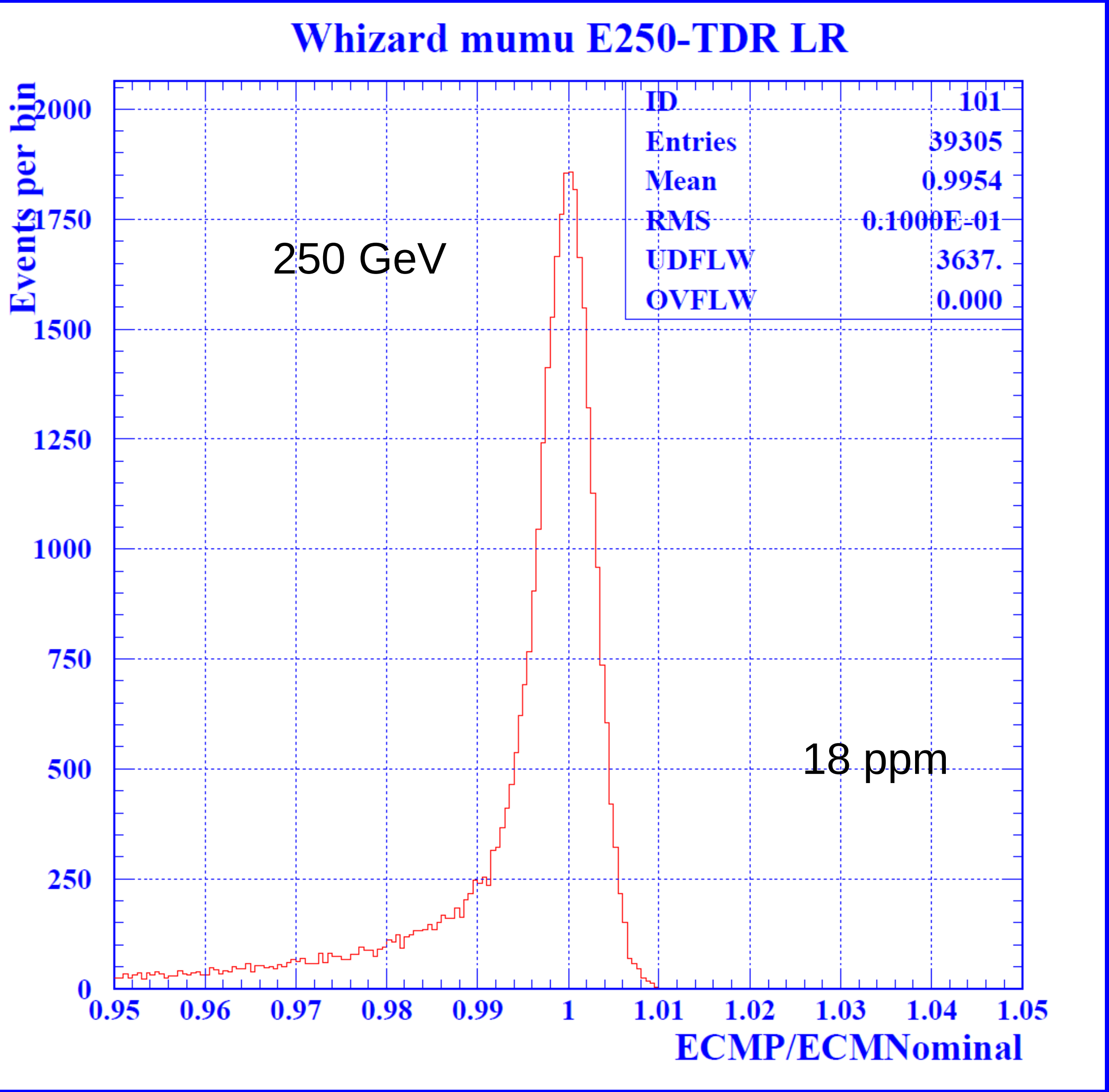


0.15% < Error < 0.30%

RMS width of peak is about 0.30%.

As expected from convolving 0.12% with something like 0.23%.

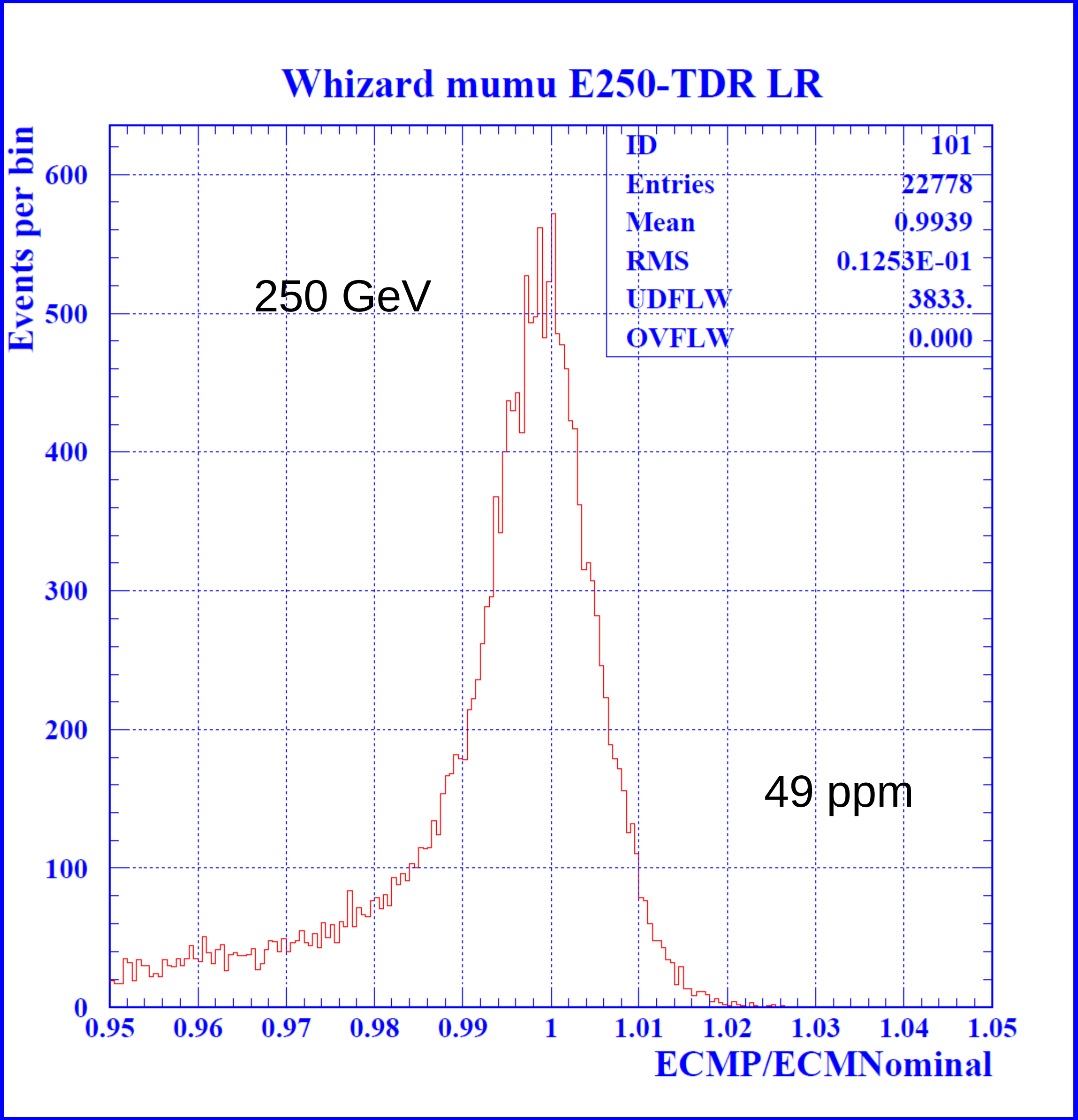
Estimate 80% in peak.



0.30% < Error < 0.80%

RMS width of peak is about 0.6%.

Estimate 80% in peak



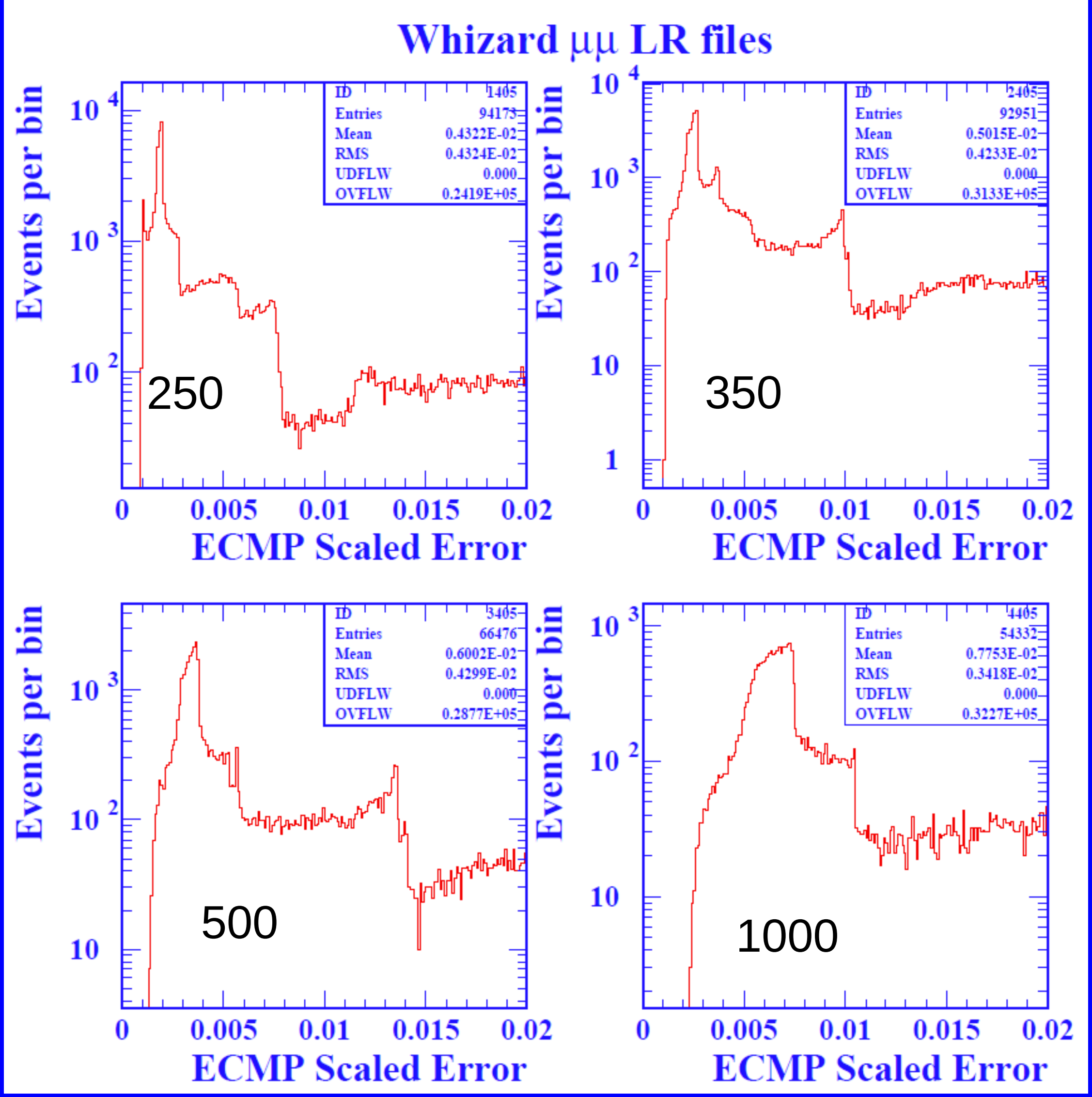
Statistical Errors

- Numbers on previous slides estimated for the statistics of 1 LR stdhep file (10.4 inv fb).
- Weighted average of the 3 bins – gives 15 ppm on peak $\sqrt{s}$.
- Canonical 250 inv fb at 250 GeV with equal weights of LR, RL and (80,30) polarization, gives 4 ppm on peak $\sqrt{s}$.
- (Remember 10 ppm on mW is 0.8 MeV)
  - Good prospects for beam energy precision at a level far better than what is required to make beam energy error for W mass measurements negligible.

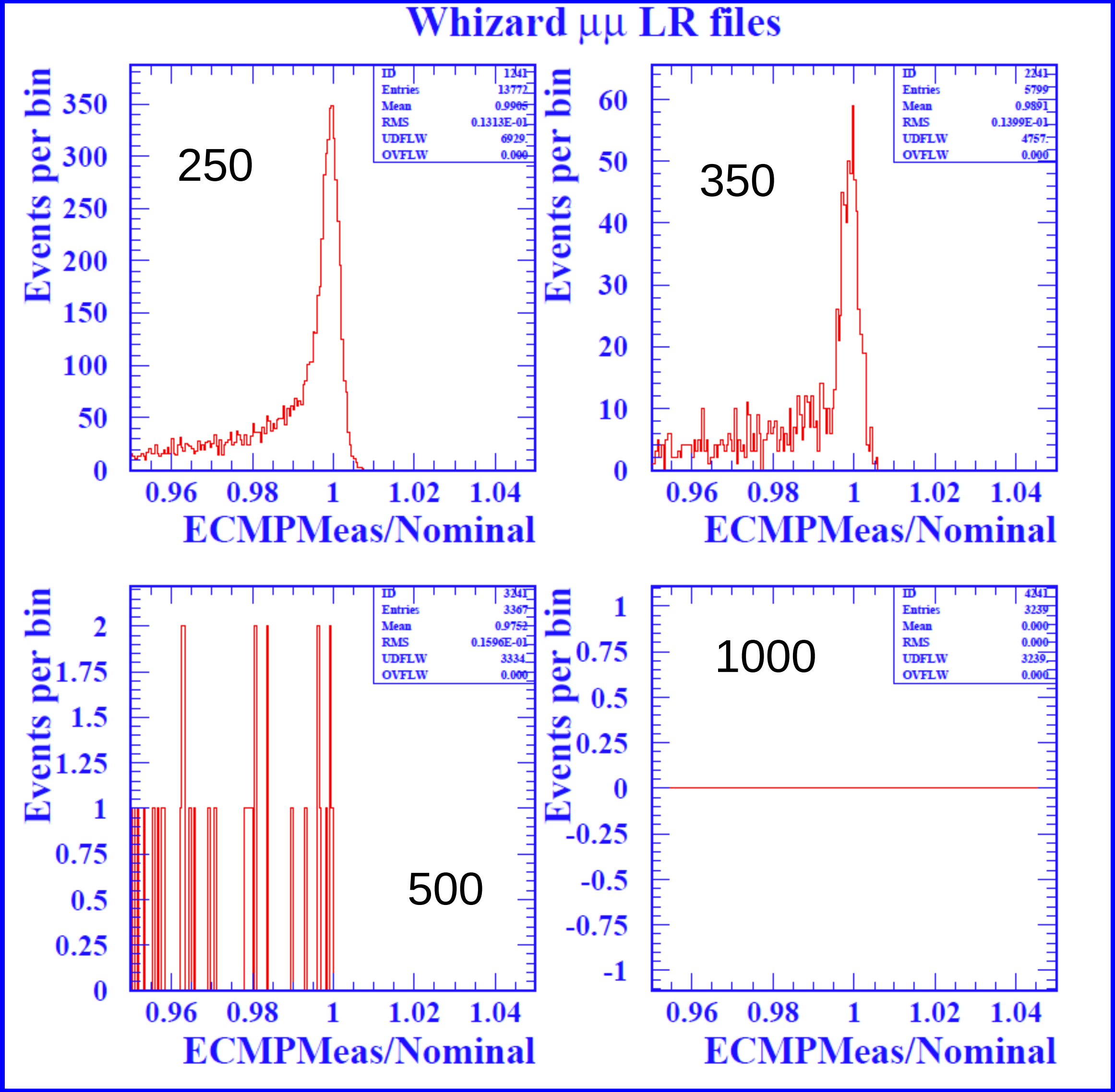
ECMP Errors at All Energies

0.8% is a sensible overall quality cut at 250 GeV.

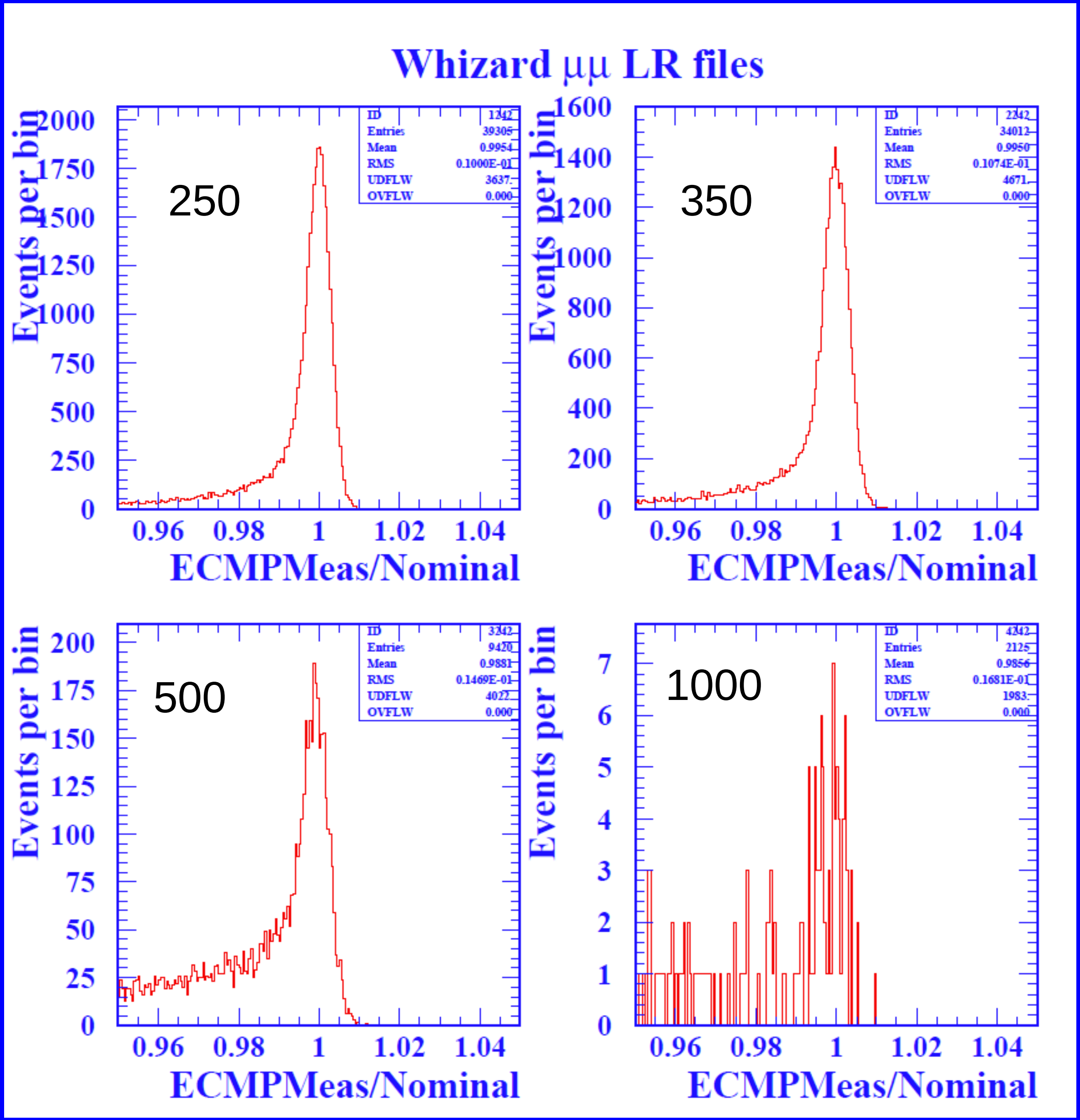
Likely need to relax requirement at higher ECM.



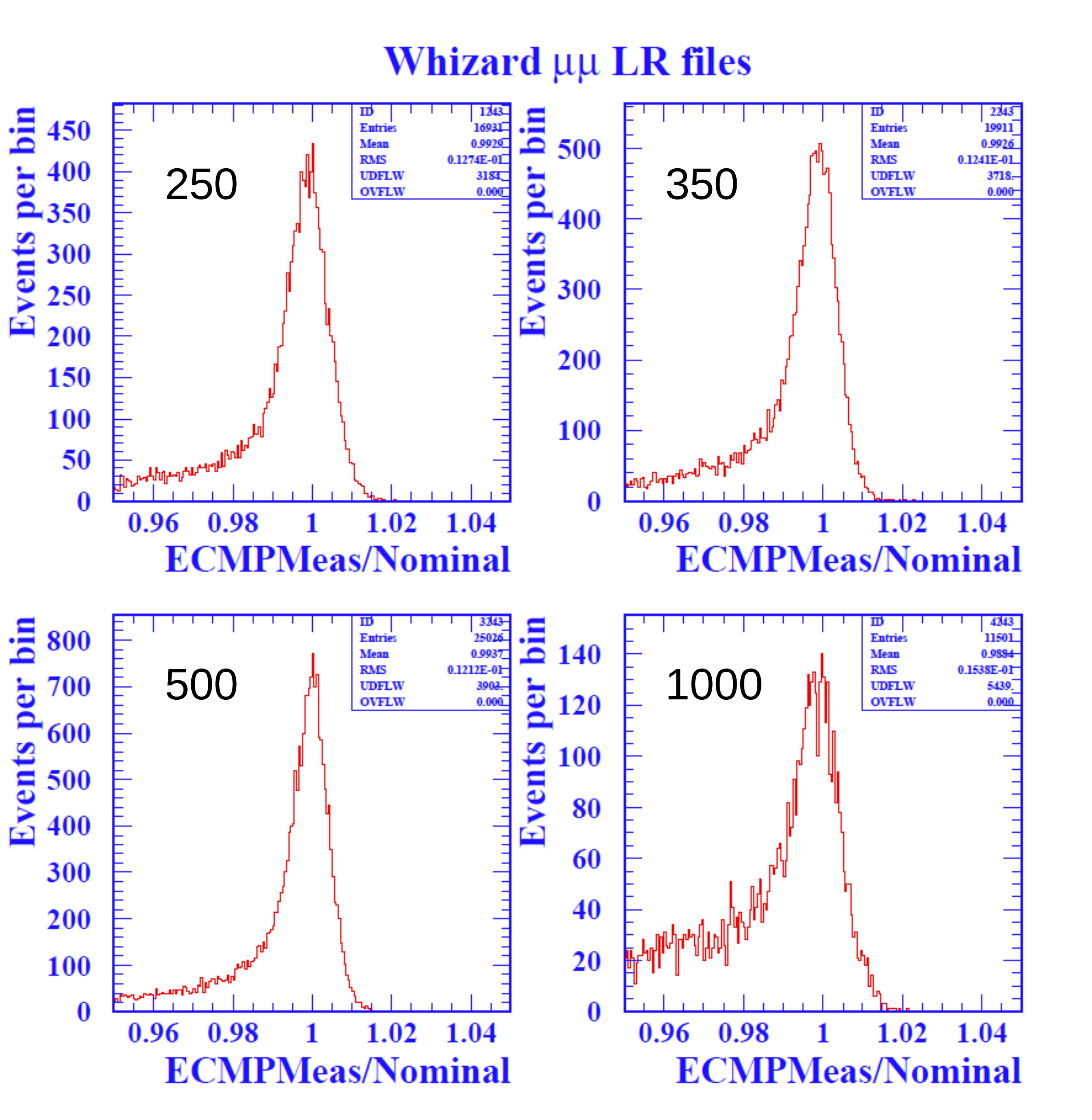
<0.15%


Whizard μμ LR files
250
350
500
1000
Events per bin
ECMPMeas/Nominal
ID 1241
Entries 13772
Mean 0.9905
RMS 0.1313E-01
UDFLW 6929.
OVFLW 0.000
ID 2241
Entries 5799
Mean 0.9891
RMS 0.1399E-01
UDFLW 4757.
OVFLW 0.000
ID 3241
Entries 3367
Mean 0.9752
RMS 0.1596E-01
UDFLW 3334.
OVFLW 0.000
ID 4241
Entries 3239
Mean 0.000
RMS 0.000
UDFLW 3239.
OVFLW 0.000

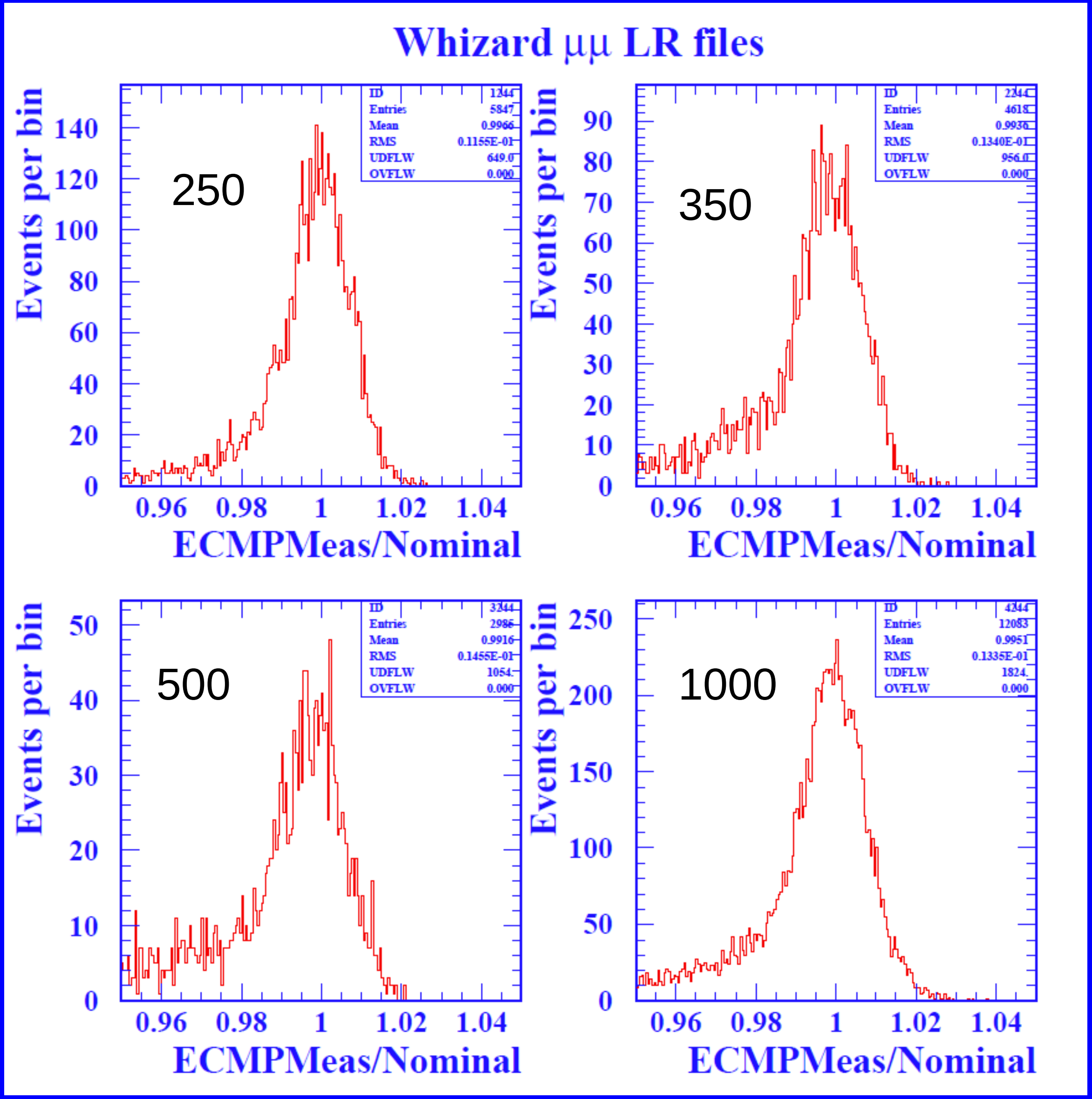
0.15 < Error < 0.30%



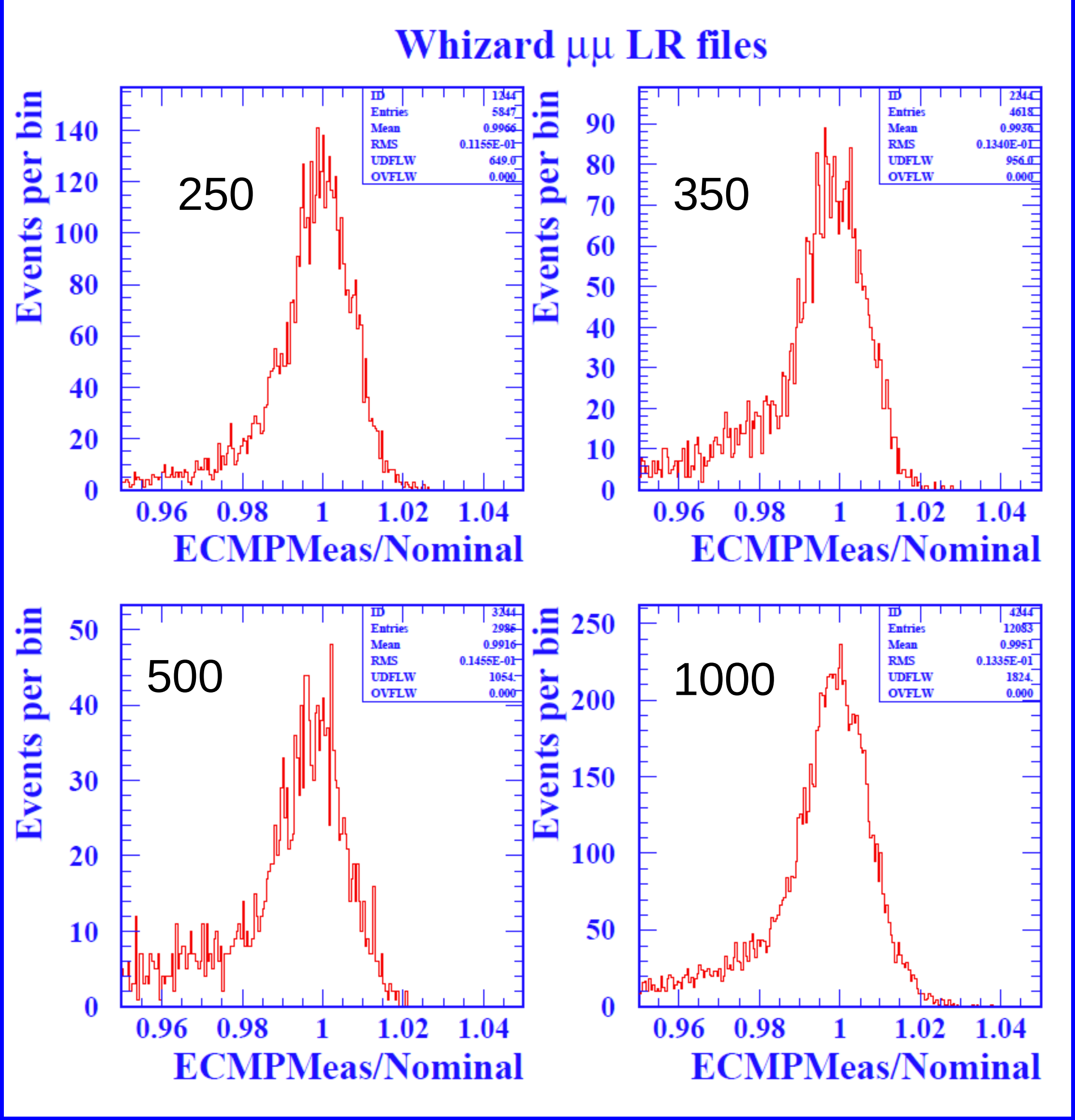
0.3 < Error < 0.6%


Whizard μμ LR files
250
ID 1243
Entries 16931
Mean 0.9929
RMS 0.1274E-01
UDFLW 3184.
OVFLW 0.000
Events per bin
0
50
100
150
200
250
300
350
400
450
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
350
ID 2243
Entries 19911
Mean 0.9926
RMS 0.1241E-01
UDFLW 3718.
OVFLW 0.000
Events per bin
0
100
200
300
400
500
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
500
ID 3243
Entries 25026
Mean 0.9937
RMS 0.1212E-01
UDFLW 3903.
OVFLW 0.000
Events per bin
0
100
200
300
400
500
600
700
800
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
1000
ID 4243
Entries 11501
Mean 0.9884
RMS 0.1538E-01
UDFLW 5439.
OVFLW 0.000
Events per bin
0
20
40
60
80
100
120
140
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal

0.6% < Error < 0.8%

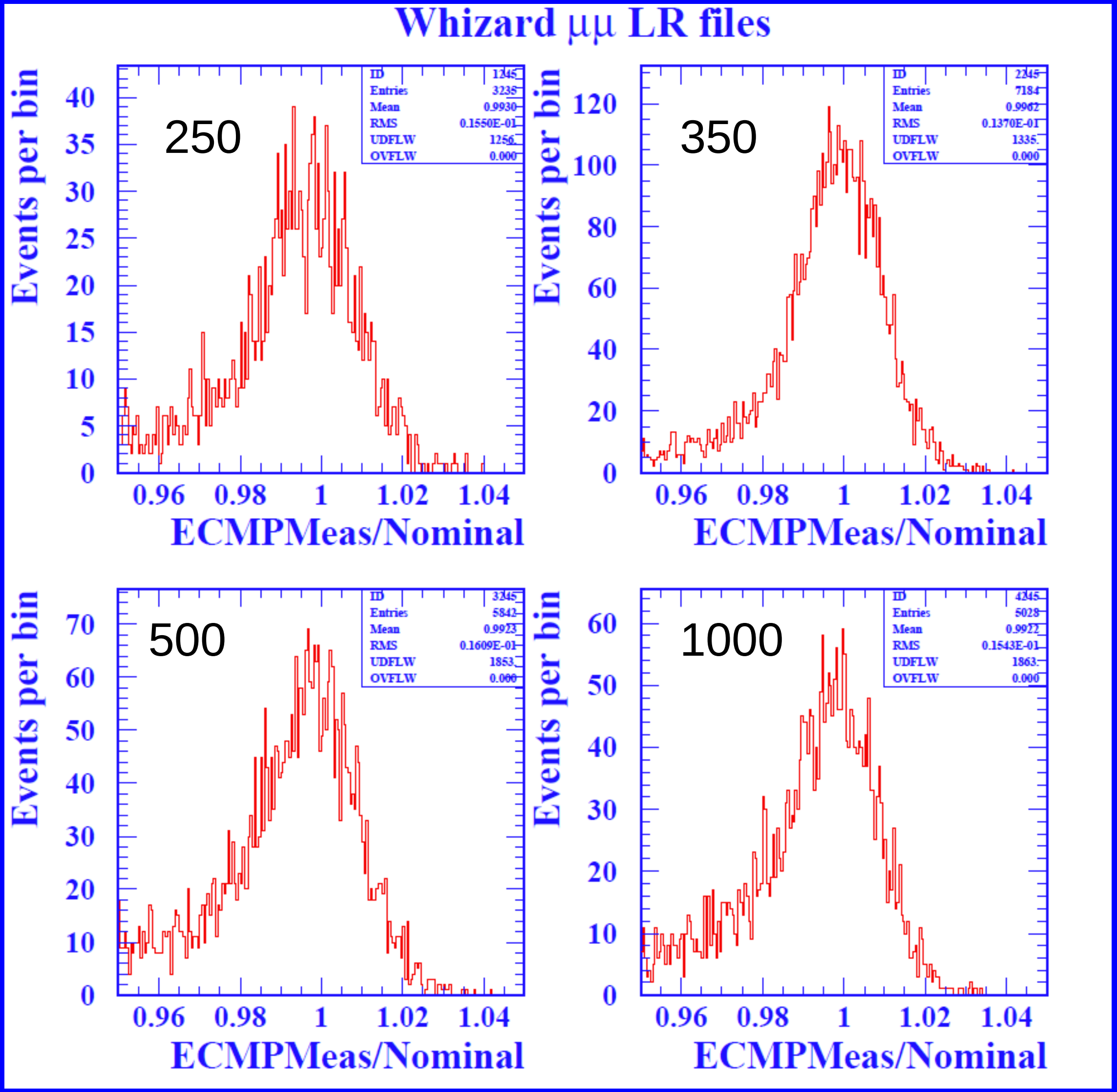

Whizard μμ LR files
250
ID 1244
Entries 5847
Mean 0.9966
RMS 0.1155E-01
UDFLW 649.0
OVFLW 0.000
Events per bin
0
20
40
60
80
100
120
140
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
350
ID 2244
Entries 4618
Mean 0.9936
RMS 0.1340E-01
UDFLW 956.0
OVFLW 0.000
Events per bin
0
10
20
30
40
50
60
70
80
90
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
500
ID 3244
Entries 2986
Mean 0.9916
RMS 0.1455E-01
UDFLW 1054.
OVFLW 0.000
Events per bin
0
10
20
30
40
50
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal
1000
ID 4244
Entries 12083
Mean 0.9951
RMS 0.1335E-01
UDFLW 1824.
OVFLW 0.000
Events per bin
0
50
100
150
200
250
0.96
0.98
1
1.02
1.04
ECMPMeas/Nominal

0.8% < Error < 1.2%



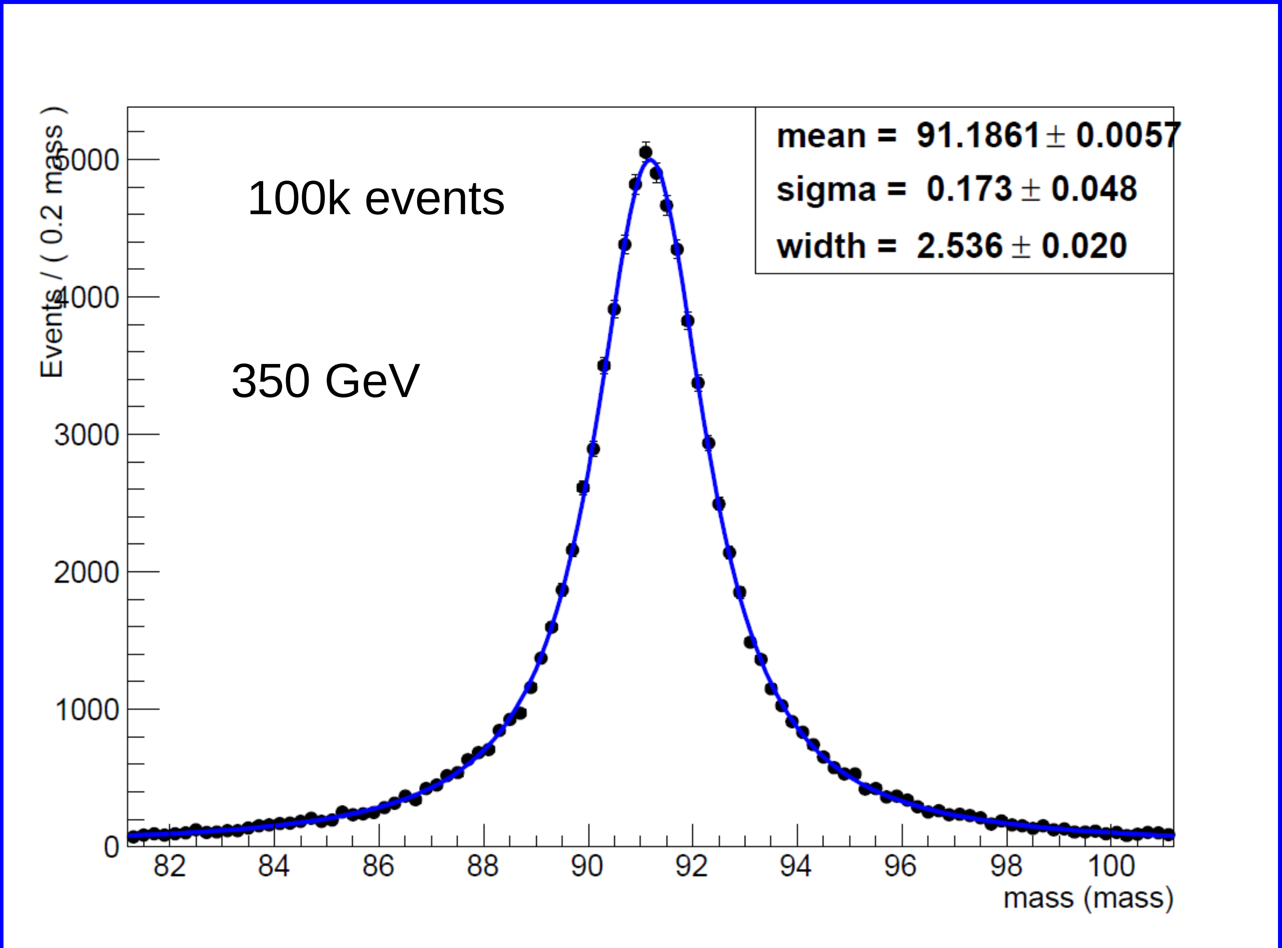
Whizard μμ LR files
Events per bin
ECMPMeas/Nominal
250
350
500
1000
ID 1244
Entries 5847
Mean 0.9966
RMS 0.1155E-01
UDFLW 649.0
OVFLW 0.000
ID 2244
Entries 4618
Mean 0.9936
RMS 0.1340E-01
UDFLW 956.0
OVFLW 0.000
ID 3244
Entries 2986
Mean 0.9916
RMS 0.1455E-01
UDFLW 1054.
OVFLW 0.000
ID 4244
Entries 12083
Mean 0.9951
RMS 0.1335E-01
UDFLW 1824.
OVFLW 0.000

1.2< Error < 2.0%



Whizard μμ LR files
250
350
500
1000
Events per bin
ECMPMeas/Nominal
0.96
0.98
1
1.02
1.04
ID 1245
Entries 3235
Mean 0.9930
RMS 0.1550E-01
UDFLW 1256.
OVFLW 0.000
ID 2245
Entries 7184
Mean 0.9962
RMS 0.1370E-01
UDFLW 1335.
OVFLW 0.000
ID 3245
Entries 5842
Mean 0.9923
RMS 0.1609E-01
UDFLW 1853.
OVFLW 0.000
ID 4245
Entries 5028
Mean 0.9922
RMS 0.1543E-01
UDFLW 1863.
OVFLW 0.000

Can control for p-scale using measured di-lepton mass



This is about 100 fb^{-1} at ECM=350 GeV.

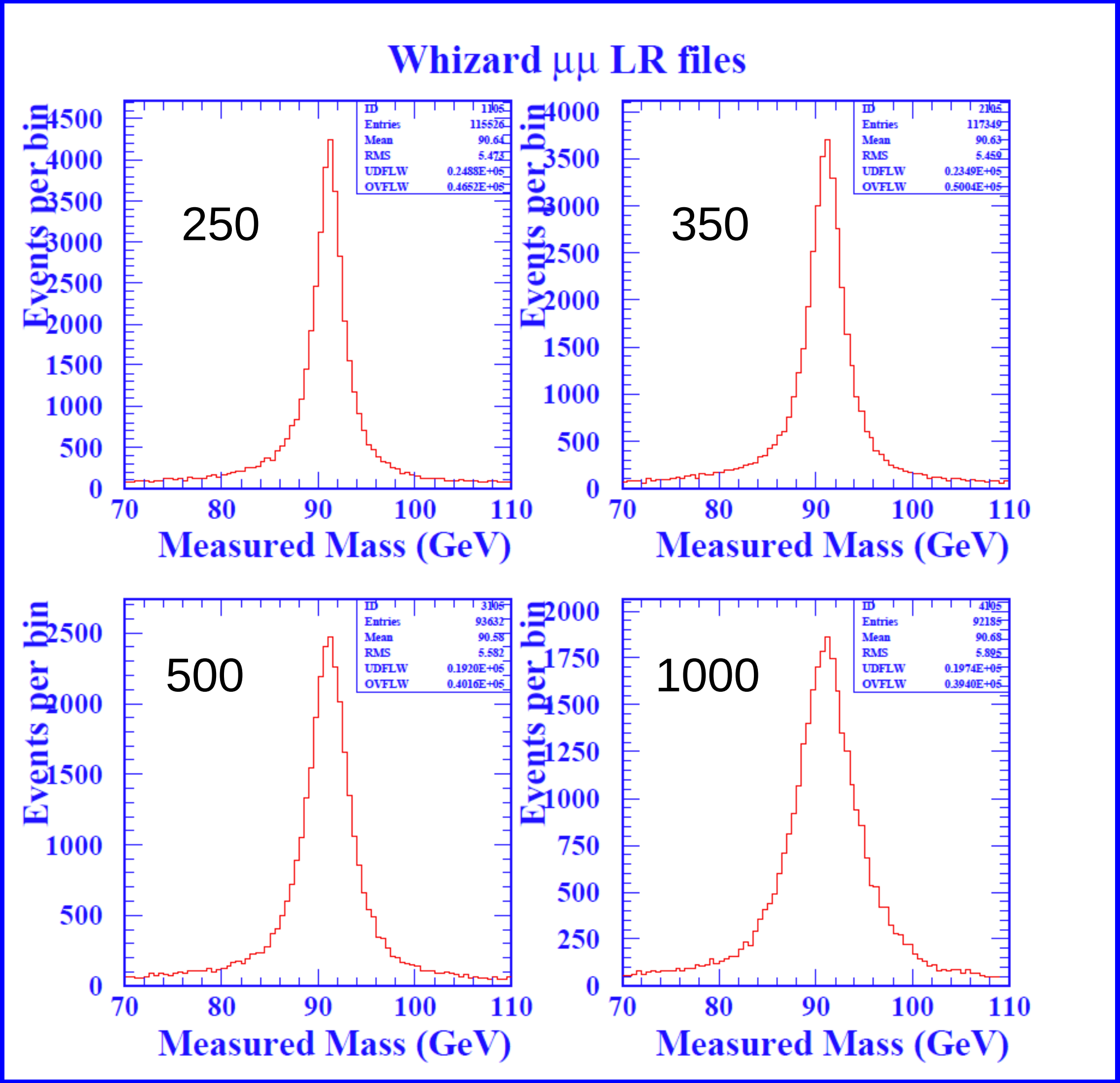
Statistical sensitivity if one turns this into a Z mass measurement (if p-scale is determined by other means) is

1.8 MeV / $\sqrt{N}$

With N in millions.

Alignment ?
B-field ?
Push-pull ?
Etc …

Z Mass distributions



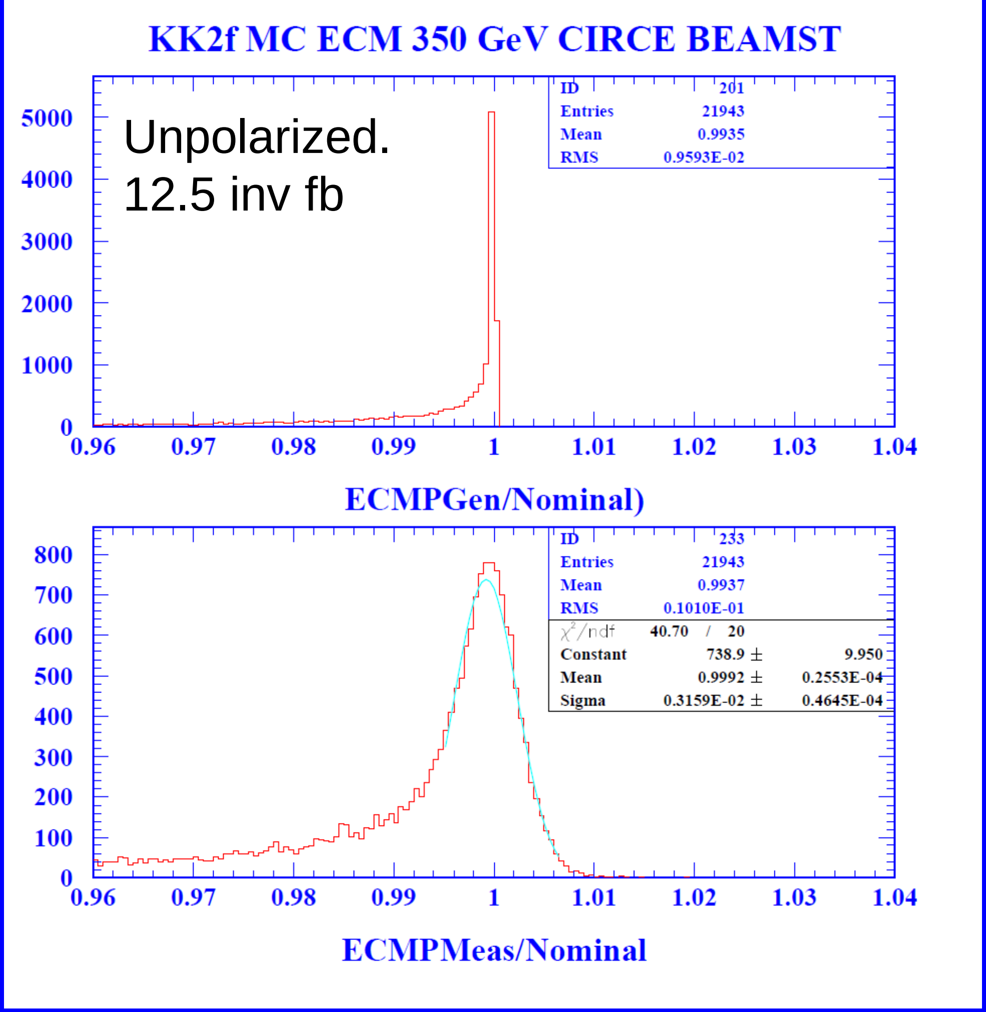
No error cuts in these plots.

Cross-check

KK2f MC v4.19b Study

Includes beamstrahlung (TESLA350 - CIRCE) but no momentum spread.

Sophisticated photon treatment including FSR and ISR+FSR interference.
Find error of about 6 ppm. (For 350 fb^{-1}, (80,30) +-, -+ assumptions as before) but just from this simple fit.



$(E_1 + E_2 + p_{12})/350$

$(E_1 + E_2 + p_{12})/350$

Resolution is about 0.32%

KKMC from Jadach, Ward, Was

Note - need to get a robust fit implemented.

Beam Energy Spread

- Current ILC Design.
- Not a big issue especially at high √s

IP RMS Energy spreads (%)

Centre of mass energy (GeV)		200	230	250	350	500	1000 A1	1000 B1B
							0,250	0,225
Damping ring @ 5GeV	e+	0,137	0,137	0,137	0,11	0,11	0,109	0,109
	e-	0,12	0,12	0,12	0,12	0,12		
RTML @ 15 GeV	e+	1,23	1,23	1,23	1,13	1,13	1,36	1,51
(assume no z-correlation)	e-	1,17	1,17	1,17	1,13	1,13		
Main linac	e+	0,185	0,160	0,148	0,097	0,068	0,041	0,045
	e-	0,176	0,153	0,140	0,097	0,068	0,014	0,014
Long. wakefield contribution		0,046	0,039	0,036	0,026	0,018		
Positron undulator contribution	e-	0,098	0,113	0,123	0,122	0,103	0,071	0,071
IP value	**e+**	**0,190**	**0,165**	**0,152**	**0,100**	**0,070**	**0,043**	**0,047**
	e-	**0,206**	**0,194**	**0,190**	**0,158**	**0,124**	**0,083**	**0,085**

LEP2 was 0.19% per beam at 200 GeV.

Summary Table

ECMP errors based on estimates from weighted averages from various error bins up to 2.0%. Assumes (80,30) polarized beams, equal fractions of +- and -+.

(Statistical errors only …)

ECM (GeV)	L (inv fb)	$\Delta(\sqrt{s})/\sqrt{s}$ Angles (ppm)	$\Delta(\sqrt{s})/\sqrt{s}$ Momenta (ppm)	Ratio
250	250	64	4.0	16
350	350	65	5.7	11.3
500	500	70	10.2	6.9
1000	1000	93	26	3.6

< 10 ppm for 200 – 500 GeV CoM energy

Conclusions

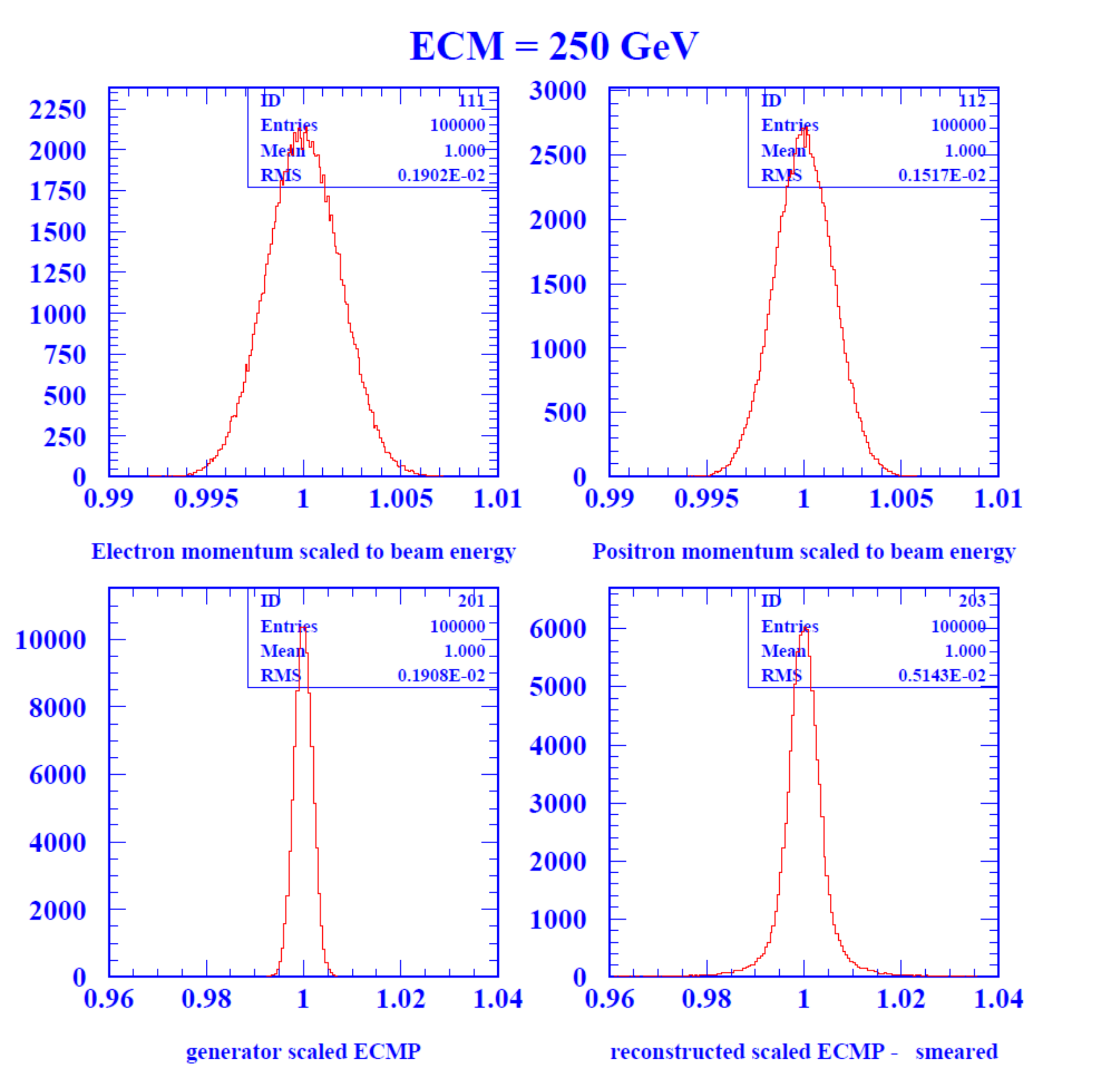
- The $\sqrt{s_P}$ method looks very promising for obtaining a high precision measurement of the peak centre-of-mass energy.
- This should work well especially for 161-500 GeV
  - Better than 10 ppm is within reach.
- A LEP2 style W mass measurement at 250-350 GeV?
- Important aspects will be
  - Luminosity spectrum determination
    - Can use μμ in addition to Bhabha events
  - Tracker-alignment, B-field
  - Momentum-scale determination (not necessarily relying on m_Z)
  - Momentum resolution understanding
  - Excellent momentum resolution in endcap

Backup Slides

Check intrinsic resolution for Method P

p(e-) / 125.0

0.19%



p(e+) / 125.0

0.15%

0.19%

0.51% (0.34% central part)

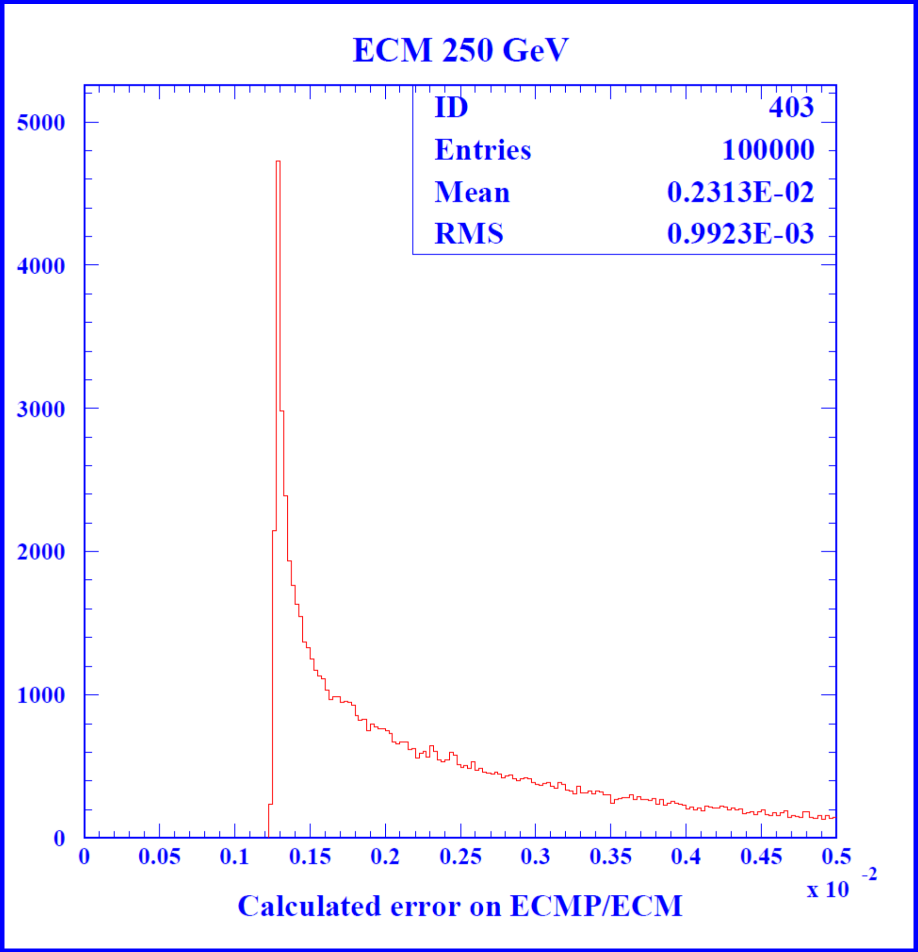
$(E_1 + E_2 + p_{12})/250$ $(E_1 + E_2 + p_{12})/250$

Contribution from Momentum Resolution.

Calculate error from the measured p_T's and polar angles of each muon.

Combined this gives a range of errors from event-to-event with symmetric events having an error of around 0.14%.

Can also use this information to improve the statistical power.



Momentum Resolution

Currently use the large polar angle parametrization from ILD LOI (blue line).

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin\theta)$$

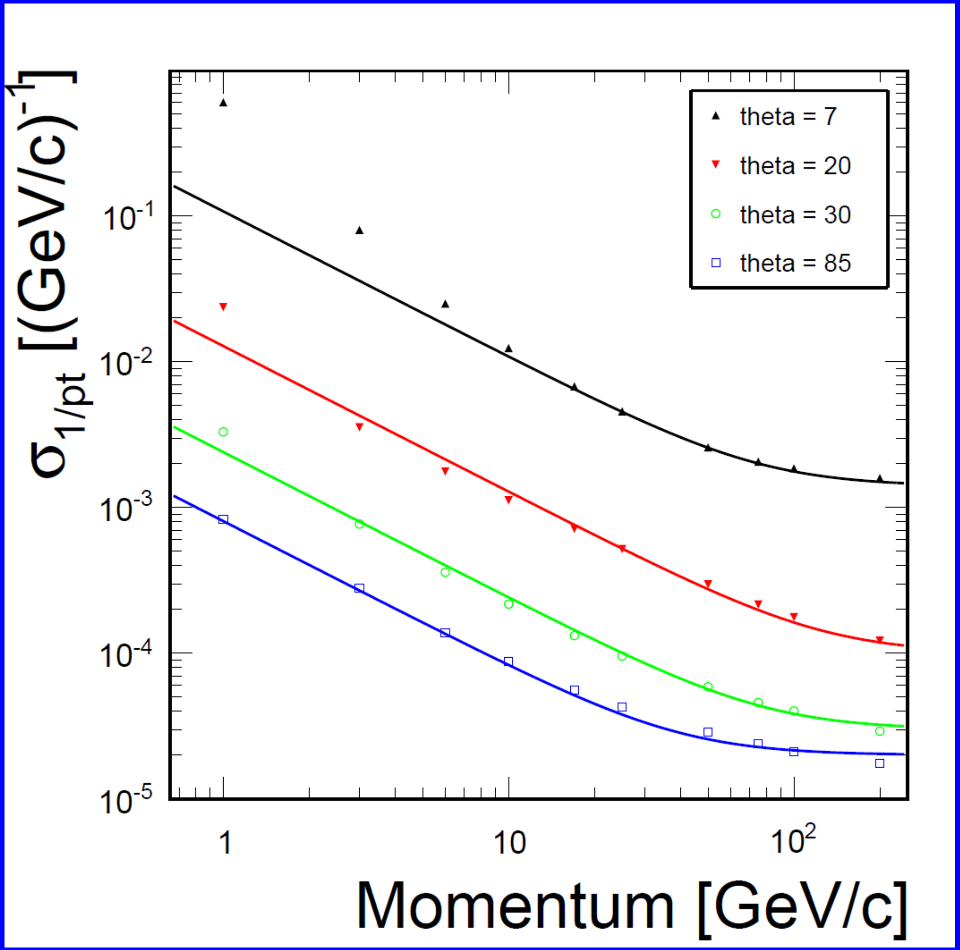
Where

$a = 2 \times 10^{-5}\,\mathrm{GeV}^{-1}$ and $b = 1 \times 10^{-3}$

Should be OK for the full TPC coverage (θ > 37°)

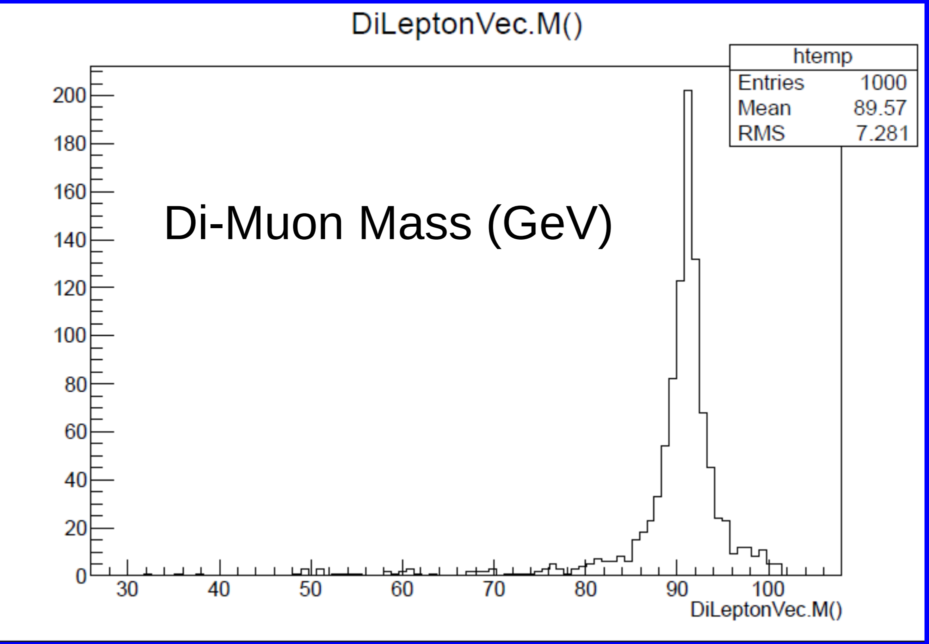
Plot is data from Steve Aplin's macro. Superimposed curves have a,b parameters tweaked for θ=7°,20°,30° to give a decent fit for p > 10 GeV.

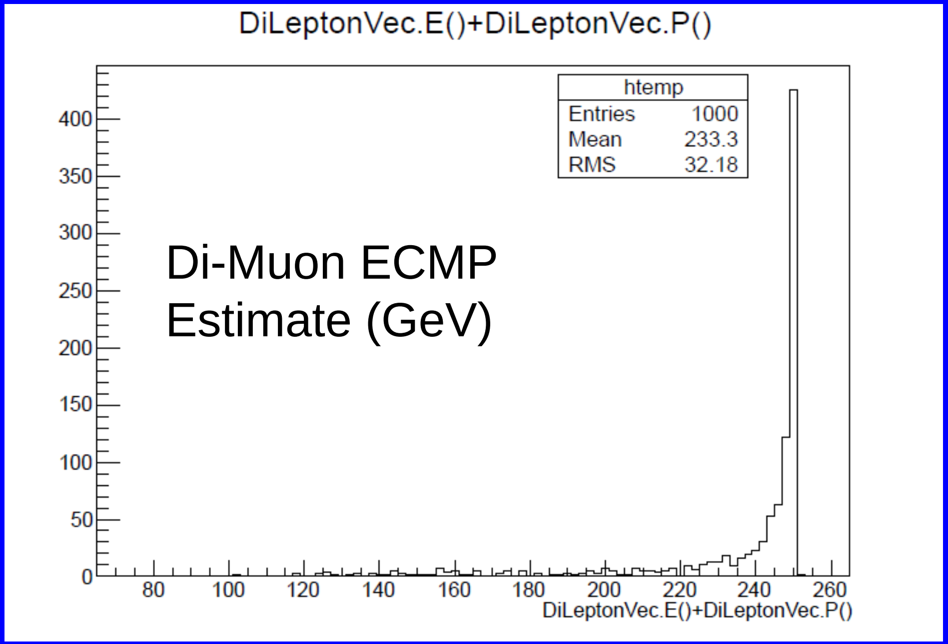
Will need good parametrized description of this and/or use SGV particularly for high √s (for highly boosted di-muons).



Whizard Generator Level Studies

ECM = 250 GeV. $e^-_L \, e^+_R \rightarrow \mu \, \mu$
Require 81.2 < M < 101.2 GeV. sinθ > 0.12. σ = 3.84 +- 0.02 pb





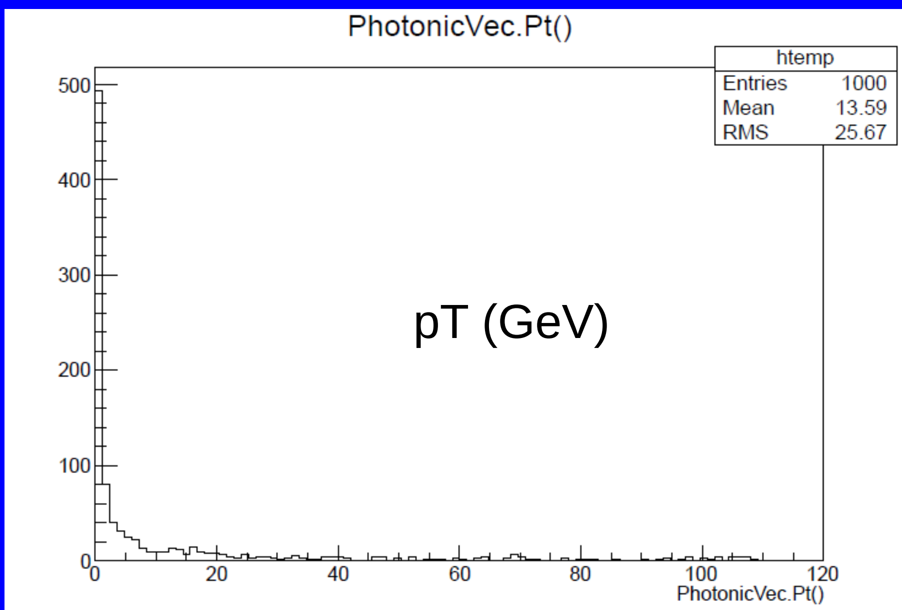
Tail to low mass from FSR

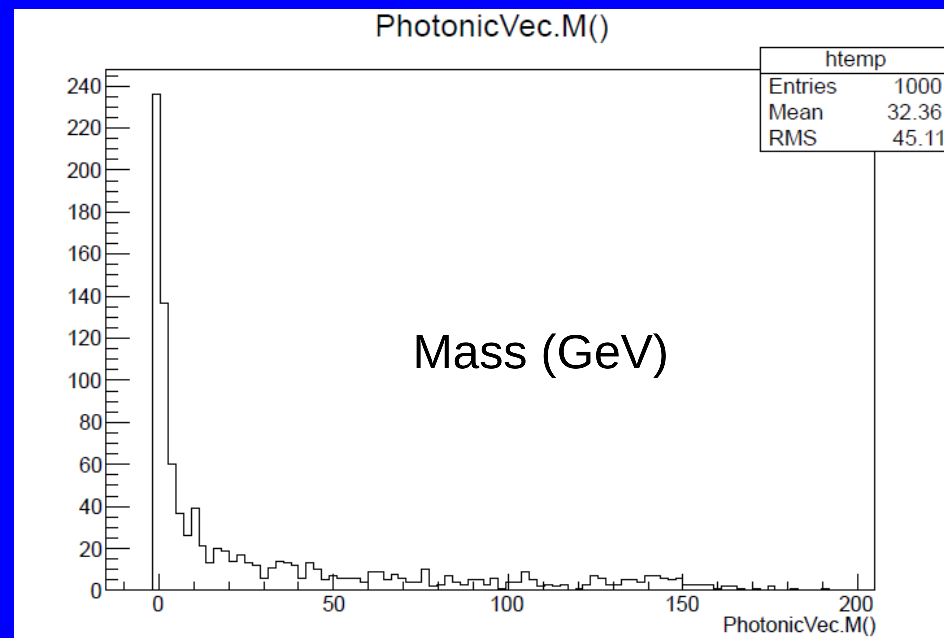
Distribution is sensitive to luminosity spectrum. Not clear to me if beam energy spread is properly included.

Whizard Generator Level Studies

ECM = 250 GeV. e-L e+R → μ μ

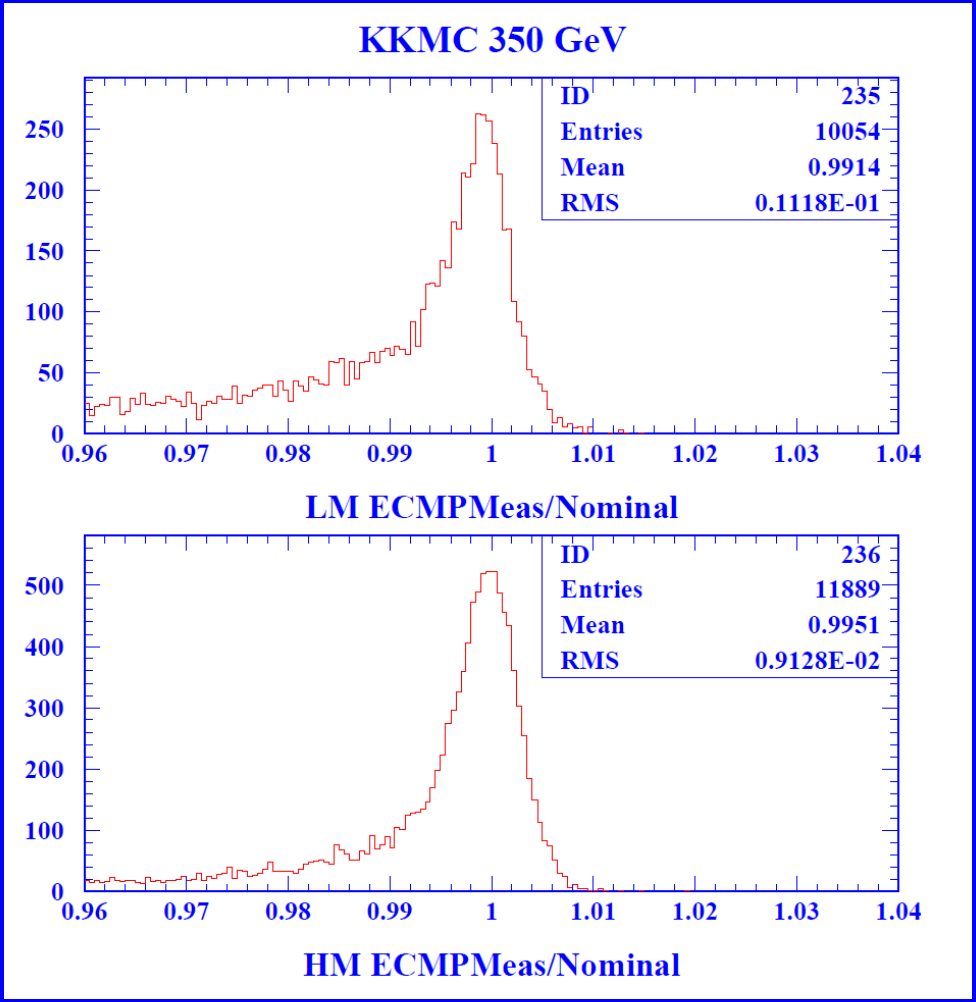
Check characteristics of photonic system (ISR + FSR).





As expected, photonic system usually has small p_T, and low mass – making 3-body assumption often plausible. But double ISR from opposite beam particles does give long tail to high mass.

KKMC Study contd.



m_{12} < 200 GeV

m_{12} > 200 GeV

High mass and low mass have similar sensitivities. High mass – more events in peak, less tail - but worse intrinsic resolution (high p_T).

Tim's Conjecture

- Slides from Tim suggest that one can fit for the tracker momentum scale without using the Z peak.
- This does not appear to be the case in my simplified tests with 3-body zero pT photon with $m \approx m_Z$ and no additional complications.
- Tests done with shifted $\sqrt{s}$ and shifted tracker momentum-scale factors
  - see no ability to distinguish a shift in one from a shift in the other.
- Because of the basic 1-1 correspondence between track pT and the $\sqrt{s_P}$ estimate, this seems to me unlikely to be correct.

$$\sqrt{s_{\mathrm{P}}} = p_{\mathrm{T}} \left(\frac{1+\cos\theta_1}{\sin\theta_1} + \frac{1+\cos\theta_2}{\sin\theta_2} \right)$$

- This is a pity – but we should have handles on the momentum scale – not least the Z mass.