Slepton Mass Measurement and the Luminosity Spectrum at CLIC

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on behalf of the CLIC Detector and Physics Studies

ECFA LC Workshop
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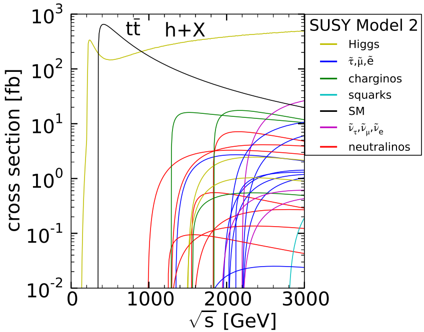
CLIC Detector Benchmark Processes

- Evaluate physics performance at 3 TeV with typical New Physics signatures
- One of the benchmarks: Slepton pair production



Smuon Pair-Production and Decay

- $e^+e^- \to \widetilde{\mu}_R^+\widetilde{\mu}_R^- \to \mu^+\mu^-\widetilde{\chi}_1^0\widetilde{\chi}_1^0$
- Event topology: Two oppositely charged leptons and missing energy
- Masses for this model:
  - $m_{\widetilde{\mu}_R} = 1011$ GeV
  - $m_{\widetilde{\chi}_1^0} = 340$ GeV
- Cross-section: 0.70 fb at $\sqrt{s} = 3$ TeV (with unpolarized beams)
- Assuming 2.0 ab^{-1} of integrated luminosity
- The two body decay leads to uniform distribution of muon energies
- Masses can be extracted from the endpoints of this distribution
- However, Initial State Radiation, Luminosity Spectrum and detector resolution modify the spectrum

Muon Energy Spectrum

- The uniform distribution has the endpoints:

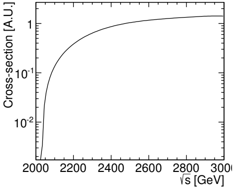
$$E_{\mathrm{H,L}} = \frac{\sqrt{s'}}{4}\left(1 - \frac{m_{\tilde{\chi}0}^2}{m_{\tilde{\mu}\pm}^2}\right)\left(1 \pm \sqrt{1 - 4\frac{m_{\tilde{\mu}\pm}^2}{s'}}\right),$$

  where $\sqrt{s'}$ is the effective centre-of-mass energy
- The uniform distribution is modified into: $f(E_\mu; m_{\tilde{\mu}}, m_{\tilde{\chi}_1^0}) =$

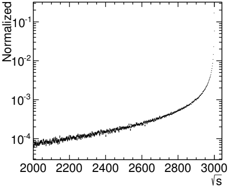
$$\left(\sigma(\sqrt{s'}) \otimes \mathrm{ISR}(\sqrt{s'}) \otimes \mathscr{L}(\sqrt{s'})\right) \times \left(\mathrm{U}(E_\mu; \sqrt{s'}, m_{\tilde{\mu}}, m_{\tilde{\chi}_1^0}) \otimes \mathrm{D}(E_\mu)\right)$$

- Cross-section and ISR is taken from WHIZARD
- Luminosity Spectrum from GUINEAPIG beam-beam simulation (for now)
- Detector resolution from full GEANT4 simulation and reconstruction

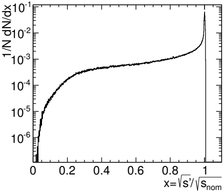
σ



ISR



$\mathscr{L}$



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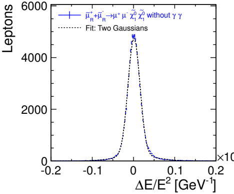


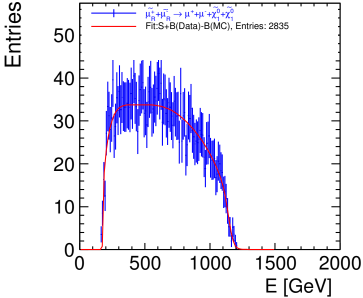
Fit to Muon Energy Spectrum

The background subtracted muon energy distribution is fit to $f(E_{\mu}; m_{\tilde{\mu}}, m_{\tilde{\chi}_1^0})$

- Result with the GuineaPig luminosity spectrum:

$$m_{\tilde{\mu}} = (1012 \pm 3(\text{stat}))\ \text{GeV},$$
$$m_{\tilde{\chi}_1^0} = (\ 343 \pm 7(\text{stat}))\ \text{GeV}$$

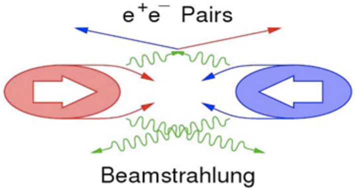
- Results compatible with generator value
- How well can the luminosity spectrum be known experimentally?
- Will this have an impact on the mass measurement?

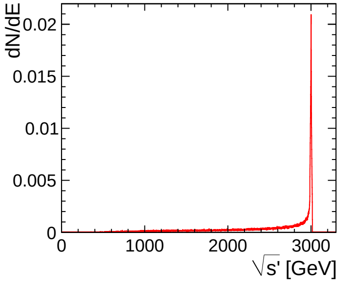


Reconstruction of the Luminosity Spectrum

Beam–Beam Interactions

- Large luminosities require high bunch charge and small beams $L \propto \frac{N^2}{\sigma_x \sigma_y}$
- Electromagnetic fields during bunch crossing $B \propto \frac{\gamma N}{\sigma_z(\sigma_x+\sigma_y)}$ cause deflection of beam particles
- Deflection of particles by the other bunch leads to synchrotron radiation (Beamstrahlung)
- Energy loss leads to luminosity spectrum
  - Still 30% of luminosity above 99% of nominal energy





Luminosity spectrum for 3 TeV CLIC

Measuring the Luminosity Spectrum

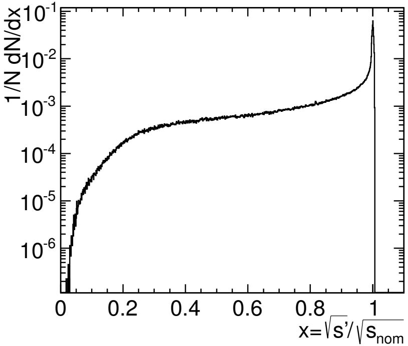
Beam–beam effects (and thus the luminosity spectrum) are highly dependent on bunch geometries

- Cannot measure bunch geometry to sufficient detail
- Bunch geometry changes over time
- If geometry is not known, simulation is not possible
- Downstream measurement of Beamstrahlung photons give no direct access to luminosity spectrum

Therefore: Have to measure luminosity spectrum at the IP with the detector

The Luminosity Spectrum: Definitions

CLIC $\sqrt{s_{\text{nom}}} = 3$ TeV luminosity spectrum as simulated by GUINEAPIG



- Given two particles with the energies E_1 and E_2 colliding head-on, the centre-of-mass energy is $\sqrt{s'} = 2\sqrt{E_1 E_2} = 2E_{\text{Beam}}\sqrt{x_1 x_2}$ $(x_{1,2} = E_{1,2}/E_{\text{Beam}})$
- The luminosity spectrum is the probability distribution of centre-of-mass energies $\mathscr{L}(\sqrt{s'})$

$$\mathscr{L}(\sqrt{s'}) = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \mathscr{L}(x_1, x_2)\,\delta\left(\frac{\sqrt{s'}}{\sqrt{s_{\text{nom}}}} - \sqrt{x_1 x_2}\right)$$

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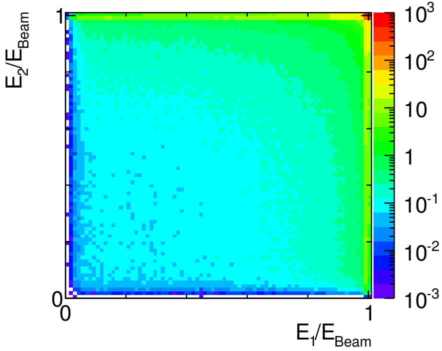

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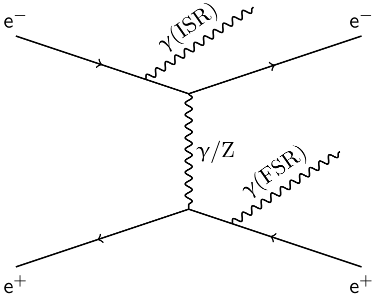
- Better: The luminosity spectrum is the probability distribution of the pairs of the particle energies $\mathscr{L}(E_1, E_2)$
  - Only reconstructing the centre-of-mass energy ignores the longitudinal boost of the system
  - Strong correlation between the two particle energies
  - Account for asymmetric beams

$$\mathscr{L}(\sqrt{s'}) = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \mathscr{L}(x_1, x_2)\,\delta\left(\frac{\sqrt{s'}}{\sqrt{s_{\text{nom}}}} - \sqrt{x_1 x_2}\right)$$

What Do We Measure in the Detector?

- Need a process with large cross-section: Bhabha scattering
- In the detector we measure the final state electron and positron distribution affected by the cross-section (initial state radiation (ISR), final state radiation (FSR), $\sqrt{s'}$ dependence)
- There is no way, for an individual event, to know if the energy was lost from initial state radiation or Beamstrahlung
- The distributions are also affected by the resolution of the respective sub-detector



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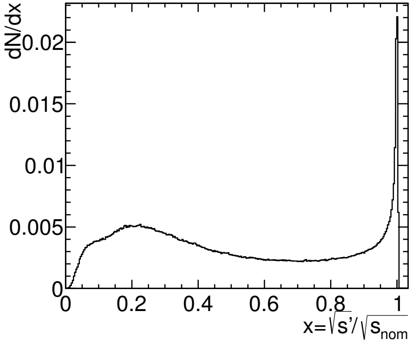
Distributions after Bhabha scattering (+ISR) and cross-section (without detector resolutions)

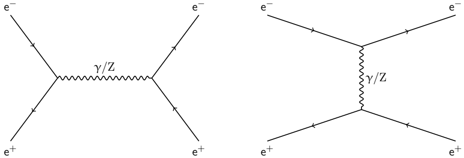


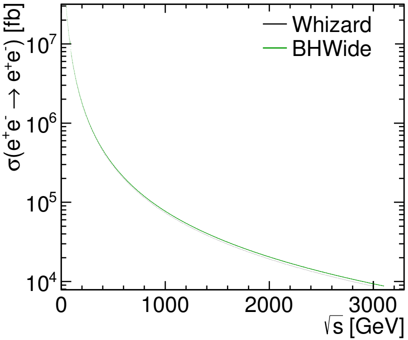
Bhabha Scattering

- Bhabha scattering $e^+e^- \to e^+e^-(\gamma)$ has:
  - Large cross-section
  - Well known cross-section (calculable to high precision)
- Cross-section: 10 000 fb at 3 TeV (with polar angle of electrons above 7°)
  - Proportional to $1/(s \sin^3 \theta/2)$
- Can reconstruct relative centre-of-mass energy from polar angle difference (acollinearity)

$$\frac{\sqrt{s'_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$$

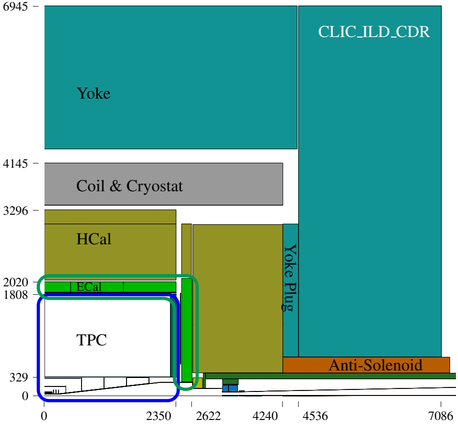
- Also measure the energy of final state electron and positron





Tracker and Electromagnetic Calorimeter

- Use the (silicon) trackers to obtain the polar angles θ_1 and θ_2
- Measure particle energies with the electromagnetic calorimeter
  - Good energy resolution for electrons and photons
  - Better than the tracker for 1.5 TeV electrons at small polar angles



How Do We Reconstruct the Luminosity Spectrum from the Measurement?

With the distribution of observables O

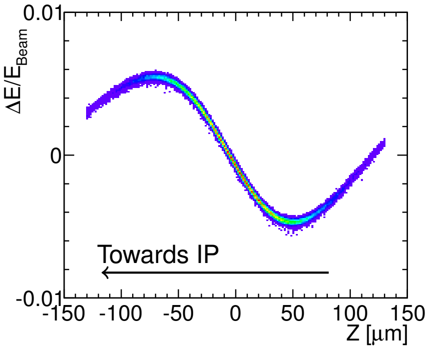
$$f(O_1, O_2, \ldots) \approx$$
$$\sigma(O_1, O_2, \ldots; E_1, E_2) \times \mathscr{L}(E_1, E_2) \otimes \mathrm{ISR}(E_1, E_2) \otimes \mathrm{FSR}(O_1, O_2, \ldots) \otimes \mathrm{D}(E_1)\mathrm{D}(E_2)$$

connected to the luminosity spectrum $\mathscr{L}(E_1, E_2)$ and measurable in the detector. One can then:

- **Model (i.e., parameterise) the luminosity spectrum**
- Let Bhabha generator take care of cross-section and initial state radiation
- Do GEANT4 simulation for detector resolutions
- Use a reweighting technique for *efficient* fitting and extract $\mathscr{L}$

Beam-Energy Spread I

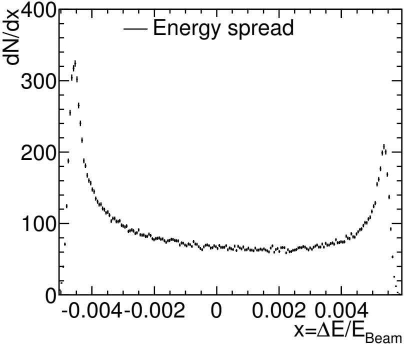
Particle energy vs. longitudinal position from the accelerator simulation



- Energy distribution in the bunch mostly due to intra-bunch wakefields and RF phase offset in main Linac
- Front of bunch gains more energy, because wakefields reduce effective gradient for the tail

Beam-Energy Spread II

- Beam-Energy spread shows two peaks
- Mean around the nominal beam-energy



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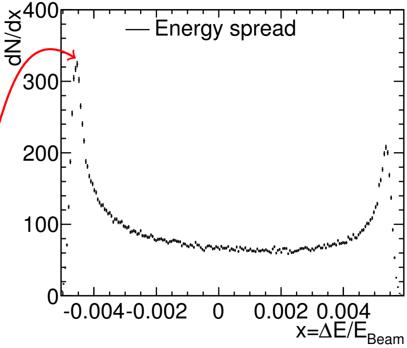
- Beam-Energy spread shows two peaks
- Mean around the nominal beam-energy
- N.B.: Lower energy peak is *back* of the bunch



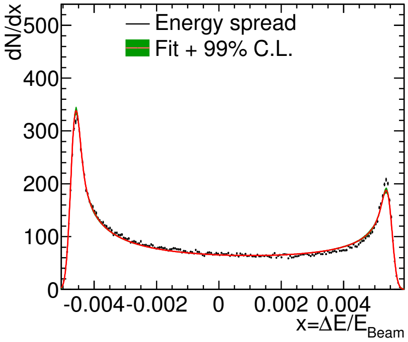
Beam-Energy Spread Function

- Beam-Energy Spread:
  Beta-distribution $b = x^{a_1}(1-x)^{a_2}$
  convoluted with Gauss function

$$\mathrm{BES}(x) = \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} b(\tau)\mathrm{Gauss}(x-\tau)\mathrm{d}\tau$$

- 5 parameters, including min. and max. of beta-distribution range
- χ^2/ndf = 764/195
- Tried many other functions (Cosh, Polynomials), none of them work as well with a limited number of parameters

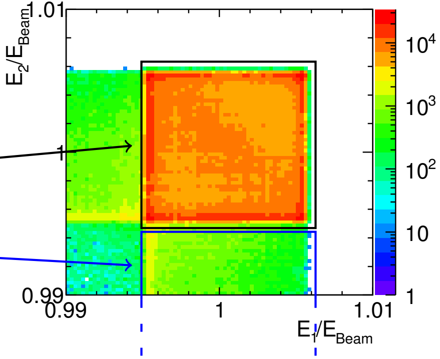
Particle energy distribution from accelerator simulation



Luminosity-weighted Beam-Energy Spread

- Due to the correlation of particle energy and longitudinal position, Beamstrahlung, and beam-beam effects, two vastly different beam-energy spread distributions emerge for the luminosity spectrum
- *Peak* Region: Both particles with $E > 0.995E_{\text{Beam}}$
- *Arms* Region: Only one of the particles with $E > 0.995E_{\text{Beam}}$
- Both can be fit with a beta-distribution convoluted with a Gauss function (keeping x_{min}, x_{max}, and σ fixed)

Peak of the luminosity spectrum



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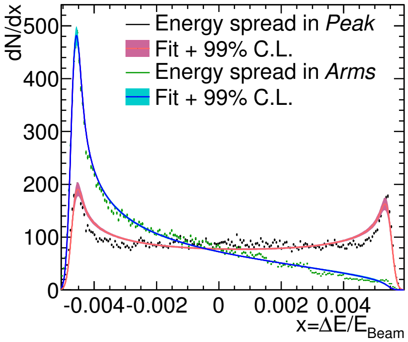
Particle energy distribution from the GuineaPig simulation

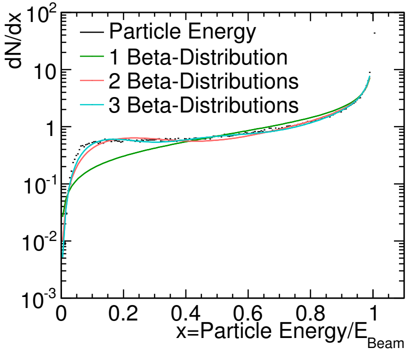


Beamstrahlung

- Second contribution to luminosity spectrum is energy loss due to Beamstrahlung
- Potentially large loss of energy for some particles

Fitting the particle Energy Spectrum

- Upper bound of $0.995 E_{\text{Beam}}$, because of impact of beam-energy spread (Particle energy is convolution of Beamstrahlung and beam-energy spread effect)
- Keep small number of parameters: Limit model to $0.5 E_{\text{Beam}}$ and a single beta-distribution, but could extend in the future



$$\int_0^{0.995 E_{\text{Beam}}} = 1$$

$$b_{\text{linear}}(x) = \sum_{i=1}^{N_{\text{Beta}}} p_i \, b(x; [p]_i)$$

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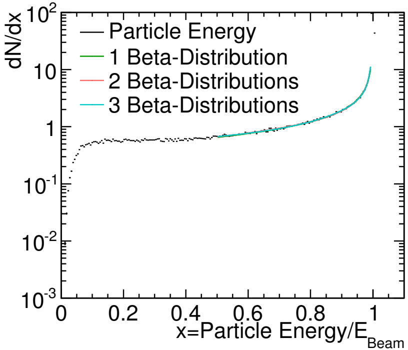

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The MODEL: Putting the Parts Together

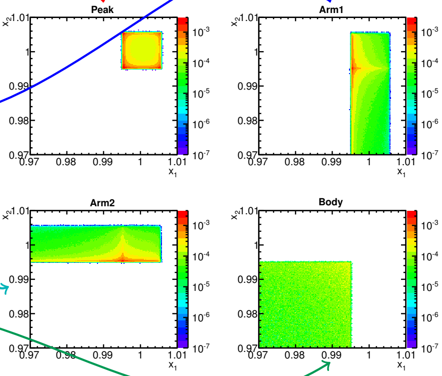
$$\mathscr{L}(x_1, x_2) =$$

$$p_{\text{Peak}}\,\delta(1-x_1)\otimes \text{BES}\left(x_1;[p]_1^{\text{Peak}}\right)\;\delta(1-x_2)\otimes \text{BES}\left(x_2;[p]_2^{\text{Peak}}\right)$$

$$+\,p_{\text{Arm1}}\,\delta(1-x_1)\otimes \text{BES}\left(x_1;[p]_1^{\text{Arm}}\right)\;\text{BB}\left(x_2;[p]_2^{\text{Arm}},\beta^1_{\text{limit}}\right)$$

$$+\,p_{\text{Arm2}}\,\text{BB}\left(x_1;[p]_1^{\text{Arm}},\beta^1_{\text{limit}}\right)\;\delta(1-x_2)\otimes \text{BES}\left(x_2;[p]_2^{\text{Arm}}\right)$$

$$+\,p_{\text{Body}}\,\text{BG}\left(x_1;[p]_1^{\text{Body}},\beta^2_{\text{limit}}\right)\;\text{BG}\left(x_2;[p]_2^{\text{Body}},\beta^2_{\text{limit}}\right)$$



With

$$\text{BES}(x) = \int_{x_{\min}}^{x_{\max}} b(\tau)\text{Gauss}(x-\tau)\,\mathrm{d}\tau$$

$$\text{BB}(x) = (b\otimes\text{BES})(x)$$

$$\text{BG}(x) = (b\otimes g)(x)$$

Model: 19 free parameters, here drawn with arbitrary parameter values

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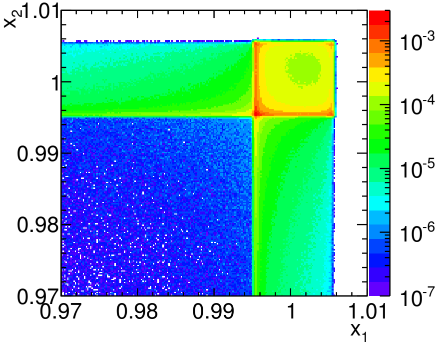

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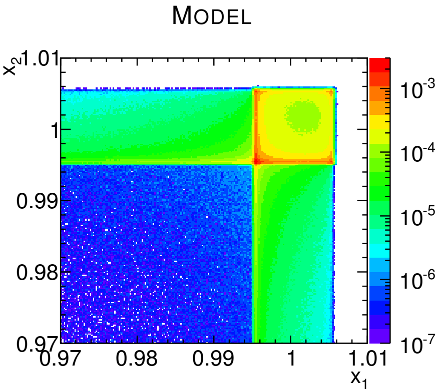
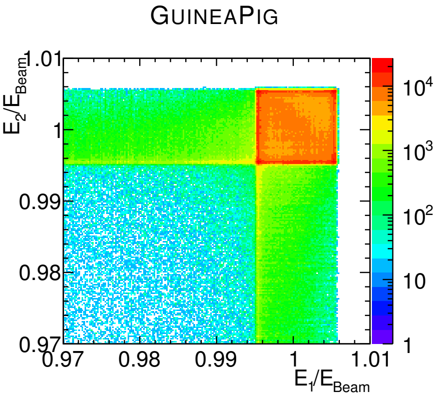
MODEL VS. GUINEAPIG





Arbitrary parameter values for the MODEL

Reweighting Fit in Words

Reweighting technique uses χ^2-fit of two histogram with a distribution like

$$f(O_1, O_2, \ldots) \approx$$

$$\sigma(O_1, O_2, \ldots; E_1, E_2) \times \mathscr{L}(E_1, E_2) \otimes \mathrm{ISR}(E_1, E_2) \otimes \mathrm{FSR}(O_1, O_2, \ldots) \otimes \mathrm{D}(E_1)\mathrm{D}(E_2)$$

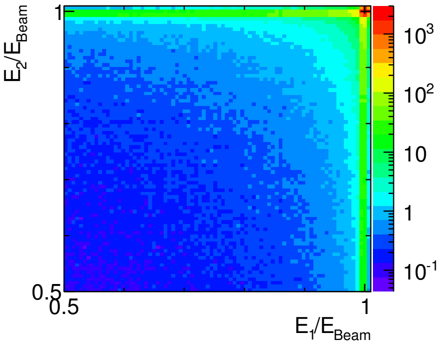
- Data histogram: measured in detector (simulated by GuineaPig) (also apply Bhabha-scattering and detector simulation)
- MC histogram: Luminosity spectrum according to the Model
  - Apply Bhabha scattering/ISR/Detector resolutions on event-by-event basis via MC Generator and detector simulation
  - Remember initial probability based on luminosity spectrum of each event $\mathscr{L}(x_1^i, x_2^i; [p]_0)$
  - Vary all event probabilities (via Model parameters $[p]_N$) until minimum χ^2 is found

$$\text{event weight: } w^i = \frac{\mathscr{L}(x_1^i, x_2^i; [p]_N)}{\mathscr{L}(x_1^i, x_2^i; [p]_0)}$$

- Advantage
  - Only have to do (very time consuming) Bhabha-scattering and detector simulation once

Binning

- Luminosity spectrum has strong peak and long tail
- χ^2-fit requires binned events and sufficient number of events in each bin
- Too coarse binning smears the peak, too fine binning leaves not enough events per bin in the tail
- Use *equiprobability* binning: Varying bin size, but the same number of entries in each bin



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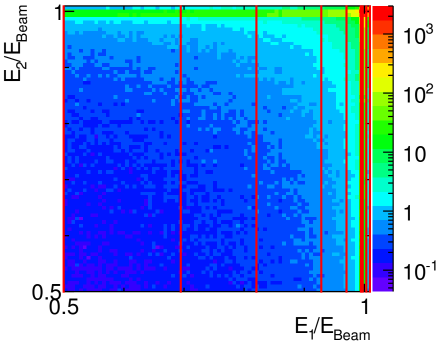

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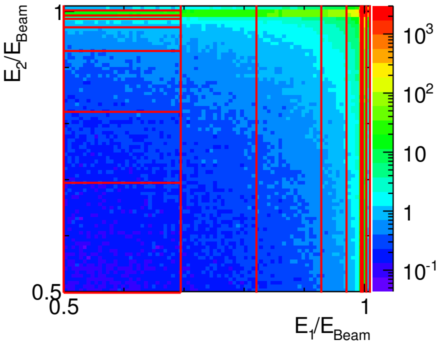

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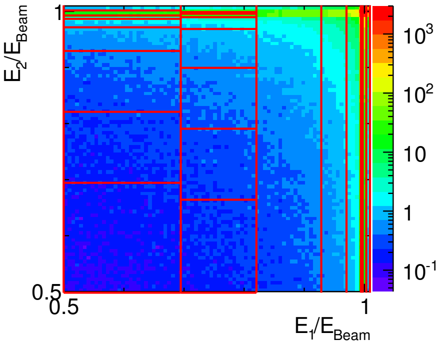

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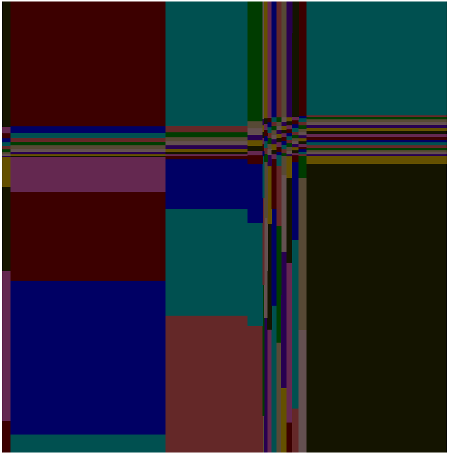
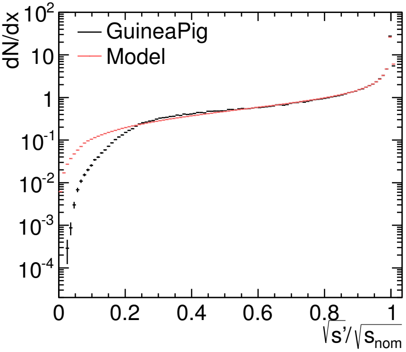
- Luminosity spectrum has strong peak and long tail
- χ^2-fit requires binned events and sufficient number of events in each bin
- Too coarse binning smears the peak, too fine binning leaves not enough events per bin in the tail
- Use *equiprobability* binning: Varying bin size, but the same number of entries in each bin
- Slice events first along dimension 1 into equal parts
- Slice parts of dimension 1 into equal parts along dimension 2
- Wrote program to create, store, and fill equiprobability in 2D and 3D



MODEL Validation

- Fit the 2D distribution of *Initial Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Difference in the width of the peak, but averages out
- Some problem with the width of the peak
  - Only statistical uncertainty from GUINEAPIG sample (1M events)
  - Uncertainty due to parameters smaller



Results for 150×150 (E_1, E_2) bins and cut $\sqrt{s'} > 1.5$ TeV

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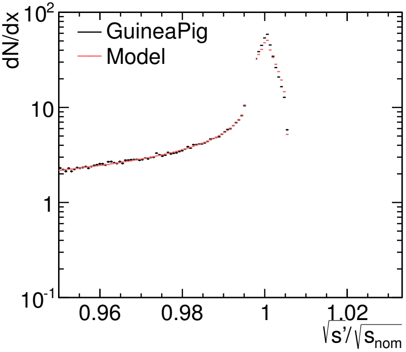

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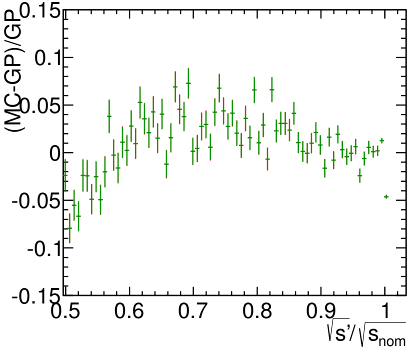

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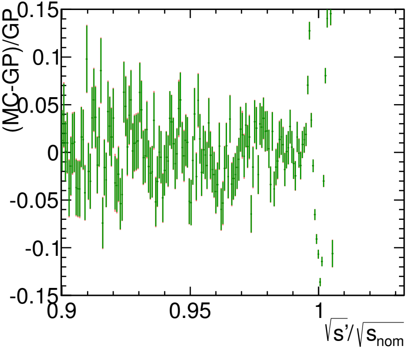

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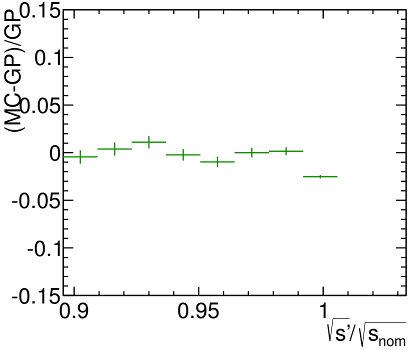
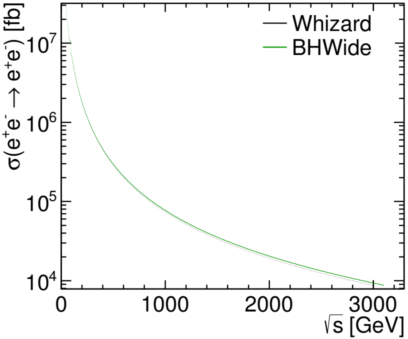


Results for 150×150 (E_1, E_2) bins and cut $\sqrt{s'} > 1.5$ TeV

Luminosity Spectrum with Cross-Section

- Bhabha cross-section proportional to $1/s$
- Cross-section calculated by WHIZARD and BHWIDE $7^\circ < \theta_{e^\pm} < 173^\circ$, without luminosity spectrum
- Need Luminosity Spectrum scaled according to cross-section
- Feed these energy pairs to BHWIDE for ISR/FSR and Bhabha-scattering



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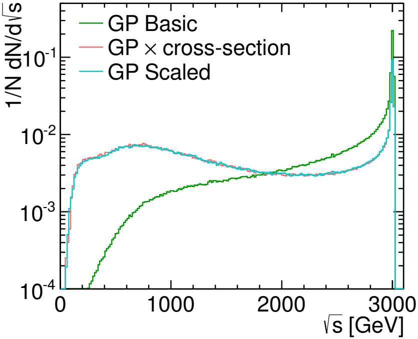
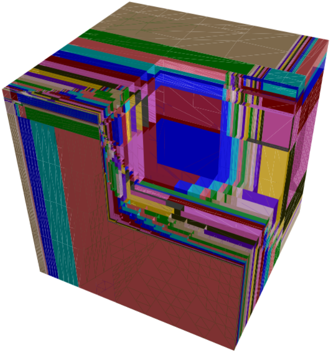


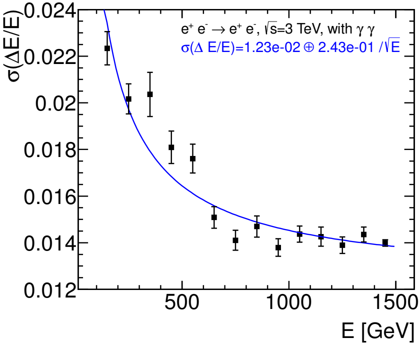
Binning 3D

- The relative centre-of-mass energy calculated from the angles $\frac{\sqrt{s'_{\text{acol}}}}{\sqrt{s_{\text{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$ gives not enough information to reconstruct 2D spectrum
- Additionally use the electron and positron energy measured with calorimeter to see which of the particles lost energy
- These three observables are filled into 3D equiprobability histogram

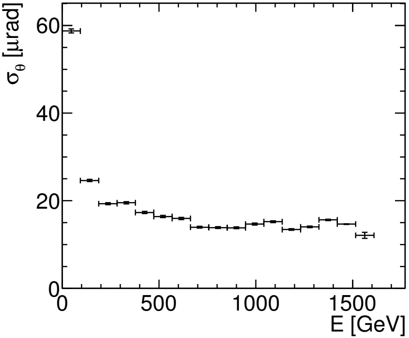


Detector Effects

Particle Energy



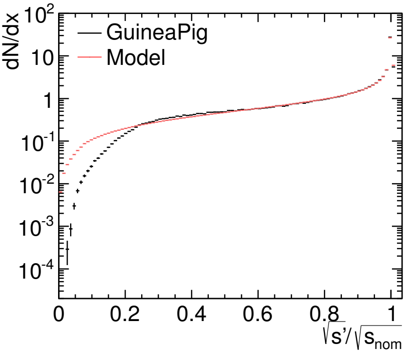
Angular Resolution ($e^{\pm}, \theta \geq 7°$)



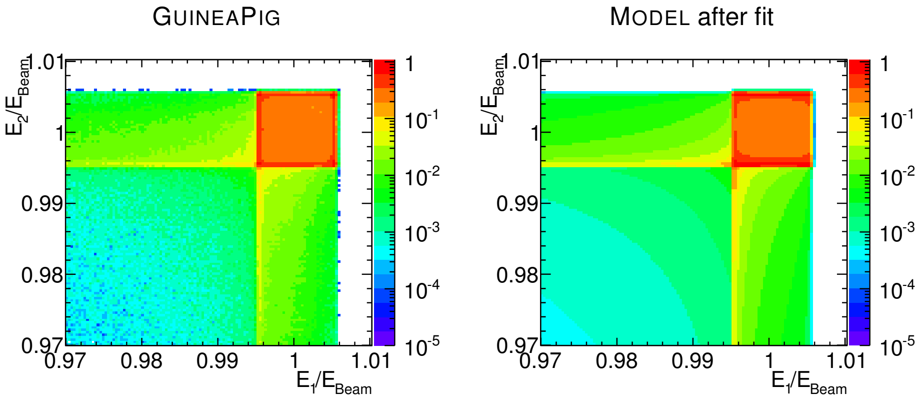
- Full simulation of millions of Bhabha events not feasible, use 4-vector smearing
- Detector resolutions obtained with full simulation/reconstruction with $\gamma\gamma \to$ hadron background overlay thanks to J.J. Blaising

Final Fit: All Effects

- Includes cross-section scaling, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$ (Rel. c.m.s., E_1, E_2)
- 2 million GP (current number of available events, approx. 400fb^{-1}) , 10 million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV

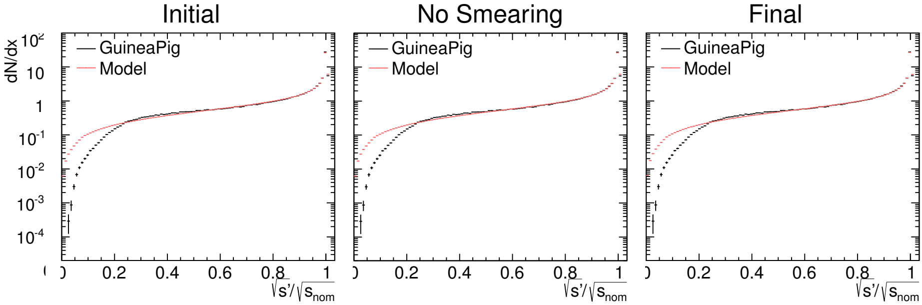


Reconstructed 2D Spectrum



Fit with all effects $60 \times 30 \times 30$ bins

Comparison: Fit Stages



- Initial: Initial Particle Energies Fit (MODEL Validation)
- No Smearing: Bhabha observables and cross-section, no detector resolutions
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- N.B.: The GUINEAPIG sample for all these plots is the same.
- The differences between the GUINEAPIG and the MODEL spectra are very similar for all stages of the reconstruction

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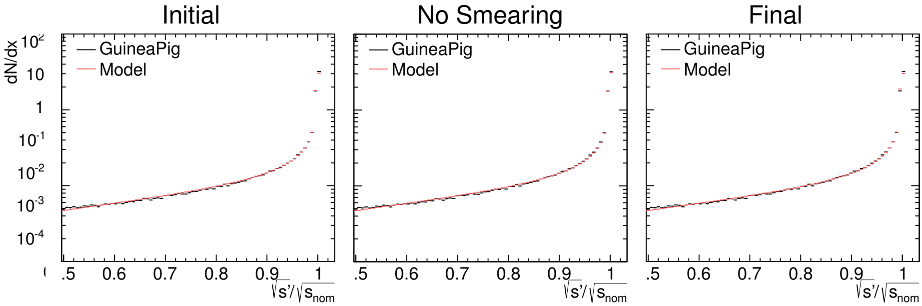

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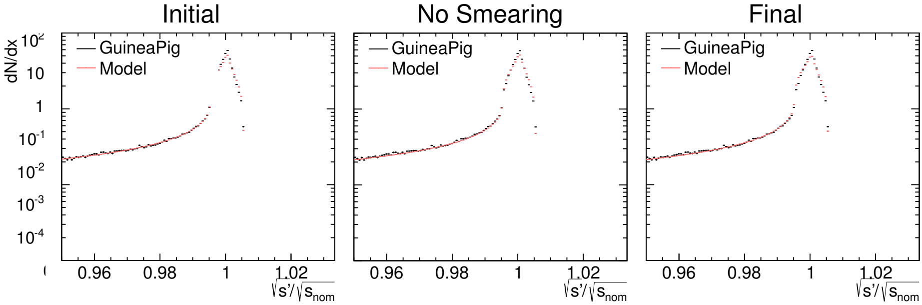

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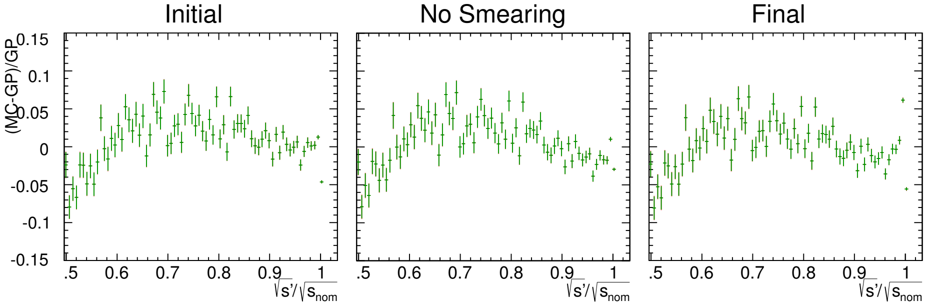

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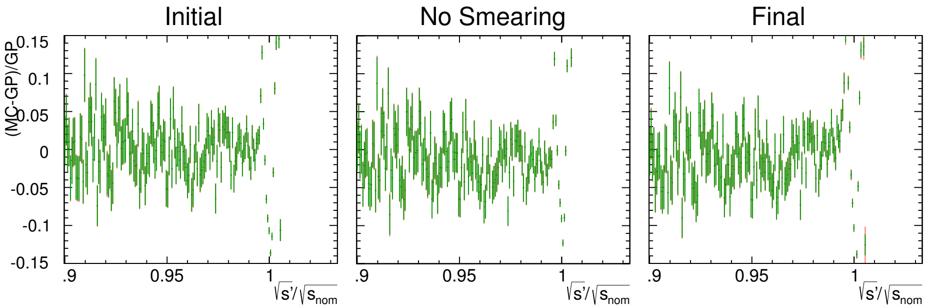

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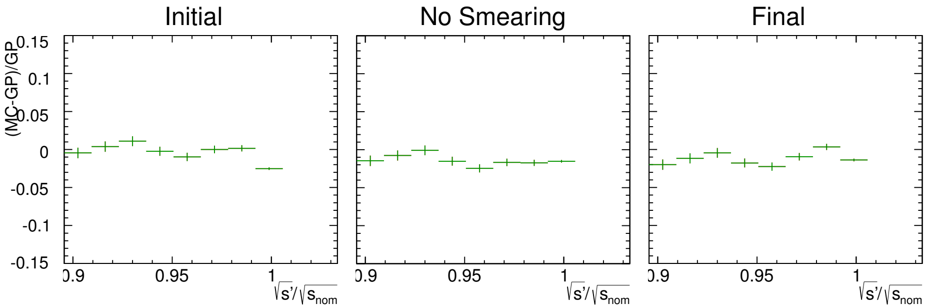

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Impact on Smuon Mass Fit

- Fit background subtracted muon energy distribution to extract Smuon and neutralino mass with $f(E_\mu; m_{\tilde{\mu}}, m_{\tilde{\chi}_1^0}) =$
$\left(\sigma(\sqrt{s'}) \otimes \mathscr{L}(\sqrt{s'}; \vec{p}) \otimes \mathrm{ISR}(\sqrt{s'})\right) \times \left(\mathrm{U}(E_\mu; \sqrt{s'}, m_{\tilde{\mu}}, m_{\tilde{\chi}_1^0}) \otimes \mathrm{D}(E_\mu)\right)$
- Fit with all parameters of luminosity spectrum varied by $\pm\sigma_p^i/2$ individually

$$m_{+_i} = f\left(\vec{p} + \vec{e}_i \frac{\sigma_{p_i}}{2}\right), \qquad m_{-_i} = f\left(\vec{p} - \vec{e}_i \frac{\sigma_{p_i}}{2}\right)$$

- Then the uncertainty on the masses are

$$\sigma_m^2 = \sum_{i,j} \delta_i C_{ij} \delta_j, \qquad \delta_i = m_{+_i} - m_{-_i}$$

with the correlation matrix

$$C = \begin{pmatrix} 1 & -0.6 & \dots & -0.02 \\ -0.6 & 1 & \dots & 0.04 \\ \dots & \dots & \dots & \dots \\ -0.02 & 0.04 & \dots & 1 \end{pmatrix}$$

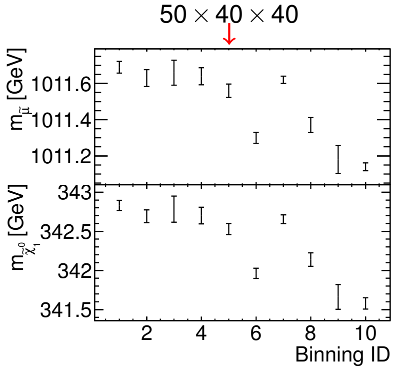
Comparison of the Results

- Using our reconstructed spectrum (e.g., Fit with $50 \times 40 \times 40$ bins):

$$m_{\tilde{\mu}} = (1011.56 \pm 3.0(\mathrm{stat}) \pm 0.04(\mathrm{par}))\,\mathrm{GeV},$$
$$m_{\tilde{\chi}_1^0} = (\ \ 342.53 \pm 6.8(\mathrm{stat}) \pm 0.07(\mathrm{par}))\,\mathrm{GeV}$$

- With GUINEAPIG luminosity spectrum

$$m_{\tilde{\mu}} = (1011.77 \pm 3.05(\mathrm{stat}))\ \mathrm{GeV},$$
$$m_{\tilde{\chi}_1^0} = (\ \ 342.98 \pm 6.82(\mathrm{stat}))\ \mathrm{GeV}$$



Systematic uncertainty from parameter reconstruction only

Conclusion:

- Small dependence on number of bins used during reconstruction, but changes smaller than statistical uncertainty
- The luminosity spectrum reconstruction has no significant impact on $\tilde{\mu}/\chi$ mass measurements

Summary & Conclusions

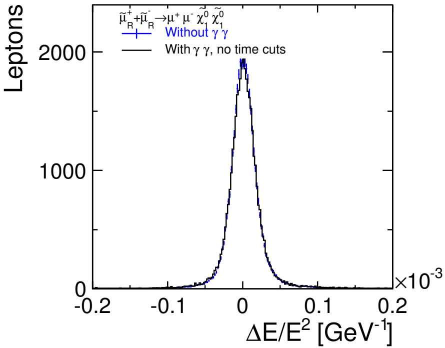
- Smuon Mass Measurement possible with sub-percent precision (see LCD-Note-2012-012)
- Measurement of Smuon and Neutralino mass not significantly affected by luminosity spectrum reconstruction
- For luminosity spectrum reconstruction:
  - Implemented sophisticated reconstruction procedure via a reweighting fit
  - Modelled the CLIC luminosity spectrum
    - ⋆ Beam-energy spread gives very peculiar shape; not an issue for 3 TeV benchmarks, but might be problematic for threshold scans (e.g., at 350 GeV)
  - Included all relevant effects: cross-section, ISR, FSR, detector resolutions
  - Reconstruction of the spectrum within 5% down to $0.5\sqrt{s_{\mathrm{nom}}}$
  - Soon to be released: LCD-Note-2011-040 '*Differential Luminosity Measurement using Bhabha Events*', then to be shortened and submitted for publication

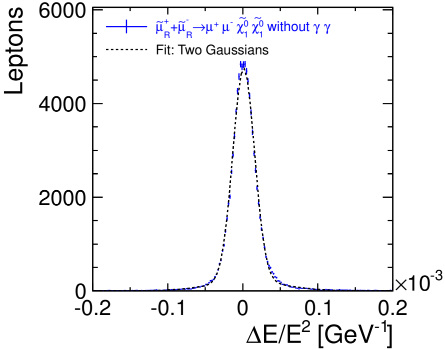
Thank you for your attention!

Backup Slides

Energy Resolution of Muons

- The $\gamma\gamma \to$ hadron background does not affect the muon energy resolution





Backgrounds, Simulation, and Reconstruction

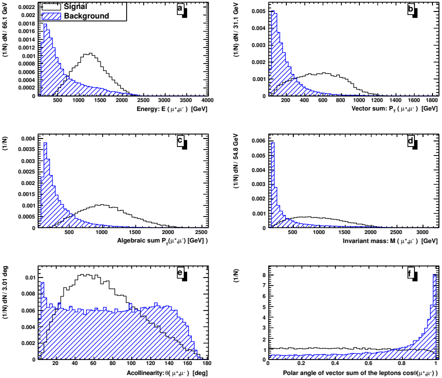
- Generator level pre-selection cuts
  - p_T(L1 and L2) > 4 GeV
  - $10^\circ < \theta$(L1 and L2) $< 170^\circ$
  - $4^\circ < \Delta\phi$(L1,L2) $< 176^\circ$
  - p_T(L1,L2) > 10 GeV
  - M(L1,L2) > 100 GeV
- All selected events are simulated in full GEANT4 simulation, and reconstructed via particle flow.
- The lepton energy is corrected for final state radiation and bremsstrahlung

Background processes with two oppositely charged muons (L1 and L2) in the final state, and their cross-sections after the pre-selection cuts.

Process	$\sigma \times$ BR [fb]
$e^+e^- \to \mu^+\mu^-$	0.65
$e^+e^- \to \mu^+\nu_e\mu^-\nu_e$	3.5
$e^+e^- \to \mu^+\nu_\mu\mu^-\nu_\mu$	2.2
$e^+e^- \to \mu^+\mu^-e^+e^-$	41.5
$e^+e^- \to W^+\nu W^-\nu$	2.4
$e^+e^- \to Z^0\nu Z^0\nu$	0.002
$e^+e^- \to \mathrm{SUSY} - (\tilde{\mu}_R^+\tilde{\mu}_R^-)$	0.31

Event Selection

- Event selection with boosted decision tree (BDT) as implemented in TMVA
- Variables
  - Dilepton energy $E(\mathrm{L1}) + E(\mathrm{L2})$
  - Vector sum $p_{\mathrm{T}}(\mathrm{L1},\mathrm{L2})$ of the two leptons,
  - Algebraic sum pT (L1) + pT (L2) of the two leptons,
  - Dilepton invariant mass M(L1,L2),
  - Dilepton velocity $\beta(L1, L2)$,
  - $\cos\theta(L1, L2)$; $\theta(\mathrm{L1},\mathrm{L2})$ is the polar angle of the vector sum of the two leptons,
  - Dilepton acollinearity $\pi - \theta_2 - \theta_1$,
  - Dilepton acoplanarity $\pi - \phi_2 - \phi_1$,
  - Dilepton energy imbalance $\Delta = |E(L1)E(L2)|/|E(L1) + E(L2)|$,

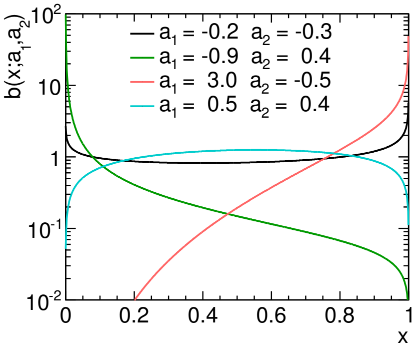


Beta-Distributions

- For the MODEL of the luminosity spectrum mostly using Beta-Distributions

$$b(x) = \frac{1}{N} x^{a_1} (1-x)^{a_2}$$

  with different parameter bounds
- Range: $0 < x < 1$
- Beta-Distribution can represent wide variety of shapes
- Two free parameters: a_1 and a_2



Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \mathrm{Cheb}_i(x)$

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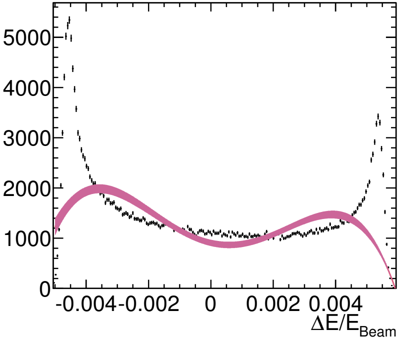

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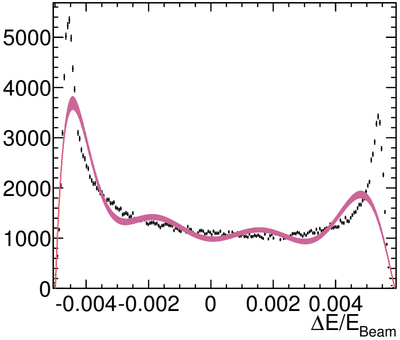
  - 10 Parameters



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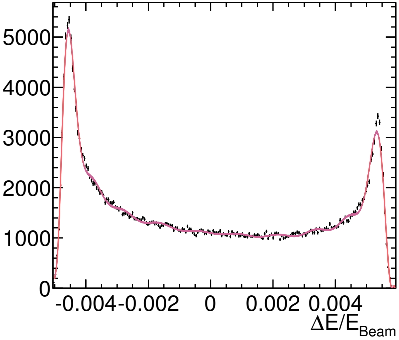
  - 26 Parameters: χ^2/ndf = 668/173



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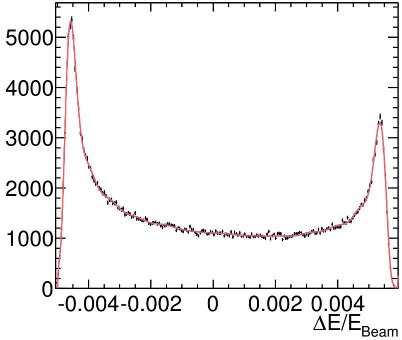

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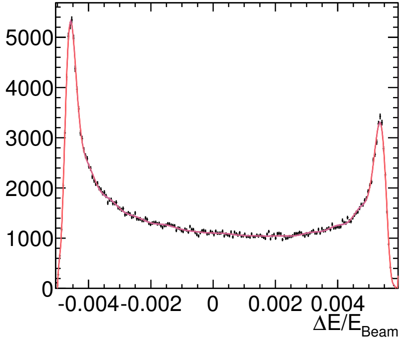
  - 26 Parameters: χ^2/ndf = 668/173
  - 35 Parameters: χ^2/ndf = 226/164
- Saves trouble of convolution, but at the cost of many parameters
- Could also fit centre only and do convolution with Gauss, but still need larger number of parameters



How Can We Extract the Luminosity Spectrum ($\mathscr{L}$) from our Measurement?

The distribution $f(E_1, E_2)$

$$f(E_1,E_2) \approx \sigma(E_1,E_2) \times \mathscr{L}(E_1,E_2) \otimes \mathrm{ISR}(E_1,E_2) \otimes \mathrm{FSR}(E_1,E_2) \otimes \mathrm{D}(E_1)\mathrm{D}(E_2)$$

is connected to the luminosity spectrum and measurable in the detector.
One can then:

- De-convolute the measured (2D) spectrum to remove the initial state radiation energy loss, and detector resolutions, un-weight cross-section dependence
- Model the measured spectrum including cross-section, initial state radiation, and luminosity spectrum
  - Create a 2D function for the complete model and fit the measured spectrum to extract the luminosity spectrum
  - Let Bhabha generator take care of cross-section and initial state radiation, do GEANT4 simulation, and only model the luminosity spectrum
    - ★ Do a template fit (normal models have 1 or 2 free parameters (e.g., mass and width), here we would need to have templates in a $\approx$ 25D phase space)
    - ★ Use a reweighting technique for *efficient* fitting
- ...?

Reweighting Fit Details

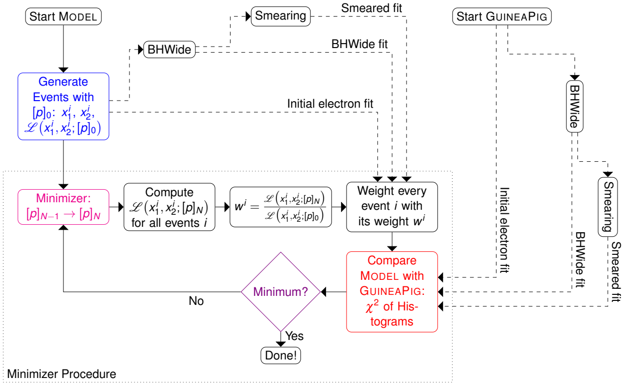
- Do not have to calculate any (numerical) convolutions:
  The distribution of a random variate (x_h), which is based on the convolution of two probability density functions (PDFs) is equal to the distribution of the sum of the individual random variates (x_f and x_g).

$$h(x) \equiv (f \otimes g)(x) \rightarrow x_h = x_f + x_g.$$

Then the new weights can be calculated from the products of the individual PDFs

$$w^i = \frac{p^N_{\text{region}}\, b\left(x^{i,1}_{\text{Strahlung}}, [p]_N\right) b\left(x^{i,1}_{\text{Spread}}, [p]_N\right) g\left(x^{i,1}_{\text{G}}, [p]_N\right) b\left(x^{i,2}_{\text{Strahlung}}, [p]_N\right) b\left(x^{i,2}_{\text{Spread}}, [p]_N\right) g\left(x^{i,2}_{\text{G}}, [p]_N\right)}{p^0_{\text{region}}\, b\left(x^{i,1}_{\text{Strahlung}}, [p]_0\right) b\left(x^{i,1}_{\text{Spread}}, [p]_0\right) g\left(x^{i,1}_{\text{G}}, [p]_0\right) b\left(x^{i,2}_{\text{Strahlung}}, [p]_0\right) b\left(x^{i,2}_{\text{Spread}}, [p]_0\right) g\left(x^{i,2}_{\text{G}}, [p]_0\right)}$$

Reweighting Fit

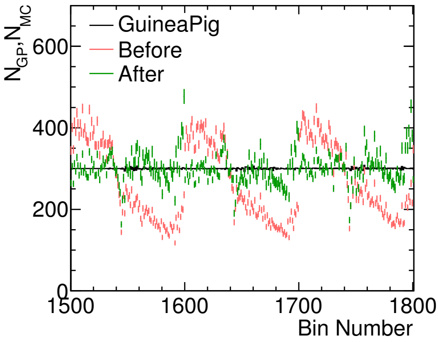


$$N^j_{\mathrm{GP}} = \sum_{\text{GP Events } i \text{ in Bin } j} 1 \qquad N^j_{\mathrm{Model}} = \sum_{\text{Model Events } i \text{ in Bin } j} w^i$$

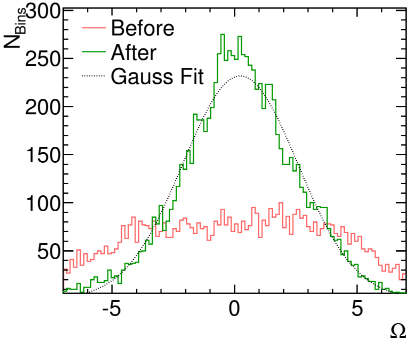
$$f_S = \frac{\sum_{\text{GP Events}} 1}{\sum_{\text{Model Events } i} w^i} \qquad \chi^2 = \sum_{\text{Bins } j} \frac{\left(N^j_{\mathrm{GP}} - f_S \cdot N^j_{\mathrm{Model}}\right)^2}{(\sigma^j_{\mathrm{GP}})^2 + (f_S \cdot \sigma^j_{\mathrm{Model}})^2}$$

Fit Validation

Section of the histograms mapped onto one dimension



Pull distribution for all the bins before and after the fit

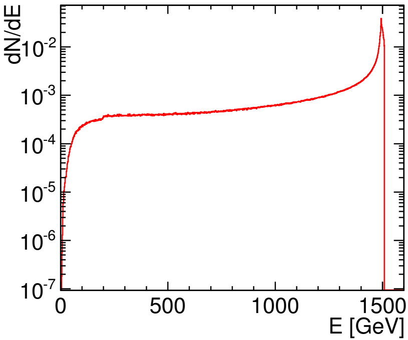


Pull distribution centred on 0, sigma larger than 1, because $\chi^2/\mathrm{ndf} \approx 4$

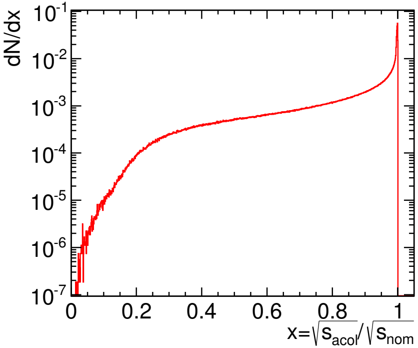
$$\Omega = \frac{\left(N^j_{\mathrm{GP}} - f_S \cdot N^j_{\mathrm{Model}}\right)}{\left((\sigma^j_{\mathrm{GP}})^2 + (f_S \cdot \sigma^j_{\mathrm{Model}})^2\right)^{1/2}}$$

Observables

Unsmeared
Energy of the electron/positron:



Relative centre-of-mass energy (c.m.e.):

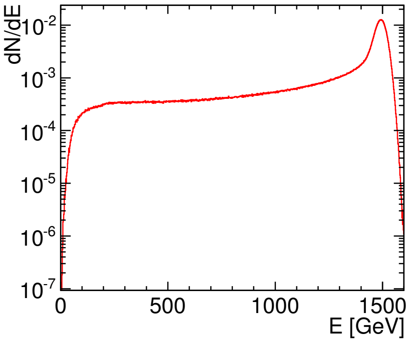


Very large effect on energy, small on relative c.m.e. because of better angular resolution

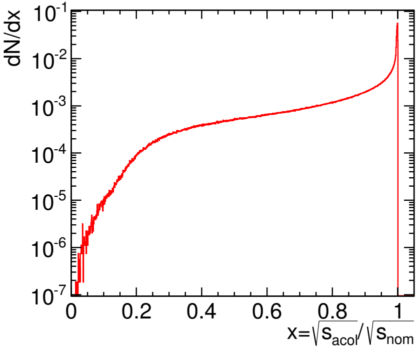
$$\frac{\sqrt{s'_{acol}}}{\sqrt{s_{nom}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}},$$

Observables

Smeared
Energy of the electron/positron:



Relative centre-of-mass energy (c.m.e.):



Very large effect on energy, small on relative c.m.e. because of better angular resolution

$$\frac{\sqrt{s'_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}},$$

Initial Parameters and Limits

Parameter	Lower Bound	Nominal Value	Upper Bound
p_{Peak}	0.0	0.25	0.4
p_{Arm1}	0.0	0.25	0.3
p_{Arm2}	0.0	0.25	0.3
$\omega^1_{\mathrm{Peak1}}$	-1.0	-0.522336	0.0
$\omega^2_{\mathrm{Peak1}}$	-1.0	-0.409289	0.0
$\omega^1_{\mathrm{Peak2}}$	-1.0	-0.522336	0.0
$\omega^2_{\mathrm{Peak2}}$	-1.0	-0.409289	0.0
ω^1_{Arm1}	-1.0	-0.522336	0.0
ω^2_{Arm1}	-1.0	0.35	10.0
ω^1_{Arm2}	-1.0	-0.522336	0.0
ω^2_{Arm2}	-1.0	0.35	10.0
η^1_{Arm1}	0.0	2.5	10.0
η^2_{Arm1}	-1.0	-0.75	0.0
η^1_{Arm2}	0.0	2.5	10.0
η^2_{Arm2}	-1.0	-0.75	0.0
η^1_{Body1}	0.0	0.15	10.0
η^2_{Body1}	-1.0	-0.55	0.0
η^1_{Body2}	0.0	0.15	10.0
η^2_{Body2}	-1.0	-0.55	0.0

Final Results

χ^2/ndf	p_{Peak}	p_{Arm1}	p_{Arm2}
63832 / 10000	0.2387 ± 0.0004	0.2672 ± 0.0004	0.2659 ± 0.0004
62697 / 10000	0.2402 ± 0.0004	0.2666 ± 0.0004	0.2642 ± 0.0004
114039 / 100000	0.2439 ± 0.0007	0.2666 ± 0.0006	0.2652 ± 0.0006
109972 / 100000	0.2479 ± 0.0009	0.2652 ± 0.0007	0.2627 ± 0.0007
100593 / 100000	0.2483 ± 0.0010	0.2681 ± 0.0009	0.2632 ± 0.0009

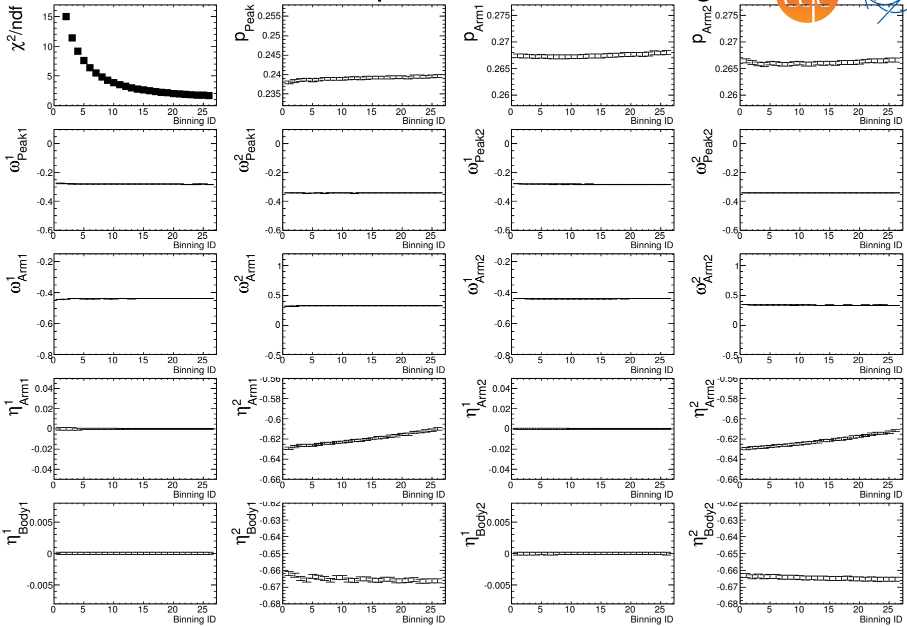
ω^1_{Peak1}	ω^2_{Peak1}	ω^1_{Peak2}	ω^2_{Peak2}
−0.2788 ± 0.0016	−0.3425 ± 0.0013	−0.2805 ± 0.0016	−0.3417 ± 0.0013
−0.2772 ± 0.0019	−0.3370 ± 0.0015	−0.2769 ± 0.0019	−0.3403 ± 0.0015
−0.2668 ± 0.0028	−0.3257 ± 0.0022	−0.2670 ± 0.0028	−0.3232 ± 0.0022
−0.2583 ± 0.0037	−0.3107 ± 0.0030	−0.2659 ± 0.0037	−0.3225 ± 0.0029
−0.3879 ± 0.0149	−0.3882 ± 0.0135	−0.3058 ± 0.0175	−0.3283 ± 0.0153

ω^1_{Arm1}	ω^2_{Arm1}	ω^1_{Arm2}	ω^2_{Arm2}
−0.4399 ± 0.0012	0.3243 ± 0.0037	−0.4399 ± 0.0012	0.3364 ± 0.0036
−0.4509 ± 0.0012	0.3581 ± 0.0039	−0.4473 ± 0.0012	0.3648 ± 0.0039
−0.4057 ± 0.0034	0.4127 ± 0.0090	−0.4078 ± 0.0033	0.4180 ± 0.0090
−0.4192 ± 0.0040	0.4762 ± 0.0112	−0.4162 ± 0.0039	0.4688 ± 0.0108
−0.4994 ± 0.0107	0.3054 ± 0.0305	−0.5501 ± 0.0098	0.1842 ± 0.0292

Final Results cont.

η^1_{Arm1}	η^2_{Arm1}	η^1_{Arm2}	η^2_{Arm2}
0.0000 ± 0.0008	-0.6253 ± 0.0011	0.0000 ± 0.0007	-0.6268 ± 0.0011
0.0000 ± 0.0004	-0.6243 ± 0.0011	0.0000 ± 0.0005	-0.6306 ± 0.0011
0.0000 ± 0.0005	-0.6021 ± 0.0020	0.0000 ± 0.0007	-0.6120 ± 0.0020
0.0000 ± 0.0003	-0.5987 ± 0.0023	0.0000 ± 0.0004	-0.6041 ± 0.0023
0.0000 ± 0.0003	-0.6054 ± 0.0027	0.0000 ± 0.0004	-0.6080 ± 0.0028
η^1_{Body1}	η^2_{Body1}	η^1_{Body2}	η^2_{Body2}
0.0000 ± 0.0002	-0.6640 ± 0.0012	0.0000 ± 0.0002	-0.6636 ± 0.0012
0.0000 ± 0.0001	-0.6643 ± 0.0010	0.0000 ± 0.0001	-0.6675 ± 0.0010
0.0000 ± 0.0005	-0.6538 ± 0.0027	0.0000 ± 0.0006	-0.6540 ± 0.0027
0.0000 ± 0.0003	-0.6550 ± 0.0025	0.0000 ± 0.0004	-0.6571 ± 0.0025
0.0000 ± 0.0004	-0.6421 ± 0.0029	0.0000 ± 0.0005	-0.6415 ± 0.0029

Initial Fit: Parameter Dependence on Binning



Initial Fit:Parameter Dependence on Binning

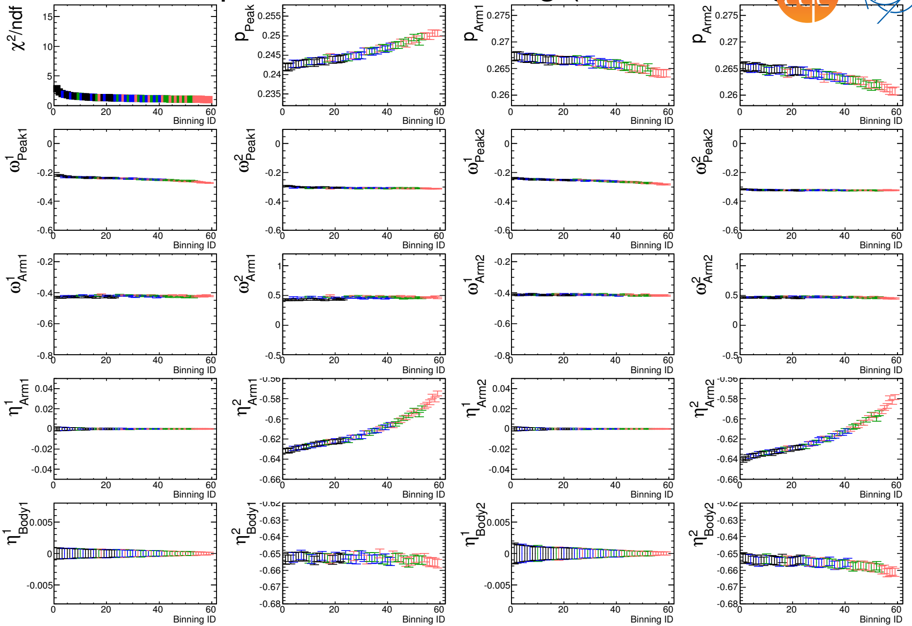
- Vary number of bins from 50×50 to 200×200
- Sorted by χ^2/ndf
- 3 million GP, 10 million MODEL
- Constant number of events, i.e., lower number of events per bin
- Some parameters show strong dependence on binning, some show (anti)correlation also seen in correlation matrix
- There is a bias in the reconstruction of the parameters, but we do not know the 'real' parameters of the spectrum
- Currently 'running'* 15000 Fits to find least biasing binning based on MODEL to MODEL fits, where we know the real parameters

*or waiting for them to run

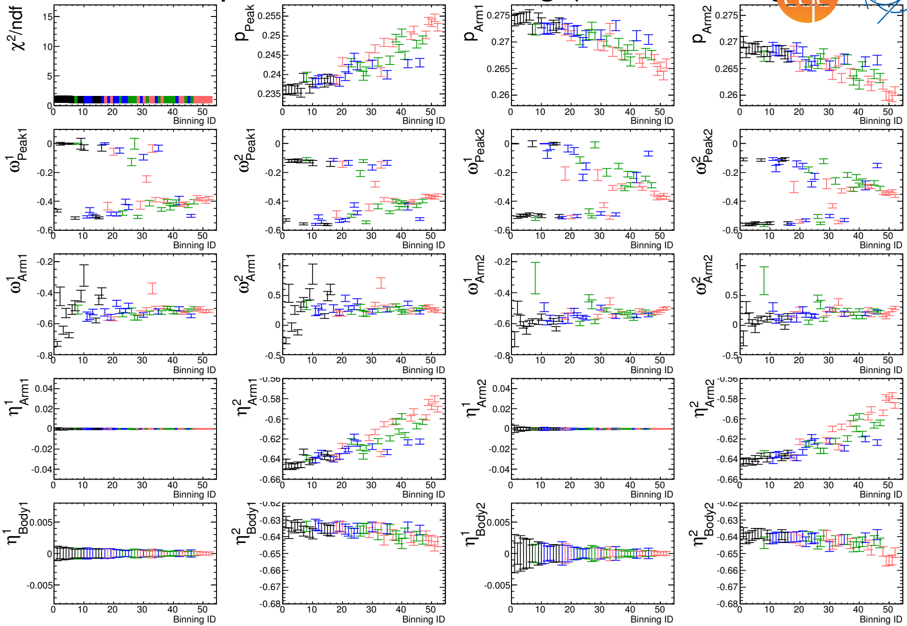
Parameter Dependence on Binning

- Vary number of bins from $10 \times 10 \times 10$ to $80 \times 50 \times 50$
- Sorted by χ^2/ndf
- Some of the binnings fail to result in converging fit
- Constant number of events, i.e., lower number of events per bin
- Some parameters show strong dependence on binning, some show (anti)correlation also seen in correlation matrix
- A minimum number of bins is necessary for proper reconstruction

Parameter Dependence on Binning (No Smearing)



Parameter Dependence on Binning (W/ Smearing)

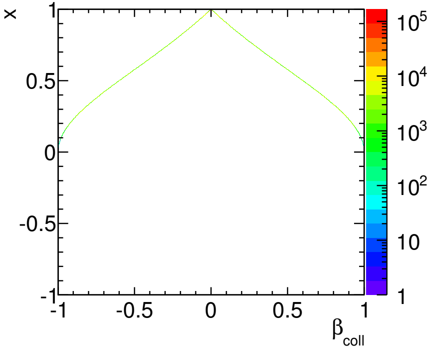


Alternate Observable

- Using: $x = \frac{\sqrt{s'_{\text{acol}}}}{\sqrt{s_{\text{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$,
- Strahinja Lukic using $\beta_{\text{Coll}} = \frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}$ for beam-beam effect corrections (LCD-Note-2012-008)
- When changing x to

$$\bar{x} = \begin{cases} 1-x & \text{for } \theta_1+\theta_2 > \pi \\ -(1-x) & \text{for } \theta_1+\theta_2 < \pi \end{cases}$$

- Two β_{coll} and $\bar{x}$ equivalent for our purpose

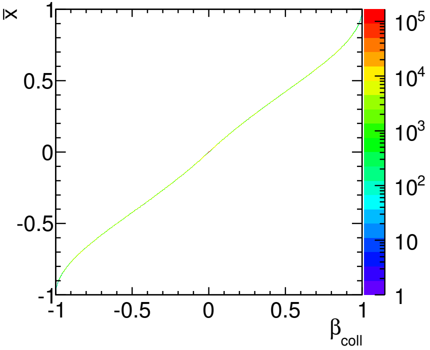


Alternate Observable

- Using: $x = \frac{\sqrt{s'_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$,
- Strahinja Lukic using $\beta_{\mathrm{Coll}} = \frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}$ for beam-beam effect corrections (LCD-Note-2012-008)
- When changing x to

$$\bar{x} = \begin{cases} 1-x & \text{for } \theta_1+\theta_2 > \pi \\ -(1-x) & \text{for } \theta_1+\theta_2 < \pi \end{cases}$$

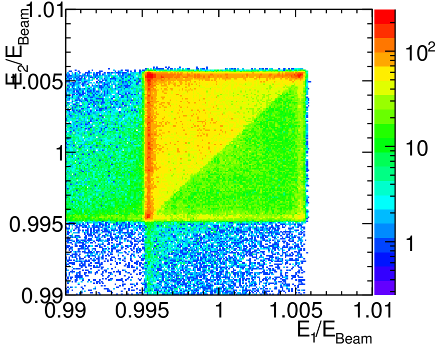
- Two β_{coll} and $\bar{x}$ equivalent for our purpose

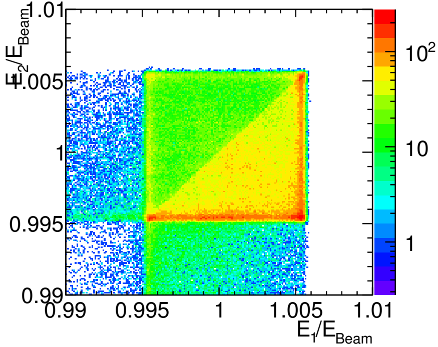


Separation of Boosts

Using the modified variables gives some separation between energies

$\bar{x}, \beta_{\mathrm{COLL}} < 0$ $\bar{x}, \beta_{\mathrm{COLL}} > 0$





Might help a little bit, but have not tried this yet