

Induced Pressure in Positron Production Target - An Analytical and Numerical Study

O. Adeyemi, A. Hartin, G. A. Moortgat-Pick, S. Riemann,
F. Staufenbiel, A. Ushakov, and V. Kovalenko

II. Institute of Theoretical Physics,
University of Hamburg
and
Deutsches Elektronen-Synchrotron (DESY)

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- The target for positron production needs to withstand induced pressure from the energy deposited by incident beam

Objective:

To determine if the target will survive the impinged incident beam

* We use fluid model to simulate the target behavior

These involves:

☞ Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

☞ Equation of Motion

$$\rho \frac{\partial u}{\partial t} + \rho \cdot (u \nabla u) = -\nabla P$$

☞ Equation of State

$$P = \frac{\Gamma}{V} Q(\mathbf{r}, t)$$

where ρ : density; u : velocity; p : pressure; Γ : Grüneisen coefficient;
 V : Volume and Q : Energy deposited

By linearizing those three equations, we arrive at:

$$\frac{\partial^2 P}{\partial t^2} - \nabla \cdot (c_s^2 \nabla P) = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$



In axially symmetric case, Pressure Wave-like Equation gives:

$$\frac{\partial^2 P(r, z, t)}{\partial t^2} - c_0^2 \left(\frac{\partial^2 P(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial P(r, z, t)}{\partial r} + \frac{\partial^2 P(r, z, t)}{\partial z^2} \right) = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$

By Definition:

$$Q_{bunch} = \int_0^T \int_0^{vol} \frac{\partial Q}{\partial t} dv dt$$

where:

$$\frac{\partial Q}{\partial t} = A \frac{z}{L} \exp\left(-\left(\frac{r}{\sigma_r}\right)^2\right) \exp\left(-\left(\frac{z-ct}{\sigma_z}\right)^2\right)$$

Q_{bunch} is the energy deposited per bunch; A is the amplitude; L is the target thickness; σ_r is the spot size; σ_z is the bunch length and T is time taken for the bunch to pass through the target.

After normalization, we have

$$A = \frac{8Q_{bunch}c}{\pi\sqrt{\pi}\sigma_r^2\sigma_zL\epsilon}$$

Energy per time deposited distributed over the volume is:

$$\frac{\partial Q}{\partial t} = \frac{8Q_{bunch}c}{\pi\sqrt{\pi}\sigma_r^2\sigma_zL\chi} \frac{z}{L} \exp\left(-\left(\frac{r}{\sigma_r}\right)^2\right) \exp\left(-\left(\frac{z-ct}{\sigma_z}\right)^2\right)$$

$\frac{\partial Q}{\partial t}$ is the energy deposited per volume per time, that is, $watt/m^3$ and ϵ is just some values that depends on R , σ_r , σ_z , L and T



Energy Deposition in Ti-Alloy



EPG Spring Meeting

O. Adeyemi

Introduction

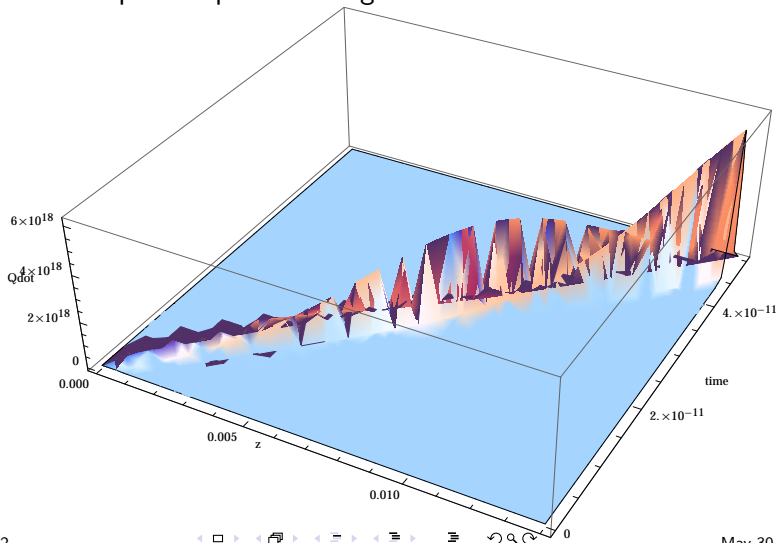
Model

Results

Summary and Outlook

See movie in the CDF file

Power deposited per m^3 along the axis:



Pressure Wake-Like Equation in (1+1)-D

$$\frac{\partial^2 P}{\partial t^2} - c_s^2 \frac{\partial^2 P}{\partial z^2} = \frac{\Gamma}{V} \frac{\partial^2 Q}{\partial t^2}$$

To solve this PDE, the problem was divided into two based on time:

- 1 during which the bunch is moving through the target (very short time!) and
- 2 after the bunch left target

Initial and Boundary Conditions for 1

$$P(z, t = 0) = 0 = \frac{\partial P(z, t = 0)}{\partial t}$$

$$P(z = 0, t) = 0 = P(z = L, t)$$

Numerical

We use a commercial software called **flexPDE**, which is a software system for obtaining numerical solutions to PDE's.

- we define the Q_{dot} with Gaussian distribution
- we describe the volume and specified the BC's
- we specify the PDE and its IC

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Analytical

- introduce a dimensionless variables;
- reduced the PDE with both the IC and BC to a dimensionless form using the dimensionless variables
- solve the dimensionless form of the PDE using Green's function

Below are the material and incident beam parameters for ILC:

Target Material Parameters

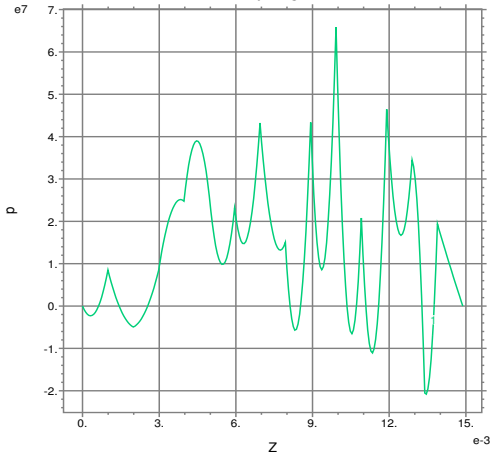
Parameters	Units	Tungsten	Ti-Alloy
Target Thickness	<i>mm</i>	1.408	14.88
Radius	<i>m</i>	0.005	0.005
Grüneisen constant	-	1.647	1.262
Sound Speed	<i>m/s</i>	5174	5072.83
Tensile Strength	<i>MPa</i>	750	880

ILC Photon Beam Parameters

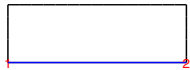
Parameters	Symbol	Units	Value
Beam size	σ_z	<i>m</i>	0.0003
Transversed length	σ_{\perp}	<i>m</i>	0.002
Energy Deposited	Q_{bunch}	<i>J</i>	1.15

Numerical Result without Error Limit:

Pressure Acoustic Waves in Ti-AlloyTarget

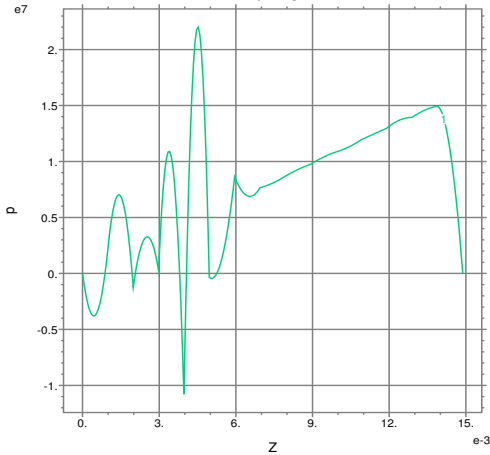


p
from(0,0)
to(Thick,0)
1: p

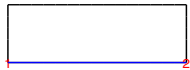


Ti-Alloy-1: Cycle=677 Time= 5.0000e-11 dt= 1.7000e-13 P2 Nodes=459 Cells=208 RMS Err= 8.1e-4
Surf_Integral= 0.012398

Pressure Acoustic Waves in Ti-AlloyTarget

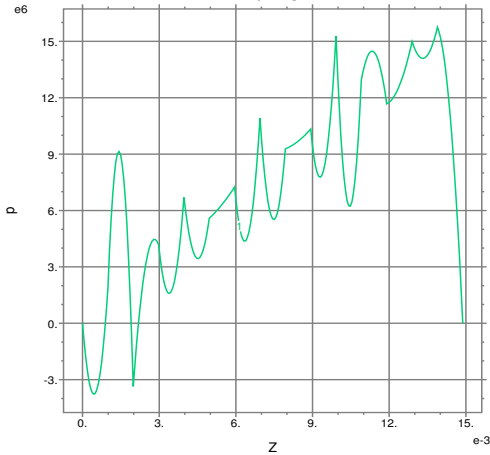


p
 from(0,0)
 to(Thick,0)
 1: p

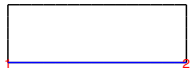


Ti-Alloy-1: Cycle=211 Time= 5.0000e-11 dt= 8.4351e-13 P2 Nodes=401 Cells=180 RMS Err= 3.e-10
 Surf_Integral= 7.575644e-3

Pressure Acoustic Waves in Ti-AlloyTarget



p
from(0,0)
to(Thick,0)
1: p



Ti-Alloy-1: Cycle=273 Time= 5.0000e-11 dt= 5.6274e-13 P2 Nodes=401 Cells=180 RMS Err= 2.e-11
Surf_Integral= 6.984817e-3

FlexPDE Professional Version 6.13.164.3D - @blade09.ifh.de

File Controls View Stop Help

Ti-Alloy-1 | Ti-Alloy(G)

Ti-Alloy-1

Elapsed Time 0:01
 Cycle 1
 Time 1.5e-22
 DT 1.5e-22
 Nodes 401
 Cells 180
 Unknowns 802
 Mem(s) 19
 RMS %Error 1
 Max %Error 0.11827
 Max Error 2.09823

STEP FAILED. RETRY

0

Nodes/Cells

log Convergence

log Error

log Timestep

Mesh

Pressure Acoustic Waves in Ti-AlloyTarget

0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5

0. 3. 6. 9. 12. 15. e-3

Error:
 Timestep has fallen below halt threshold!
 You may have a temporal discontinuity in parameters
 (see Help/TechNotes/Avoid Discontinuities)
 or you may simply need a smaller BY clause
 in the TIME section.
 Run Halted.
 -----Called from fmsolve:startup
 -----Called from fmsolve:evolve
 -----Called from control:do_pargob
 -----Called from PDEThread:zrun
 -----Called from PageStack:Glicreatejob
 -----Called from PageStack:openfile
 -----Called from fpedtsim
 -----Called from MAIN

OK

p
 from(0,0)
 to(Thick,0)
 1: p

02:21:54 5/30/13
 FlexPDE 6.13

Ti-Alloy-1: Cycle=0 Time= 0.0000 dt= 3.0000e-13 P2 Nodes=401 Cells=180 RMS Err= 1.
 Surf_Integral= 0.000000

2 Shell - Konsole
 3 4 Mie-Grüneisen equation Dictionary: C Evaluate: 02:22 Thursday 30.05.2013

FlexPDE Professional Version 6.13/L64.3D <@blade69.ifh.de>

File Controls View Stop Tools Help

Ti-Alloy-1 Ti-Alloy(G)

Ti-Alloy-1

Elapsed Time 0.01
 Cycle 1
 Time 1.5e-22
 Dt 1.5e-22
 Nodes 401
 Cells 180
 Unknowns 802
 Min(r0) 1.5
 RMS XError 1
 Max XError 0.11827
 RMS TError 2.02653
 Max TError

STEP FAILED. RETRY

Nodes/Cells

Convergence

Error

TimeStep

Mesh

Pressure Acoustic Waves in Ti-Alloy Target

Ti-Alloy-1: Cycle=0 Time= 0.0000 dt= 3.0000e-13 P2 Nodes=401 Cells=180 RMS Err= 1
 Surf_Integral= 0.000000

02:21:54 5/30/13
 FlexPDE 6.13

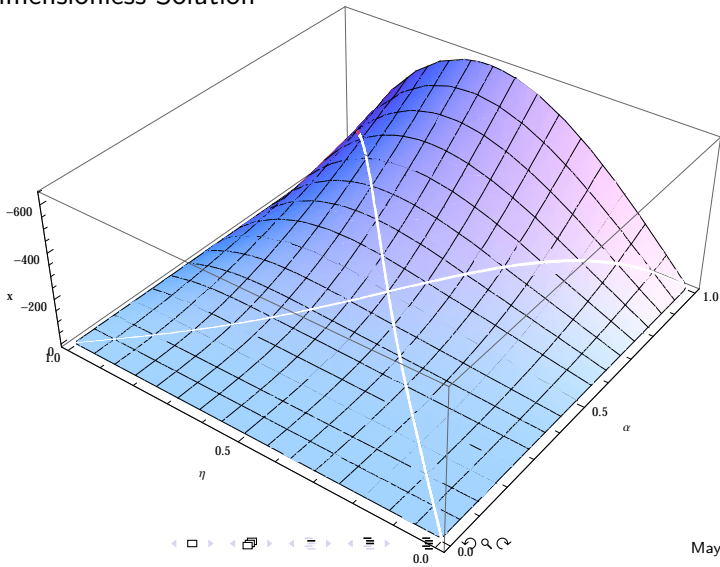
P
 from(0,0)
 to(Thick,0)
 1: p

Error:
 Timestep has fallen below halt threshold!
 You may have a temporal discontinuity in parameters
 (see Help/TechNotes/Avoid Discontinuities)
 or you may simply need a smaller BY clause
 in the TIME section.
 Run halted.
 -----Called from tinsolve:startup
 -----Called from tinsolve:evolve
 -----Called from control:dt_nupjob
 -----Called from PDEThread:zrun
 -----Called from PageStack:GUICreateJob
 -----Called from PageStack:openfile
 -----Called from fpedmain
 -----Called from MAIN

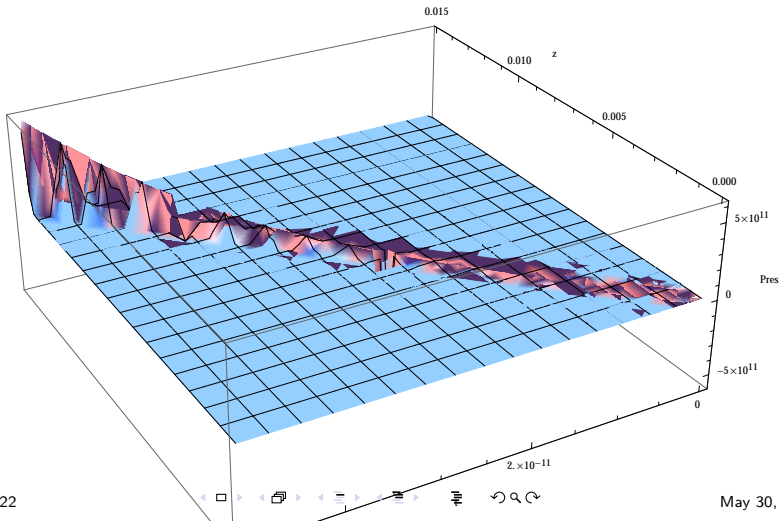
OK

Shell - Konsole Mie-Grüneisen equation Dictionary: C Evaluate: 02:22 Thursday 30.05.2013

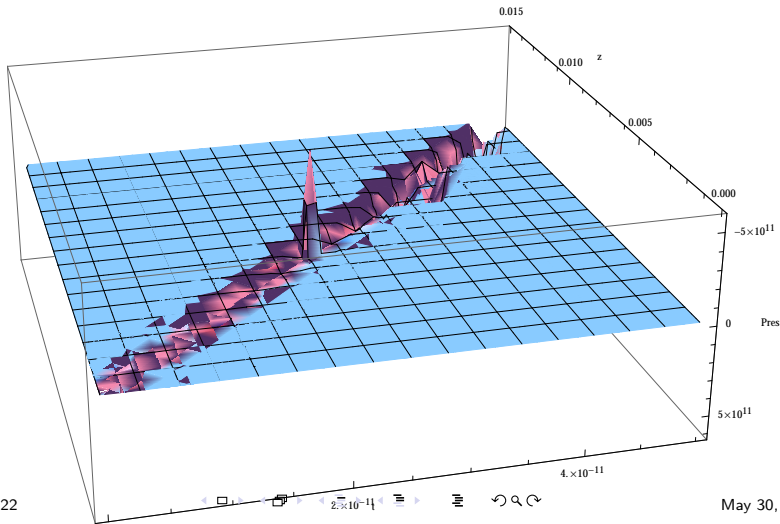
Dimensionless Solution



Dimensional Solution (all units are in SI units)



Dimensional Solution





see movie in the CDF file



Summary and Outlook



We have a solution to the induced pressure in target material during the time bunch travel through the target.

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extend the model to non-linear waves e.g. solitons, shock waves etc

Question Please

