Induced Pressure in Positron Production Target - An Analytical and Numerical Study

O. Adeyemi, A. Hartin, G. A. Moortgat-Pick, S. Riemann, F. Staufenbiel, A. Ushakov, and V. Kovalenko

II. Institute of Theoretical Physics,
University of Hamburg
and
Deutsches Elektronen-Synchrotron (DESY)

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Introduction

- The target for positron production needs to withstand induced pressure from the energy deposited by incident beam

Objective:

To determine if the target will survive the impinged incident beam

Model

- We use fluid model to simulate the target behavior

These involves:

- Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

- Equation of Motion

$$\rho \frac{\partial u}{\partial t} + \rho \cdot (u \nabla u) = -\nabla P$$

- Equation of State

$$P = \frac{\Gamma}{V} Q(\mathbf{r}, t)$$

where ρ: density; u: velocity; p: pressure; Γ: Grüneisen coefficient; V: Volume and Q: Energy deposited

Linear Approximation

By linearizing those three equations, we arrives at:

$$\frac{\partial^2 P}{\partial t^2} - \nabla \cdot (c_s^2 \nabla P) = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$

In axially symmetric case, Pressure Wave-like Equation gives:

$$\frac{\partial^2 P(r,z,t)}{\partial t^2} - c_0^2 \left(\frac{\partial^2 P(r,z,t)}{\partial r^2} + \frac{1}{r} \frac{\partial P(r,z,t)}{\partial r} + \frac{\partial^2 P(r,z,t)}{\partial z^2} \right) = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$

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Energy Deposition

By Definition:

$$Q_{bunch} = \int_0^T \int_0^{vol} \frac{\partial Q}{\partial t} \, dv \, dt$$

where:

$$\frac{\partial Q}{\partial t} = A \frac{z}{L} \exp\left(-\left(\frac{r}{\sigma_r}\right)^2\right) \exp\left(-\left(\frac{z - ct}{\sigma_z}\right)^2\right)$$

Q_{bunch} is the energy deposited per bunch; A is the amplitude; L is the target thickness; σ_r is the spot size; σ_z is the bunch length and T is time taken for the bunch to pass through the target.

Energy Deposition

After normalization, we have

$$A = \frac{8Q_{bunch}c}{\pi\sqrt{\pi}\sigma_r{}^2\sigma_z L\epsilon}$$

Energy per time deposited distributed over the volume is:

$$\frac{\partial Q}{\partial t} = \frac{8Q_{bunch}c}{\pi\sqrt{\pi}\sigma_r^2\sigma_z L\chi}\frac{z}{L}\exp\left(-\left(\frac{r}{\sigma_r}\right)^2\right)\exp\left(-\left(\frac{z-ct}{\sigma_z}\right)^2\right)$$

$\frac{\partial Q}{\partial t}$ is the energy deposited per volume per time, that is, $watt/m^3$ and ϵ is just some values that depends on R, σ_r, σ_z, L and T

Energy Deposition in Ti-Alloy

See movie in the CDF file

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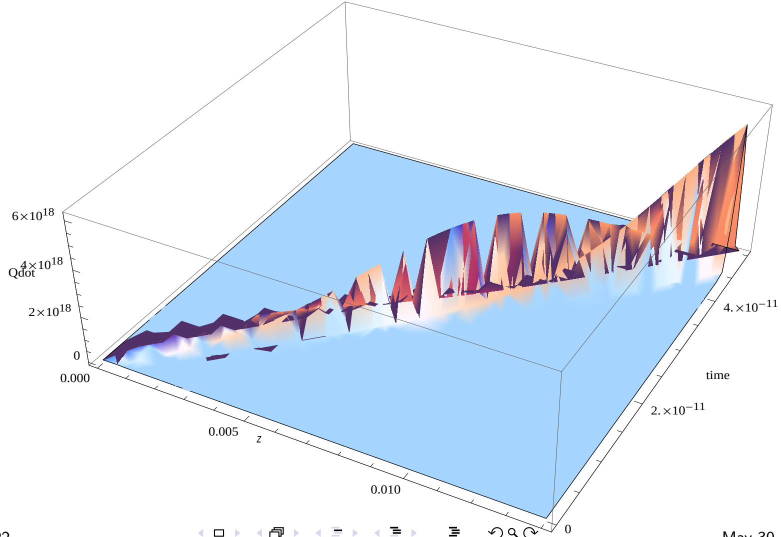
Energy Deposition in Ti-Alloy

Power deposited per m^3 along the axis:



Partial Differential Equation

Pressure Wake-Like Equation in (1+1)-D

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \frac{\partial^2 P}{\partial z^2} = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$

To solve this PDE, the problem was divided into two based on time:

1. during which the bunch is moving through the target (very short time!) and
2. after the bunch left target

Initial and Boundary Conditions for 1

$$P(z, t = 0) = 0 = \frac{\partial P(z, t = 0)}{\partial t}$$

$$P(z = 0, t) = 0 = P(z = L, t)$$

Solution Procedure

Numerical

We use a commercial software called flexPDE , which is a software system for obtaining numerical solutions to PDE's.

- we define the Qdot with Gaussian distribution
- we describe the volume and specified the BC's
- we specify the PDE and its IC

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Analytical

- introduce a dimensionless variables;
- reduced the PDE with both the IC and BC to a dimensionless form using the dimensionless varibles
- solve the dimensionless form of the PDE using Green's function

Parameters

Below are the material and incident beam parameters for ILC:

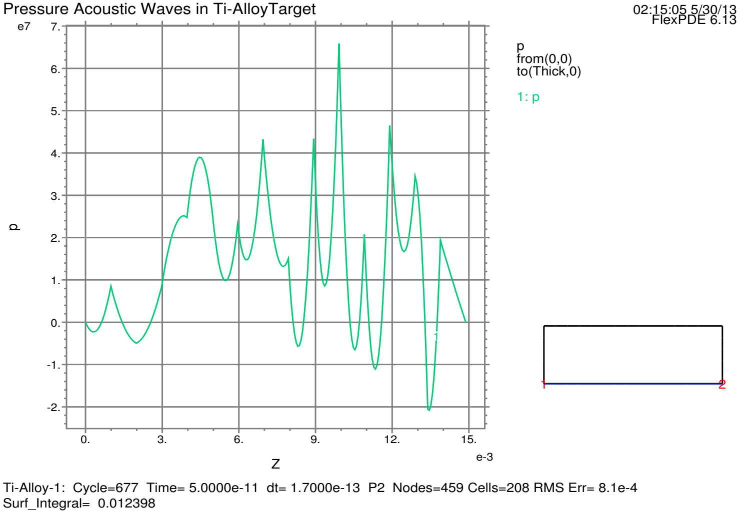
Target Material Parameters

Parameters	**Units**	**Tungsten**	**Ti-Alloy**
Target Thickness	mm	1.408	14.88
Radius	m	0.005	0.005
Grüneisen constant	-	1.647	1.262
Sound Speed	m/s	5174	5072.83
Tensile Strength	MPa	750	880

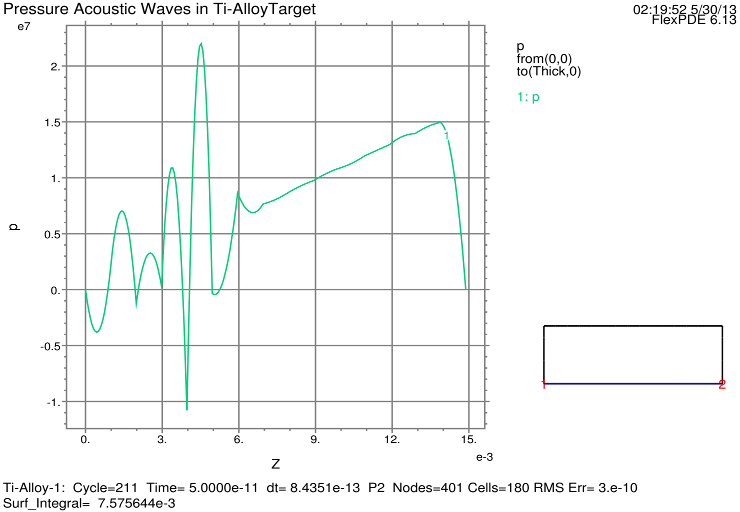
ILC Photon Beam Parameters

Parameters	**Symbol**	**Units**	**Value**
Beam size	σ_z	m	0.0003
Transerved length	$\sigma_\perp$	m	0.002
Energy Deposited	Q_{bunch}	J	1.15

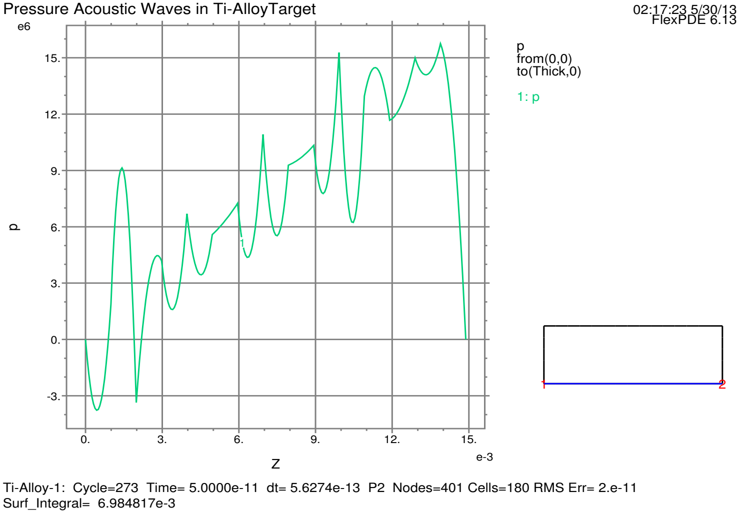
Numerical Result without Error Limit:



Numerical Result- Errlim: 0.5



Numerical Result- Errlim: 0.1



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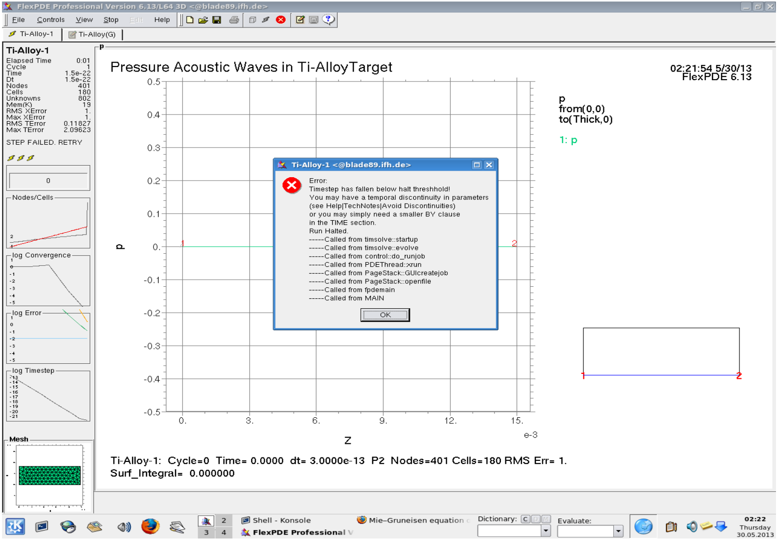
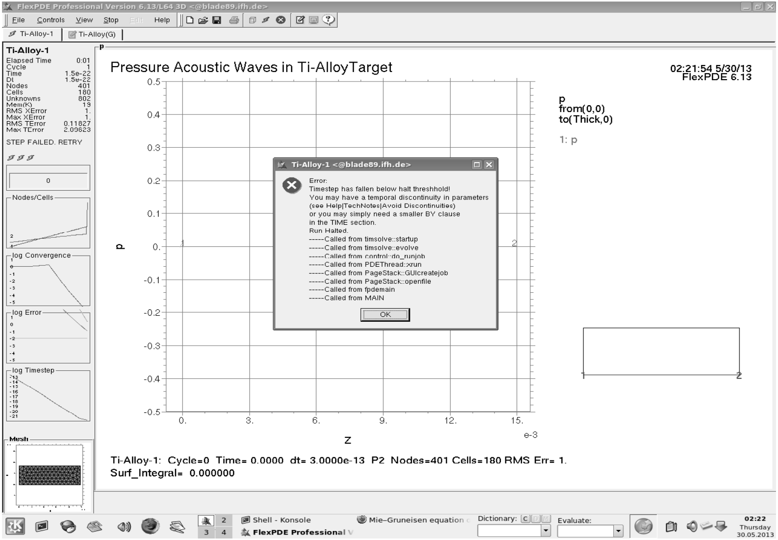
Numerical Result- errlim: 0.01

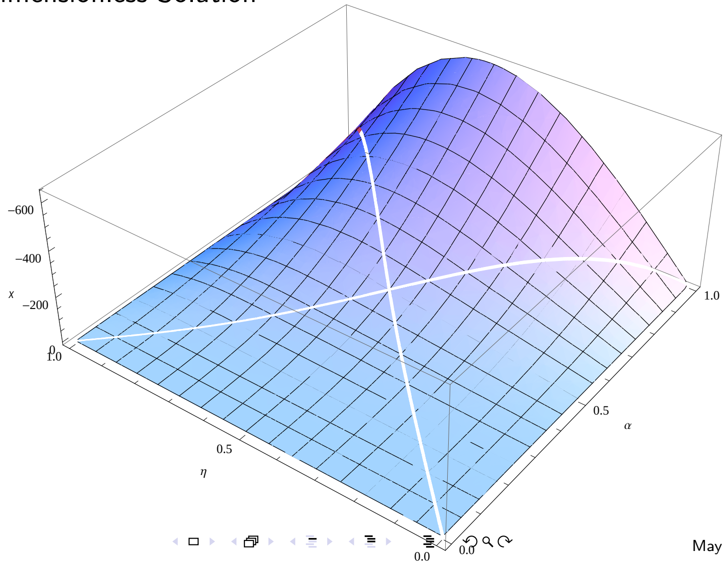


Numerical Result- errlim: 0.01



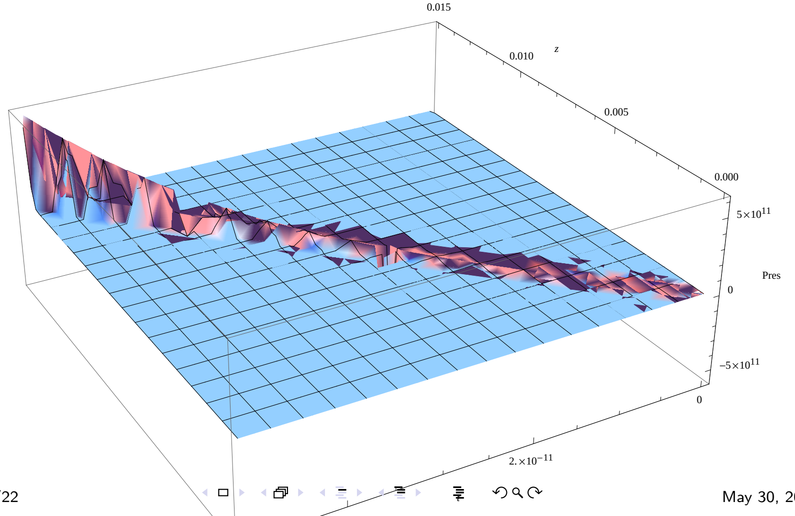
Analytical Results for Ti-Alloy

Dimensionless Solution



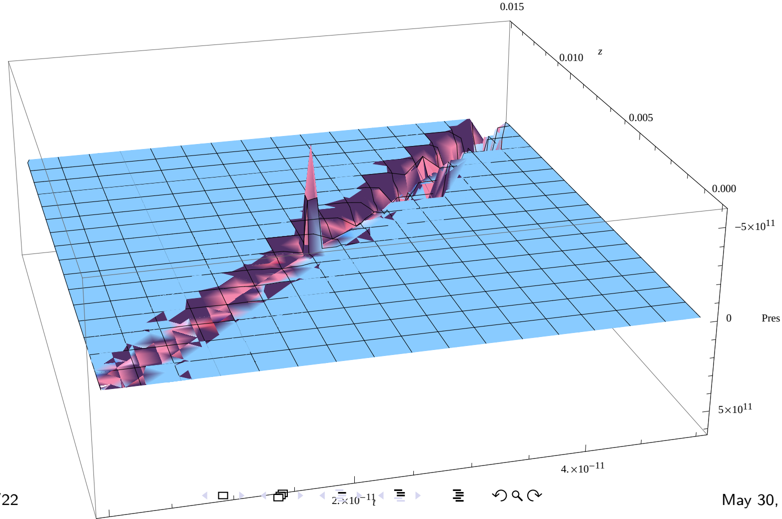
Analytical Results for Ti-Alloy

Dimensional Solution (all units are in SI units)



Analytical Results for Ti-Alloy

Dimensional Solution



Analytical Results for Ti-Alloy- Evolution of Pressure in Time

see movie in the CDF file

Summary and Outlook

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- study in details, the singularity
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extend the model to non-linear waves e.g. solitons, shock waves etc

THANK YOU FOR LISTENING

Question Please