Towards the exact calculation of strong field effects on polarized particles at future linear colliders

A. Hartin

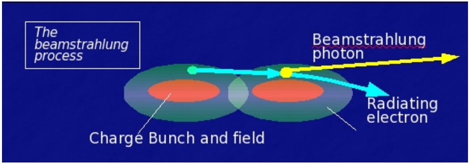
DESY

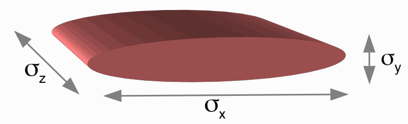
ECFA LC2013 Workshop
Mar 28, 2013

Synopsis

- Strong fields in collider interactions
- Definition of a strong field
- Furry picture - Exact interaction with strong fields
- Furry picture wavefunction solutions
  - known solutions
  - general method to obtain new solutions
  - new solutions in collider fields
- Furry picture transition probabilities - Beamstrahlung
- **IPstrong:** a new event generator to produce strong field events

Strong fields at the collider interaction point





$$\Upsilon = \frac{e}{m^3}|F_{\mu\nu}p^\nu| \equiv \frac{\text{field strength in rest frame}}{\text{Schwinger critical field}} \approx \frac{5}{6}\frac{N\ \gamma\ r_e^2}{\alpha(\sigma_x+\sigma_y)\sigma_z}$$

Υ is a natural parameter that sets the strong field scale

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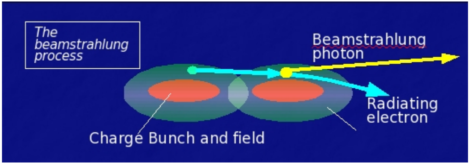

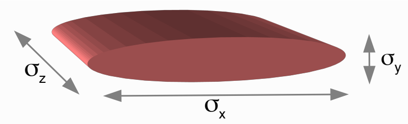

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Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
Υ_{av}	0.00015	0.001	0.24	4.9

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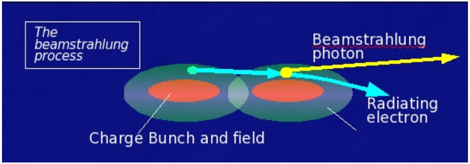

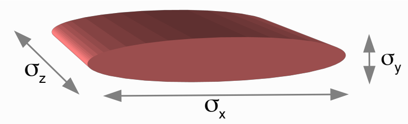

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We must develop a theory which takes into account the strong field(s) **exactly**

A quasi-nonperturbative QFT - the Furry Picture

Separate Maxwell field into external (A^{ext}) and quantised (A) parts

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}} A\!\!\!/\,\psi^{\text{FP}}$$

Solve, **exactly** equations of motion coupled to the external field

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

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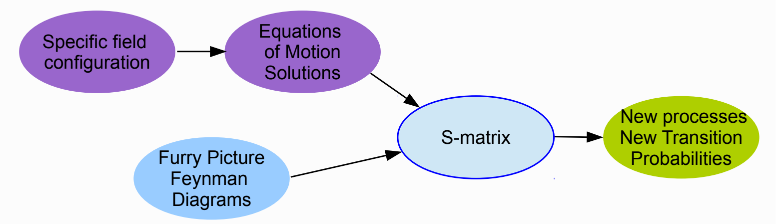
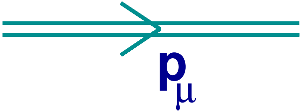


Lepton Volkov (1935) Solution

- Solution of the 2nd order Dirac equation with external potential



$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\mathsf{FP}} = 0, \quad D_\mu = \partial_\mu + ieA^{\mathsf{ext}}_\mu(k\cdot x)$$

with solution $\psi^{\mathsf{FP}} = E_p\, e^{-ip\cdot x}\, u_p$

where $E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(e\not{A}^{\mathsf{ext}}\not{k} + i2e(A^e\cdot p) - ie^2 A^{\mathsf{ext}\,2}\right)\right]$

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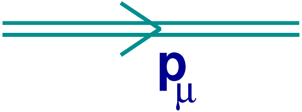

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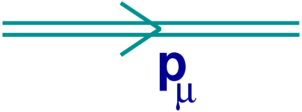

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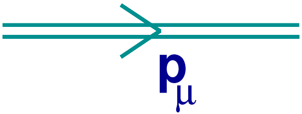

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- Volkov spinor
- Volkov phase
- Simplify x_μ dependence: $\psi^{\text{FP}} = \int dr\; e^{-i(p.x-rk)\cdot x}\; \mathcal{FT}^{-1}[E_p(r)]\, u_p$

  to get "shape function", $\mathcal{FT}^{-1}[E_p(r)]$

Exact Furry picture solutions

known solutions

- Single plane wave field - Volkov [1935]
  - Circ/Linearly polarised field, constant field - Nikishov & Ritus [1964]
  - Magnetic field - Sokolov & Ternov [1974], Baier & Katkov [1973]
  - Elliptically polarised field - L'yulka [1975]
- 2 collinear orthogonal fields - L'yulka [1975], Pardy [2004]
- n collinear fields - Hartin, Moortgat-Pick [2011]
- 2 **non**-collinear fields - Hartin [in progress]

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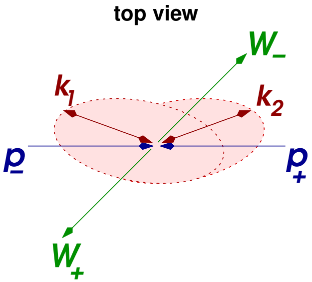
General procedure for obtaining solutions

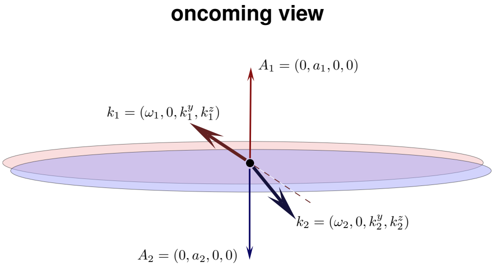
Klein-Gordon: $\left(D^2 + m^2\right) \phi_e = 0$ $\rightarrow$ phase part

2nd order Dirac: $\left(D^2 + m^2 \pm \frac{ie}{2} F^{\mu\nu} \sigma_{\mu\nu}\right) \psi_e = 0$ $\rightarrow$ spinor part

Dirac: $\left(i\not{D} - m\right) \psi_e = 0$ $\rightarrow$ particular solution

Deriving the exact solution in two non-collinear fields





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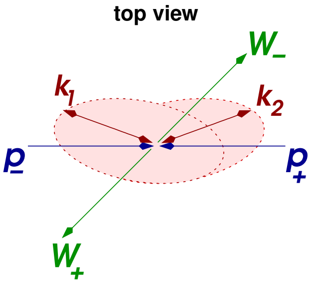

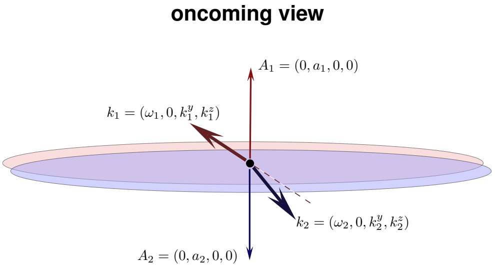

$$\left(D^2 + m^2\right)\phi_e = 0,$$

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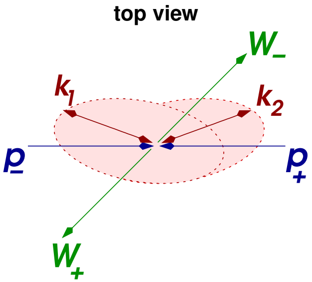

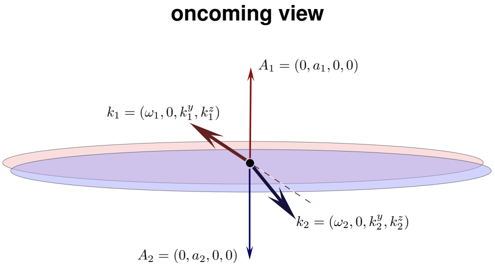

$(D^2 + m^2)\,\phi_e = 0,$ propose $\phi^e = e^{-ip\cdot x} F(\phi_1, \phi_2), \quad \phi_1 = k_1 \cdot x, \quad \phi_2 = k_2 \cdot x$

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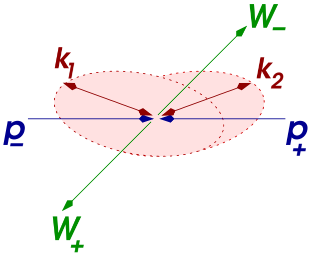

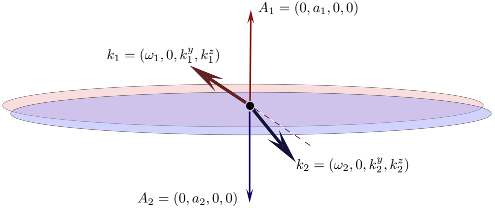

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$$-2(k_1 \cdot k_2)\frac{\partial F^2}{\partial\phi_1\partial\phi_2} + 2i\left[k_1 \cdot (p - eA_2)\frac{\partial F}{\partial\phi_1} + k_2 \cdot (p - eA_1)\frac{\partial F}{\partial\phi_2}\right] - (eA_1 + eA_2)^2 F = 0$$

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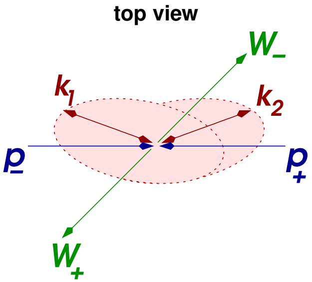

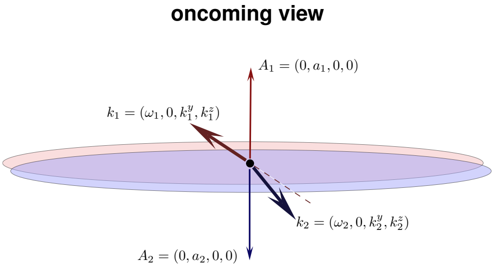

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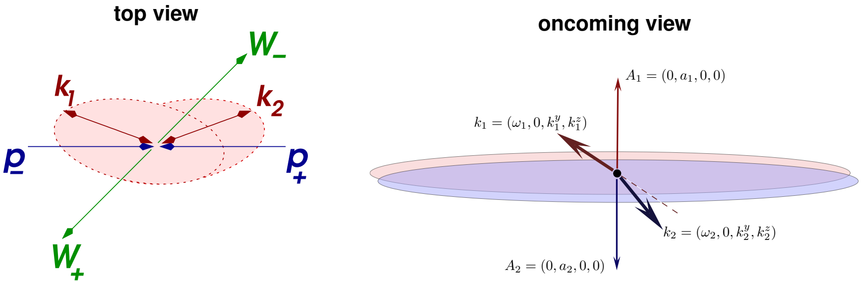
Equation	1 field Volkov	2 fields anti-collinear A	2 fields general case
Klein-Gordon	exponential	1D Gaussian	
2nd order Dirac	✓	✓	
Dirac Equation	✓	✓	
Proca equation	✓		

Deriving the exact solution in two non-collinear fields



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2nd order Dirac	✓	✓	✓
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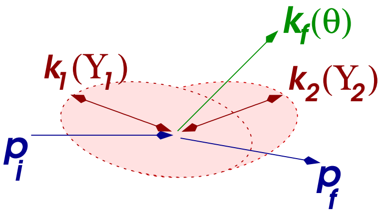
Furry Picture calculations

Field	4-momentum	Shape function
LASER	$A_\mu^{\text{ext}} = a_{1\mu}\cos(k\cdot x) + a_{2}\mu\sin(k\cdot x)$	Bessel
1 constant crossed	$A_\mu^{\text{ext}} = a_\mu \; k\cdot x$	Airy
2 non-collinear crossed	$A_\mu^{\text{ext}} = a_{1\mu}k_1\cdot x + a_{2\mu}k_2\cdot x$	Gaussian

particle process	LASER fields	1 constant crossed field	2 non-collinear crossed fields
1st Order 1γ Compton 1γ pair prod $l^- \to W^-\nu_l$	Nikishov, Ritus [1964]	Ritus [1971] Kurilin[2002]	
2nd order Moller 2γ Compton Trident process Self energy Vacuum polarisation	Oleinik[1967], Bos[1982] Hartin [2006] Hu, Müller, Keitel [2010] Becker, Mitter [1976]		
Higher orders photon splitting			

A more exact Beamstrahlung calculation



- first order Furry picture process
- 1-vertex permitted $\delta\left(p_i + rk - p_f - k_f\right)$
- Coherent pair production crossing symmetry
- helicity amplitude gives rate of spin-flip
- In general two bunches, two Υ values

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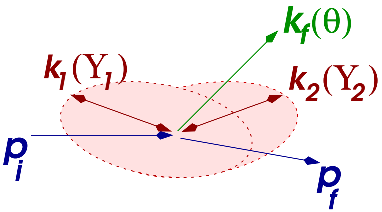

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- **Quasi-classical method:** electron motion ultra-relativistic, classical, interacts with one bunch, no crossing angle
- **Furry picture 1 crossed field solution:** ultra-relativistic, neglect one of the bunches, any crossing angle
- **Furry picture new non-collinear solutions:** any kinematics, both bunches contribute

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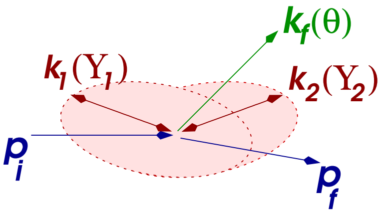

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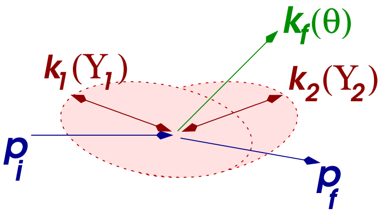

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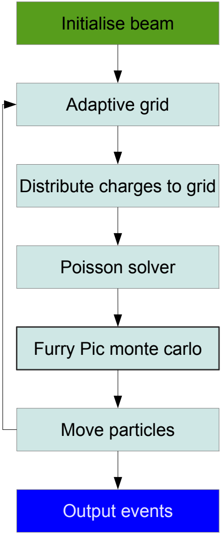
Upsilon values vary independently throughout the bunch collision

IPstrong, A new Furry picture event generator



Requirements:

We need to calculate the upsilon value at each point of a complex interaction of intense charge bunches at a collider interaction point

- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs

Summary

- **Physically**, (polarised) particle processes at the IP occur in two intense non-collinear fields
- It is more **accurate** to calculate collider processes in the Furry picture

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- Υ is a natural parameter indicating the scale of external field effects. Highly nonlinear at $\Upsilon \approx 1$, ($\Upsilon \approx$ 0.24 ILC, 4.9 CLIC)

Summary

- **Physically**, (polarised) particle processes at the IP occur in two intense non-collinear fields
- It is more **accurate** to calculate collider processes in the Furry picture
- The Furry picture is a quasi nonperturbative QFT which predicts new phenomenology
- Υ is a natural parameter indicating the scale of external field effects. Highly nonlinear at $\Upsilon \approx 1$, ($\Upsilon \approx$ 0.24 ILC, 4.9 CLIC)
- New solutions for charged particles in two non-collinear crossed fields being developed
- All physics processes are to be examined using these new solutions - starting with the beamstrahlung
- A new EM solver/generic event generator, **IPstrong** is being developed to model these intense field processes

2nd international workshop on

Physics in Intense Fields

DESY, Hamburg, 9-11 July 2013

High intensity LASERs and FELs, interactions in crystalline lattices, intense charge bunch collisions at the next generation of energy-frontier colliders, heavy-ion collisions, plasma acceleration and magnetars, all involve physics processes in ultra-intense electromagnetic fields. It is of critical importance to consider interactions with such strong fields as precisely as possible in order to understand experimental outcomes, provide additional tests of our theoretical models, and to aid the development of new experimental technologies.

The purpose of this workshop is to review the state of the art in strong-field physics. Theoretical calculations, experimental tests and simulation of physics in high intensity fields will all be examined. PIF2013 continues a series of related workshops held over previous years.

We welcome participation from all interested parties.

3 days of talks, social events and keynote speakers. The workshop will publish Proceedings

more information and registration at:

https://indico.desy.de/event/pif2013