

# ECFA LC2013

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## NNLO non-resonant contributions to $t\bar{t}$ production at threshold: endpoint-singular terms



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# Outline

- I Top-pair production at linear colliders near threshold
- II Non-resonant (electroweak) NLO contributions
- III Non-resonant NNLO contributions
- IV Results & comparisons
- V Summary

# I. Top-pair production near threshold

## Future linear colliders (ILC/CLIC)

with  $\sqrt{s} \gtrsim 2m_t \simeq 350$  GeV will produce lots of  $t\bar{t}$  pairs, allowing for a **threshold scan** of the top cross section

→ **Precise determination** of the top mass  $m_t$ , the width  $\Gamma_t$  and the Yukawa coupling  $\lambda_t$

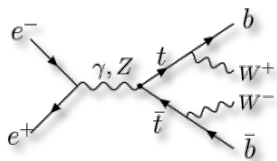
$$\rightarrow \delta m_t \simeq 40 \text{ MeV}$$

→  $m_t$  is a crucial input for electroweak precision observables!

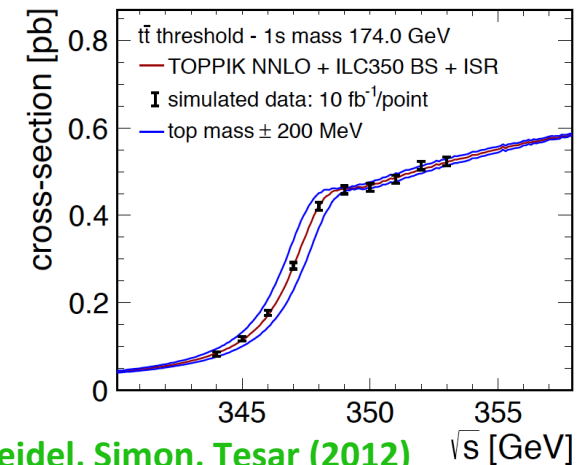
**Requires also precise theoretical prediction**  $\Rightarrow \delta\sigma/\sigma \sim 3\%$

QCD corrections are known (almost) up to NNLL/NNNLO, but **electroweak (NLO) contributions due to top decay** missing until recently!

**Note:** once EW effects are turned on, the **physical final state** is  $W^+W^-b\bar{b}$



$\Rightarrow \sigma(e^+e^- \rightarrow W^+W^-b\bar{b})$  in the  $t\bar{t}$  resonance region and allow for **invariant-mass cuts** on reconstructed  $t, \bar{t}$



Seidel, Simon, Tesar (2012)

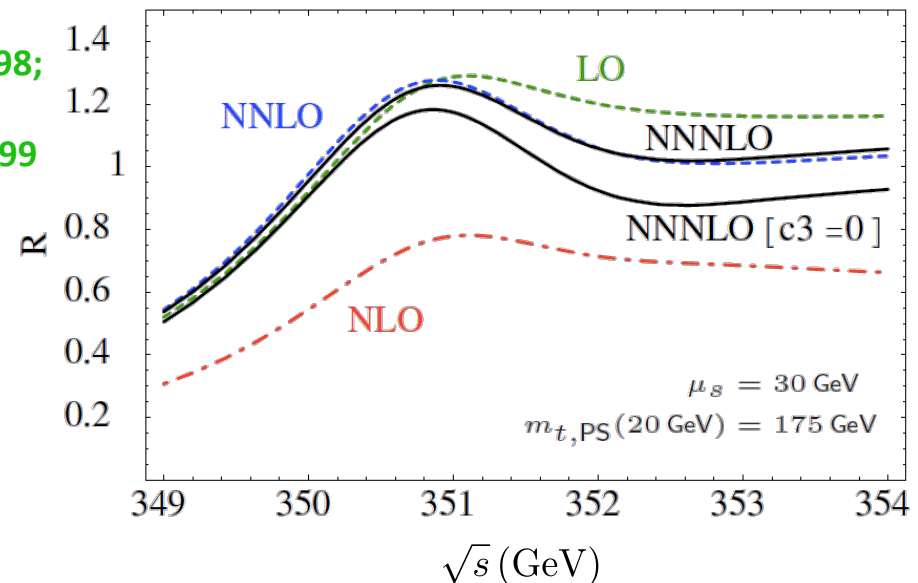
Decay  $t \rightarrow bW^+$  with  $\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$   
 $\Rightarrow t\bar{t}$  is **perturbative** at threshold

Top quarks move slowly near threshold:  $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$   
 $\hookrightarrow$  sum  $\left(\frac{\alpha_s}{v}\right)^n$  from “**Coulomb gluons**” to all orders  $\rightarrow$  **NRQCD**

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left( \{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \dots \right)$$

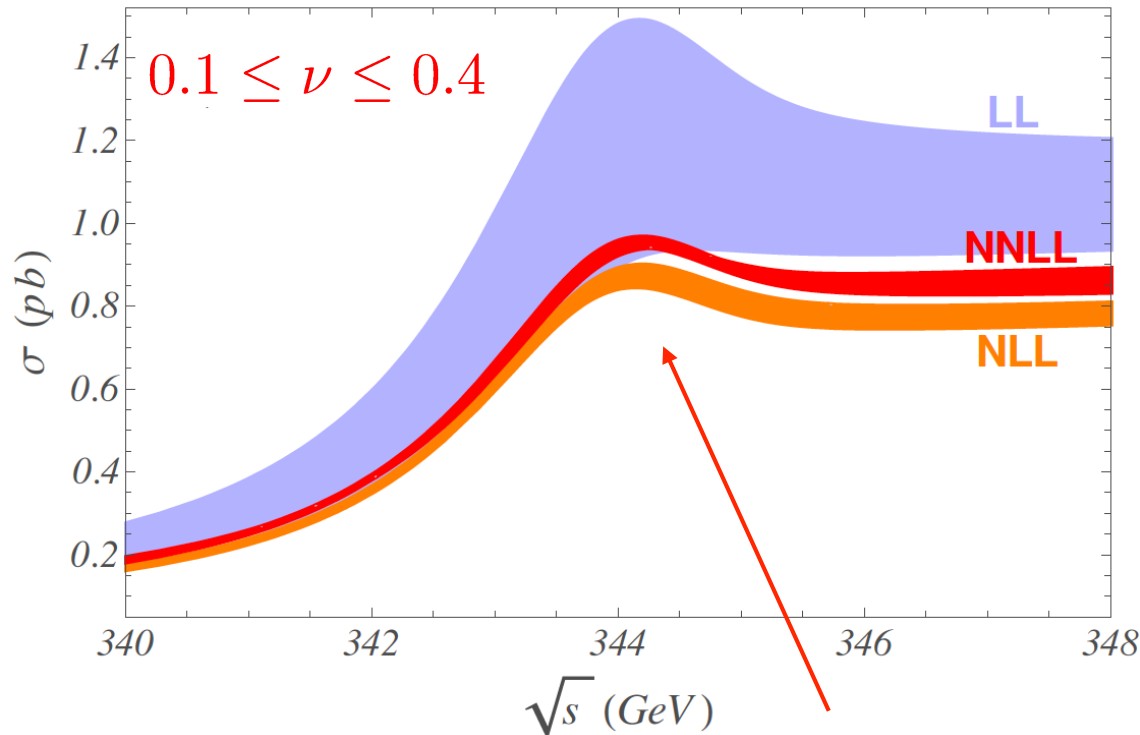
Further RG improvement by summing also  $(\alpha_s \ln v)^m$ : **LL, NLL, ...**  $\rightarrow$  **vNRQCD**  
**pNRQCD**

- **NNLO** QCD corrections  
 Hoang, Teubner '98-99; Melnikov, Yelkhovsky '98;  
 Yakovlev '98; Beneke, Signer, Smirnov '99;  
 Nagano, Ota, Sumino '99; Penin, Pivovarov '98-99
- (almost) **NNLL**  
 Hoang, Manohar, Stewart, Teubner '00-01;  
 Hoang '03; Pineda, Signer '06;  
 Hoang, Stahlhofen '06-11
- **NNNLO** (full analysis soon...)  
 Beneke, Kiyo, Schuller '05-08  $\rightarrow$  see figure



→ [see M. Stahlhofen's talk](#)

Hoang, Stahlhofen (2011)



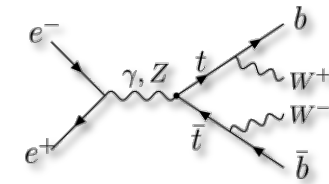
$$\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm(2-4)\%$$

$$M_{\text{peak}} \simeq 2M_{1S} = 2m_t^{\text{pole}} - 0.22 \alpha_s^2 m_t^{\text{pole}}$$

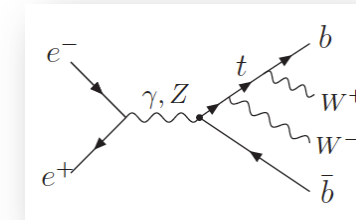
**“threshold masses”**

- missing QCD soft NNLL contributions small
- EW effects beyond LO and non-resonant effects not included  
(at LO:  $E = \sqrt{s} - 2m_t \rightarrow E + i\Gamma_t$ )

- **Resonant contributions** to  $e^+e^- \rightarrow W^+W^-b\bar{b}$   
top and antitop close to the mass-shell

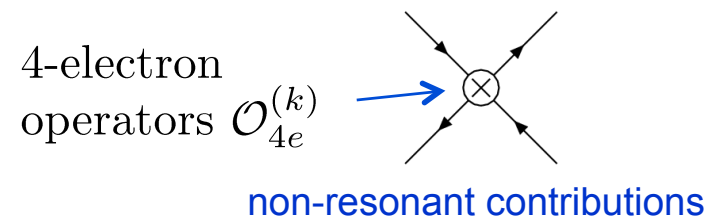
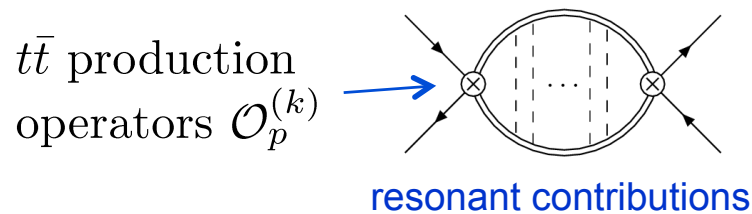


- **Non-resonant (hard) corrections:** account for the production of the  $bW$  pairs by highly virtual tops or diagrams with only one or no top



- **power counting for finite width effects:**  $\frac{\Gamma_t}{m_t} \sim \alpha_{EW} \sim \alpha_s^2 \sim v^2 \ll 1$

**Effective field theory (EFT)** for pair production of unstable particles near threshold, based on separation of resonant and nonresonant fluctuations



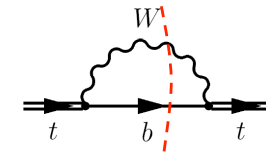
→ Extract cross section for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  from appropriate cuts of the  $e^+e^- \rightarrow e^+e^-$  **forward-scattering amplitude**

## Electroweak effects at LO Fadin, Khoze (1987)

- Replacement rule:  $E = \sqrt{s} - 2m_t \rightarrow E + i\Gamma_t$

$\Rightarrow$  unstable top propagator

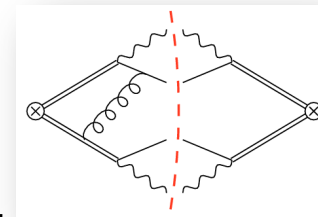
$$\frac{i}{p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2} \leftarrow \delta\mathcal{L} = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger i \frac{\Gamma_t}{2} \psi_{\mathbf{p}}$$



## Electroweak effects at NLO

- Exchange of “Coulomb photon”: trivially extension of QCD corrections

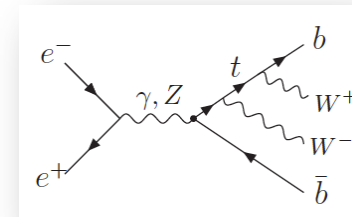
- Gluon exchange involving the bottom quarks in the final state  $\Rightarrow$  these contributions vanish at NLO for the total cross section, Fadin, Khoze, Martin; Melnikov, Yakovlev (1994)  
also negligible if loose top invariant-mass cuts are applied; remains true at NNLO Hoang, Reisser (2005); Beneke, Jantzen, RF (2010)



- Non-resonant (hard) corrections to  $e^+e^- \rightarrow W^+W^-b\bar{b}$  which account for the production of the  $Wb$  pairs by highly virtual tops or with only one or no top

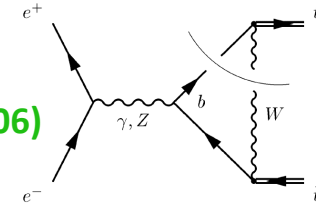
$$\hookrightarrow \Delta\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \text{Im} \left[ C_{4e}^{(k)} \right] \langle e^+e^- | \mathcal{O}_{4e}^{(k)} | e^+e^- \rangle$$

Beneke, Jantzen, RF (2010)

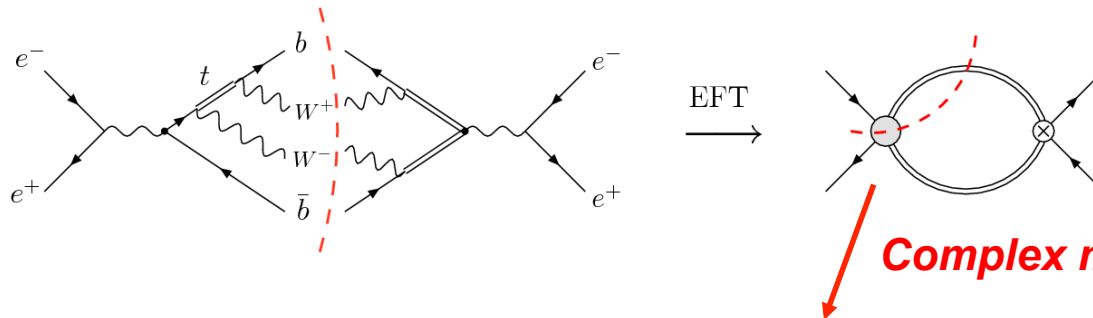


## Electroweak (non-trivial) effects at NNLO

- **absorptive parts** in the 1-loop matching coeffs. of the production operators (arising from  $bW$  cuts) **Hoang, Reisser (2006)**



⇒ reproduce interferences between double and single resonant amplitudes



**Complex matching conditions**

$$J_p = \left[ C_{LL}^{\text{born}} + C_{NLL}^{\text{QCD}} + C_{NNLL}^{\text{QCD}} + iC_{NNLL}^{bW, \text{abs}} + C_{NNLL}^{\text{EW}} + \dots \right] \left( \begin{array}{cc} e^+ & t \\ e^- & \bar{t} \end{array} \right)$$

- **real part** of hard one-loop EW corrections **Kuhn, Guth (1992); Hoang, Reisser (2006)**
- **NNLO non-resonant** contributions (**gluon corrections** to NLO ones)  
Exact computation is hard, but can compute dominant terms for moderate invariant mass cuts  
→ this talk



## II. Non-resonant (electroweak) NLO contributions

# II. Non-resonant NLO contributions

Beneke, Jantzen, RF (2010)

⇒ cuts through  $bW^+t$  (see diagrams) and  $\bar{b}W^-t$  (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:

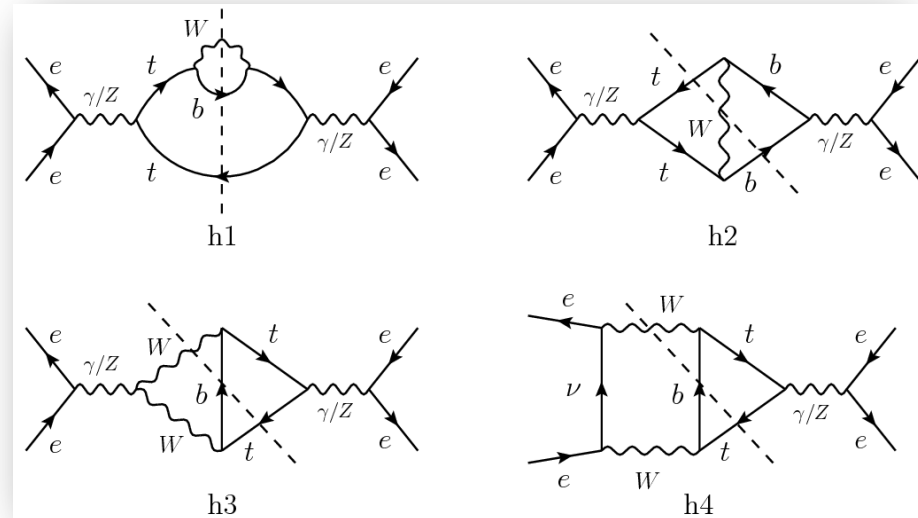
$$p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \gg \Sigma(p_t^2) \sim m_t^2 \alpha_{EW}$$

$$\rightarrow \Gamma_t = 0$$

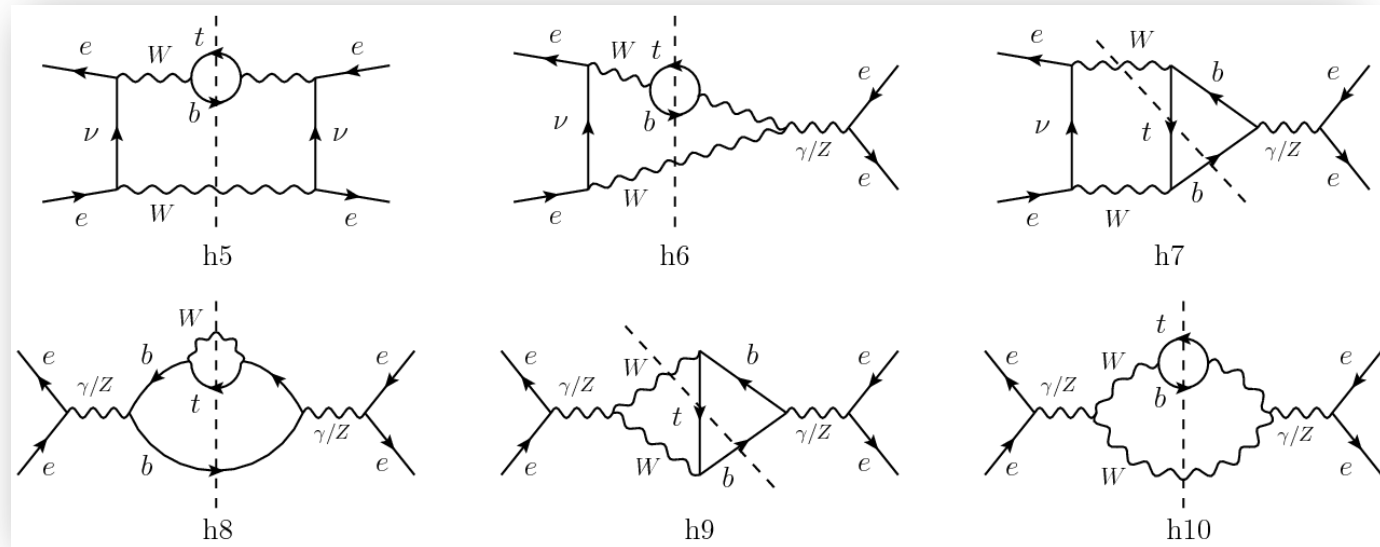
- suppressed w.r.t. LO ( $\sim v$ ) by

$$\alpha_{EW}/v \sim \alpha_s$$

$bW^+$  from highly virtual top



$bW^+$  without intermediate top



In terms of the invariant mass of the  $bW^+$  system,  $p_t^2 = (p_b + p_{W^+})^2$ ,  
 ( $p_t \rightarrow$  also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 (m_t^2 - p_t^2)^{1/2-\epsilon} H_i \left( \frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2} \right)$$

with  $\Delta^2 = M_W^2$  for the total cross section

## Applying top invariant-mass cuts

Restrict invariant masses of the reconstructed  $t, \bar{t}$ :  $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

$\hookrightarrow$  lower integration limit  $\Delta^2 = m_t^2 - \Lambda^2$  where  $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

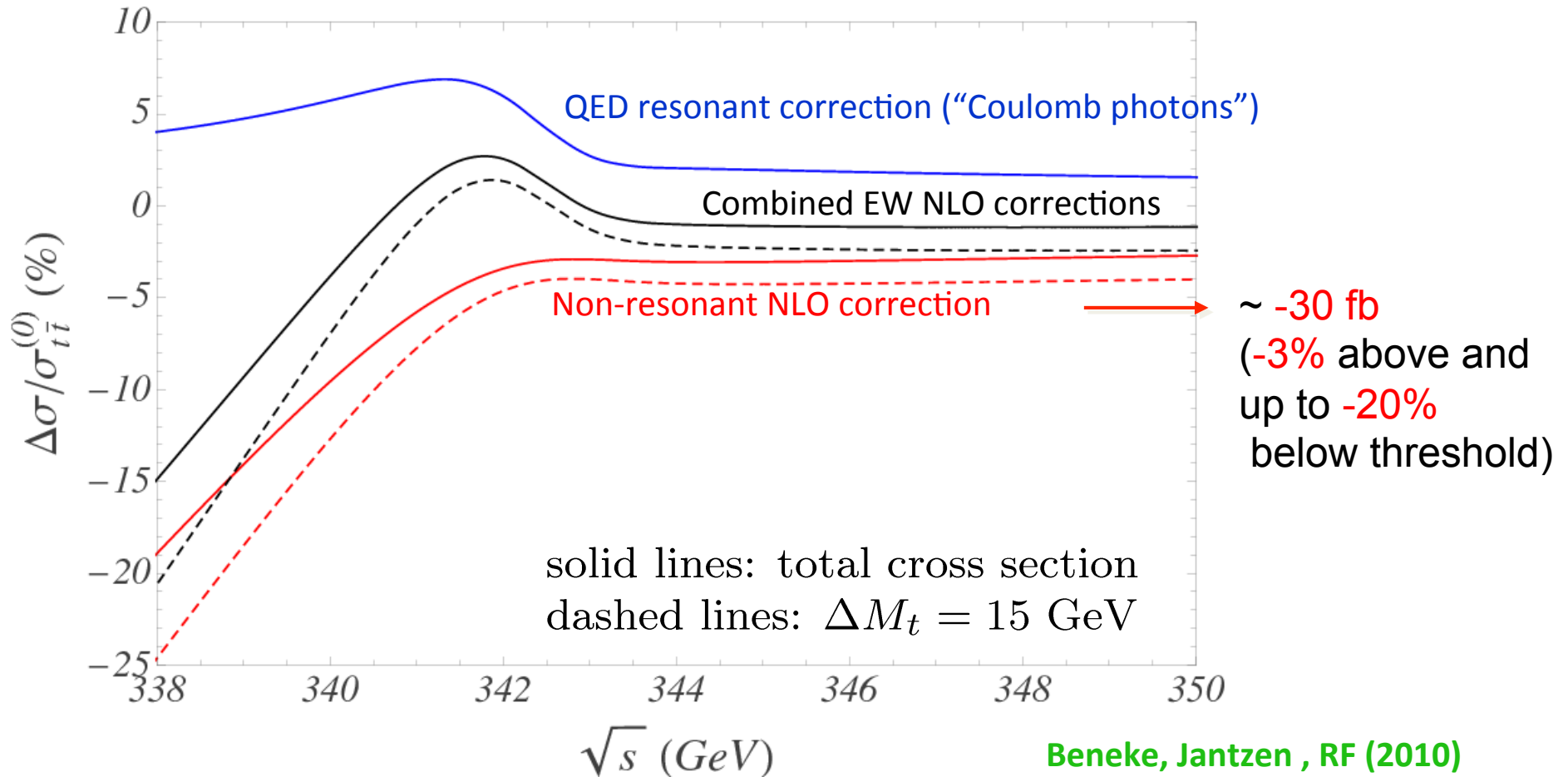
We focus on **loose cuts** with  $\Lambda^2 \gg m_t \Gamma_t$  (corresponding to  $\Delta M_t \gg \Gamma_t$ )

$\rightsquigarrow$  **cut has no effect in the resonant contributions**

[In contrast: for **tight cuts** with  $\Lambda^2 \sim m_t \Gamma_t$  ( $\Delta M_t \sim \Gamma_t$ ), non-resonant contributions vanish and cuts only affect the resonant contributions]

## Relative sizes of EW NLO corrections with respect LO

LO includes resummation of Coulomb gluons  $\propto (\alpha_s/v)^n$  [ $\alpha_s^{\overline{\text{MS}}}(30 \text{ GeV}) = 0.142$ ]



# Phase space matching

## Alternative approach to compute non-resonant contributions

Hoang, Reisser, RF (2010)

- Non-resonant contributions obtained for **moderate invariant-mass cuts**,  $m_t \Gamma_t \ll \Lambda^2 \ll m_t^2$ , as a series:

$$\frac{\Gamma_t}{\Lambda} \sum_{n,\ell,k} \left[ \left( \frac{m_t \Gamma_t}{\Lambda^2} \right)^n \times \left( \frac{\Lambda^2}{m_t^2} \right)^\ell \right] \times \left( \alpha_s \frac{m_t}{\Lambda} \right)^k \quad n, \ell, k = 0, 1, \dots$$

- NLO, NNLO and (partial) N<sup>3</sup>LO contributions obtained (counting  $\Lambda \sim m_t$ ) ✓
- Beyond NLO, phase space matching approach cannot be applied to larger cuts up to the total cross section ✗
- Expansion of full NLO non-resonant contributions in  $(\Lambda/m_t)^n$  agrees with first two terms in series above  
[higher powers receive contributions from **diagrams h5-h10 with no top**, not taken into account in the psm approach → **remainder contributions**, small at NLO ✓ ]

## III. Non-resonant NNLO contributions

# Finite-width divergences in the resonant contributions

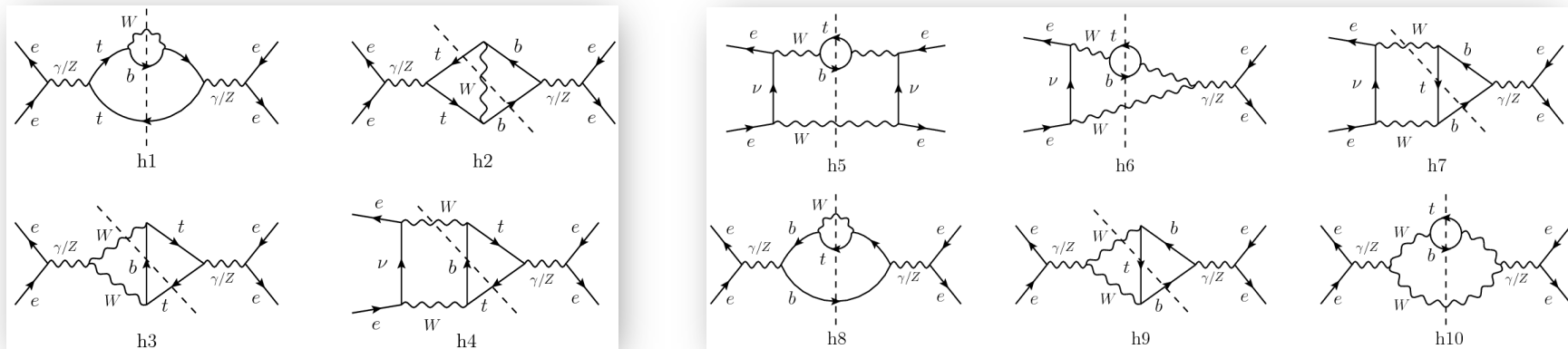
**Resonant contributions** obtained by assuming the top quarks are nearly on-shell (**potential**), but integrated over all momenta

→ uncanceled **UV-singularity** from **hard** momenta:  $\Delta\sigma^{\text{NNLO}} \sim m_t^2 \frac{\alpha_s \Gamma_t}{\epsilon}$

Related to **finite top width** in EFT cut propagator

- for stable top  $\rightarrow \pi \delta(p^0 - \frac{\vec{p}^2}{2m_t})$ ,
- for unstable top  $\rightarrow \frac{\Gamma_t/2}{(p^0 - \vec{p}^2/2m_t)^2 + (\Gamma_t/2)^2}$  Breit-Wigner, UV-behaviour changed!

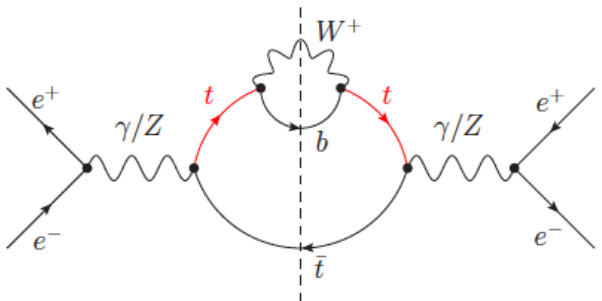
These divergences must **cancel with non-resonant (hard) NNLO terms**, which arise from **gluon corrections** to NLO non-resonant diagrams h1-h10



# Endpoint divergences in the non-resonant contributions

Endpoint divergences of the phase-space integration at  $p_t^2 \rightarrow m_t^2$  (because  $\Gamma_t = 0$  here):

**NLO:**

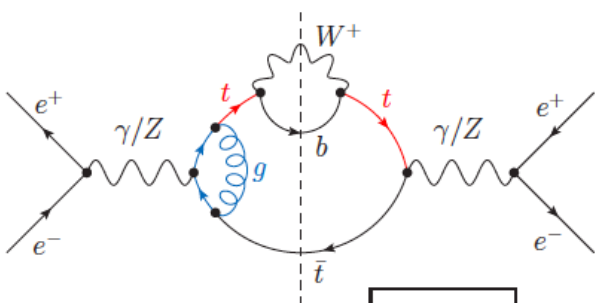


$$\sim \int \frac{dp_t^2}{(m_t^2 - p_t^2)^{n+\epsilon}} \text{ with } n = \frac{3}{2}, \frac{1}{2}, \dots$$

$\hookrightarrow$  endpoint divergence finite in dim. reg.:

$$\int_{m_t^2 - \Lambda^2}^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{\frac{3}{2}+\epsilon}} = -\frac{2}{\Lambda} + \mathcal{O}(\epsilon)$$

**NNLO:**



$$\sim \int \frac{dp_t^2}{(m_t^2 - p_t^2)^{n+a\epsilon}} \text{ with } n = 2, \frac{3}{2}, \mathbf{1}, \frac{1}{2}, \dots$$

$\hookrightarrow$  endpoint divergence  $\propto \alpha_s \frac{\Gamma_t}{\epsilon}$  from  $n = 1$ :

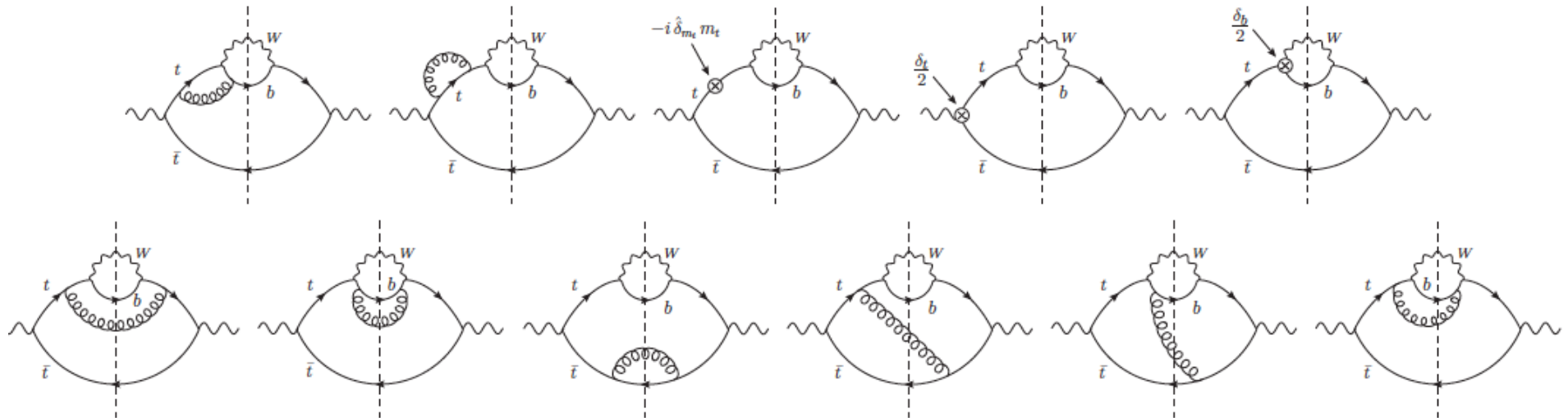
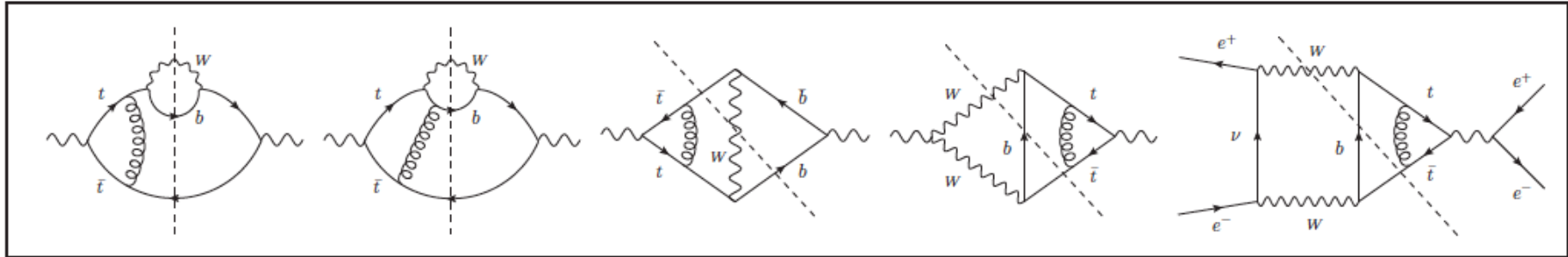
$$\mu^{4\epsilon} \int_{m_t^2 - \Lambda^2}^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{1+2\epsilon}} = -\frac{1}{2\epsilon} + \ln \frac{\Lambda^2}{\mu^2} + \mathcal{O}(\epsilon)$$

Expand integrand in  $(m_t^2 - p_t^2)/m_t^2 \iff$  asymptotic expansion of result in  $\Lambda/m_t$



# ENDPOINT-DIVERGENT NON-RESONANT NNLO DIAGRAMS

→ expanded near endpoint  $\rightsquigarrow$  potential top momentum  $p_t = p_b + p_W (+p_g)$



**boxed diagrams**  $\rightsquigarrow$  endpoint-singular  $\frac{1}{\epsilon} - 2 \ln \frac{\Lambda^2}{\mu^2}$  terms from potential gluons  
 + “finite” endpoint-divergent  $\frac{m_t}{\Lambda}$  &  $\frac{m_t^2}{\Lambda^2}$  terms from hard & potential gluons

# Endpoint-divergent non-resonant NNLO contribution

↪ dominant contribution for small  $\Lambda$  (or small  $\Delta M_t$ )

$$\sigma_{\text{non-res}}^{(2),\text{ep}} = \frac{64\pi^2\alpha^2}{s} \frac{\Gamma_t^{\text{Born}}}{m_t} \quad \left[ C_{\dots}(s) = \gamma/Z\text{-prop. \& } e^\pm\text{-coupl.} \right]$$

$$\begin{aligned} & \times \left\{ \left[ Q_t^2 C_{\gamma\gamma}(s) - 2Q_tv_t C_{\gamma Z}(s) + v_t^2 C_{ZZ}(s) \right] \left\{ 4N_c C_F \frac{\alpha_s}{\pi} \frac{m_t^2}{\Lambda^2} + \frac{6\sqrt{2}}{\pi^2} \left( \delta\Gamma_t^{(1)} - \frac{4C_F}{\pi} \alpha_s \right) \frac{m_t}{\Lambda} \right\} \right. \\ & + N_c C_F \frac{\alpha_s}{4\pi} \left( \frac{1}{\epsilon_{\text{ep}}} + 2 \ln \frac{\mu_s^2}{\Lambda^2} \right) \left\{ \left[ Q_t^2 C_{\gamma\gamma}(s) - 2Q_tv_t C_{\gamma Z}(s) + v_t^2 C_{ZZ}(s) \right] \frac{7 + 7x + 22x^2}{6(x-1)(1+2x)} \right. \\ & + \frac{1}{3} a_t^2 C_{ZZ}(s) + \frac{1}{2} Q_t a_t C_{\gamma Z}(s) - \frac{1}{2} v_t a_t C_{ZZ}(s) \\ & + \left[ Q_t Q_b C_{\gamma\gamma}(s) - (Q_t(v_b + a_b) + Q_b v_t) C_{\gamma Z}(s) + v_t(v_b + a_b) C_{ZZ}(s) \right] \frac{1 - 5x - 2x^2}{6(1+x)(1+2x)} \\ & + \left[ Q_t C_{\gamma\gamma}(s) - \left( v_t + Q_t \frac{c_w}{s_w} \right) C_{\gamma Z}(s) + v_t \frac{c_w}{s_w} C_{ZZ}(s) \right] \frac{2 + 5x - 2x^2}{6x(1+2x)} \\ & \left. \left. - \left[ Q_t C_\gamma(s) + v_t C_Z(s) \right] \left[ \ln \left( \frac{2}{x} - 1 \right) + \frac{(1-x)(1-2x-23x^2)}{12x^2} \right] \frac{x}{4(1-x)^3(1+2x)} \right\} \right\} \end{aligned}$$

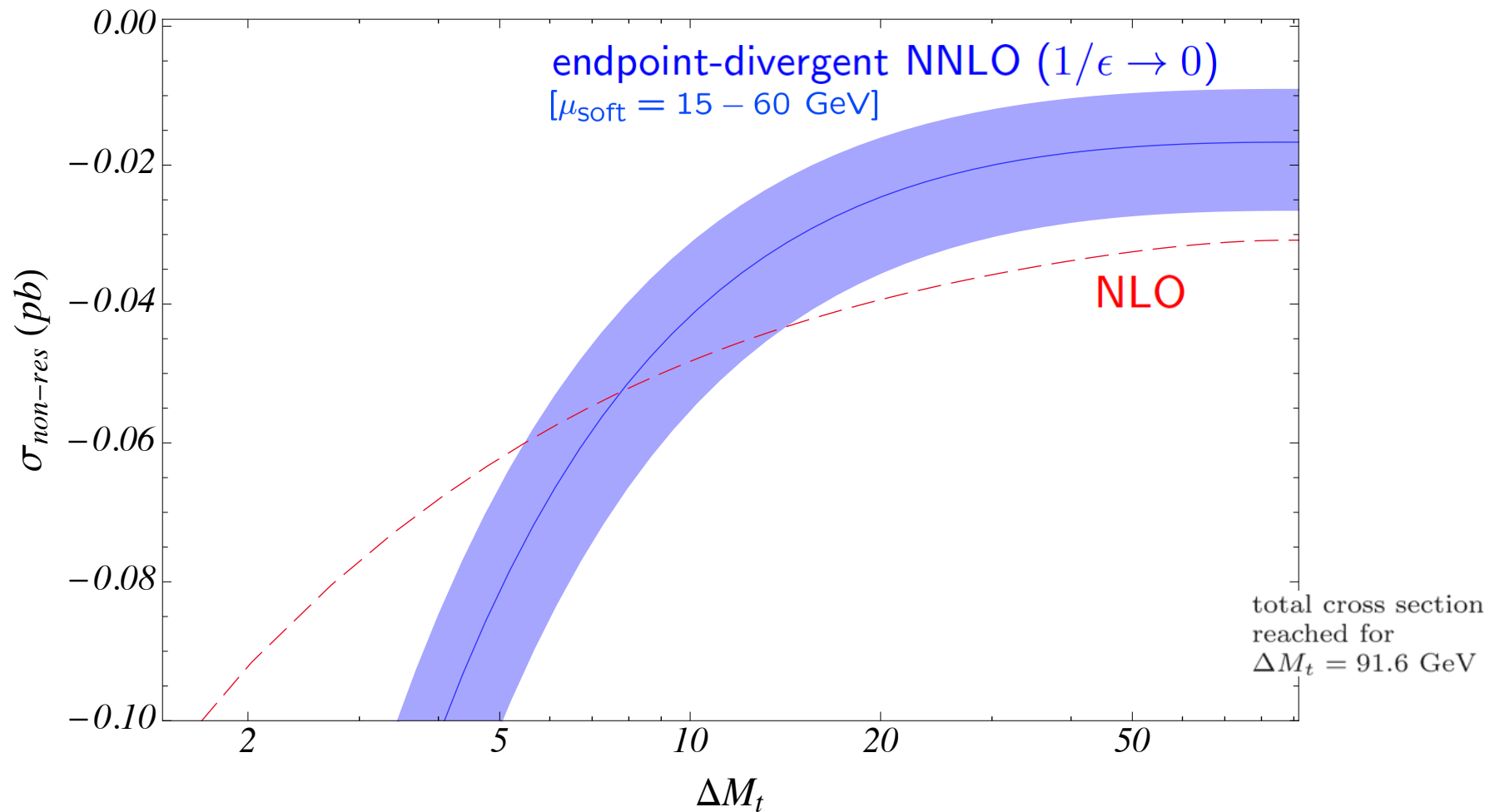
+ finite  $\Lambda$ -independent terms +  $\mathcal{O}(\Lambda/m_t)$

Jantzen, RF

- UV and IR singularities cancelled between diagrams ✓
- $1/\epsilon$  endpoint singularities & finite-width divergences cancel each other ✓
- comparison to HRR result:  $m_t^2/\Lambda^2$  ✓,  $m_t/\Lambda$  ✓,  $\Lambda^0 \ln(\Lambda^2)$  ✓

## IV. Results & comparisons

# Results for the non-resonant NNLO contributions



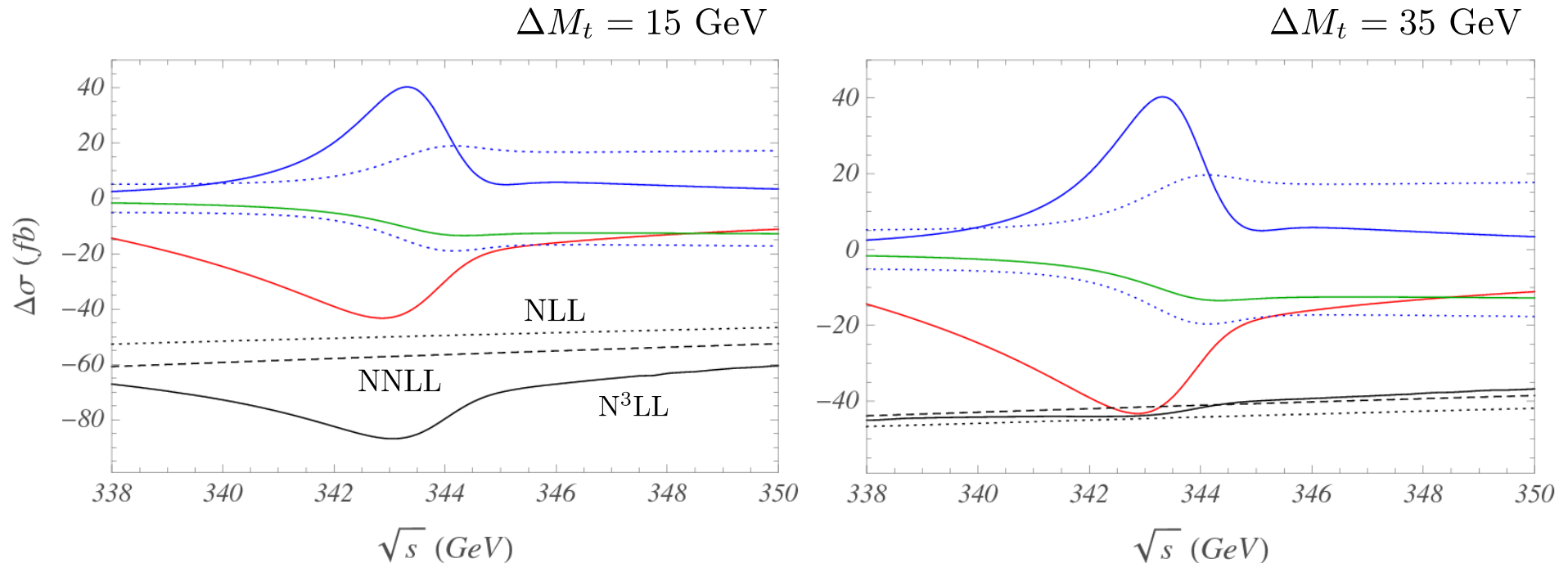
Perturbative expansion converges for **loose cuts**:

$$\alpha_s \frac{m_t^2}{\Lambda^2} @ \text{NNLO} \ll \frac{m_t}{\Lambda} @ \text{NLO} \iff \Lambda^2 \gg m_t \Gamma_t \sim m_t^2 \alpha_{\text{EW}} \sim m_t^2 \alpha_s^2$$

# Non-QCD corrections beyond NNLO

## Sizes of NNLL EW and non-resonant corrections

Hoang, Reisser, RF (2010)



**NNLL QED effects**

**NNLL hard one-loop EW effects**

**NNLL finite lifetime corrections**

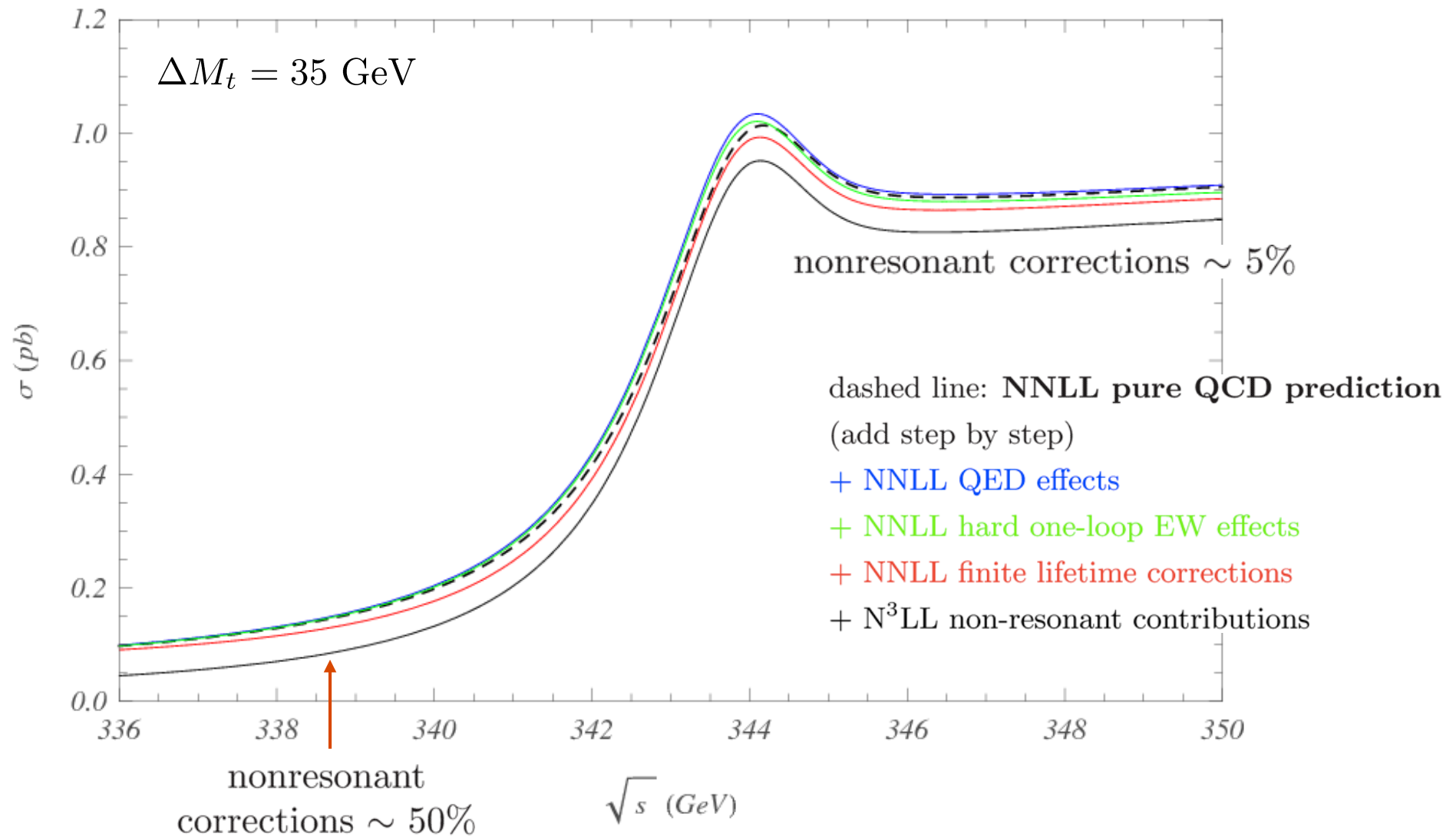
**Non-resonant corrections**

(NLL, NNLL, N<sup>3</sup>LL phase space matching contributions)

- psm contributions are the largest of the 4 classes of EW effects
- almost constant (small linear  $\sqrt{s}$ -dependence from  $\gamma, Z$  propagators)
- convergence of the psm procedure particularly good for larger  $\Delta M_t$

# Inclusive top-pair production cross section

Cut on  $bW$  invariant masses of the form  $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$



# IV. Summary

## Resonant corrections (top and antitop close to mass shell)

- QCD contributions:
  - ✓ **fixed-order approach**: all **N<sup>3</sup>LO** pieces known (compilation of all contributions shall appear soon...)
  - ✓ **RG improved calculation**: **NNLL** almost complete
- Electroweak contributions known to **NNLL** accuracy
- ✗ Detailed error analysis of **theoretical uncertainties** in preparation...

**3% theoretical uncertainty on the total cross section here seems realistic...**

## Non-resonant corrections (bW pairs from virtual tops or with only one or no top)

- ✓ **computed at NLO** for the total cross section and with **top invariant-mass cuts**
- ✓ Beyond: **dominant NNLO and NNNLO terms known only** when top invariant mass are included
- ✗ **In progress**: NNLO corrections to total cross section, expected to be small (few percent at most), but **can become very important below the peak region**
  - **include non-resonant corrections in future ILC top-quark mass measurement study**

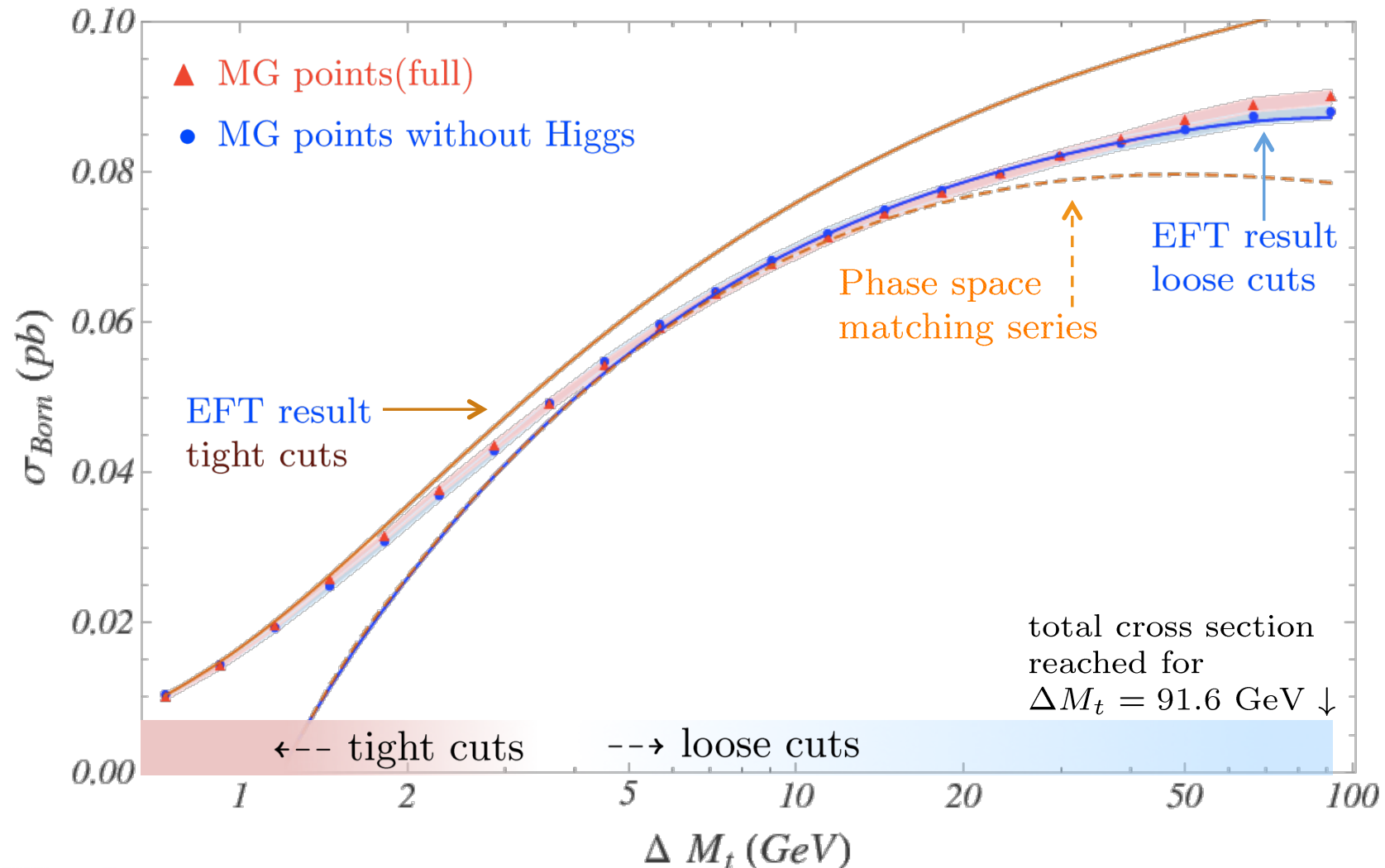
# Backup slides



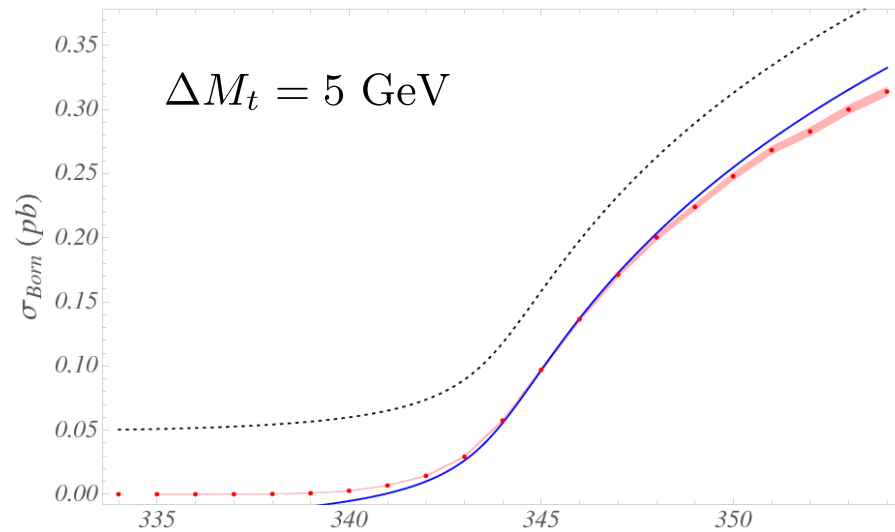
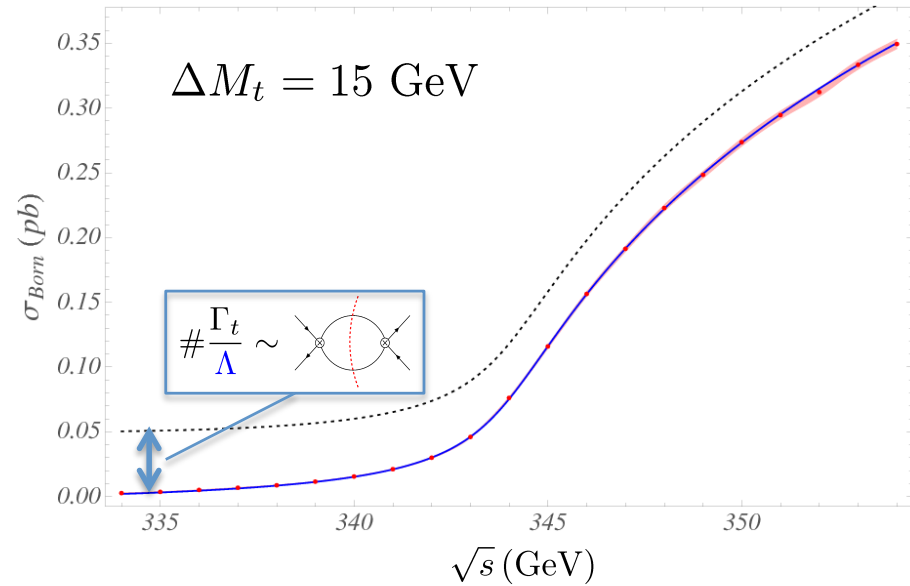
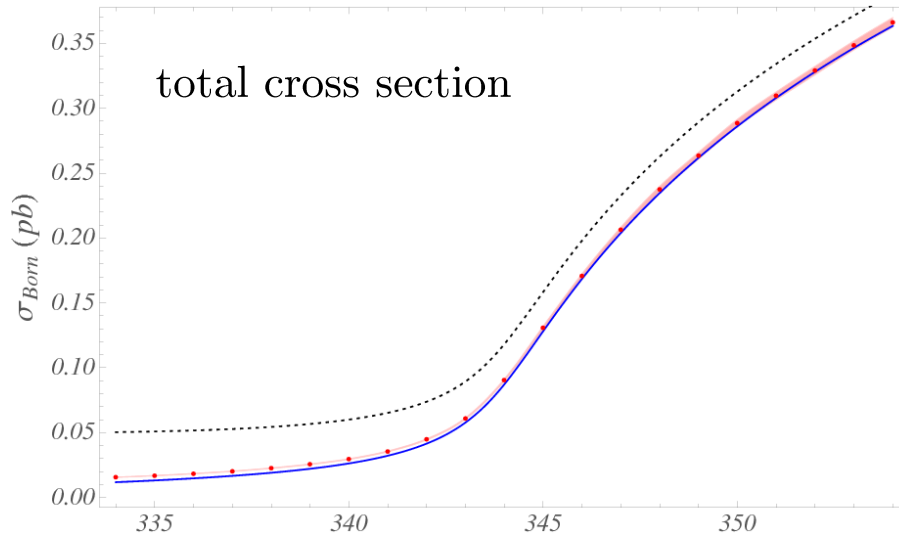
# COMPARISON TO MADGRAPH/MADEVENT

→ generated  $10^4$  events for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  with MadGraph (MG) for  $s = 4m_t^2$ , and analyzed dependence on the  $bW$  invariant-mass cut  $\Delta M_t$

EFT result: resonant LO+NNLO ( $\alpha_s = 0$ ) + non-resonant NLO



$e^+e^- \rightarrow W^+W^-b\bar{b}$  tree-level cross section: energy dependence for different  $\Delta M_t$  invariant-mass cuts



MG (full) points & error band,  
 EW NNLO tree-level contributions  
 (solid-blue) [resonant + non-resonant],  
 only resonant contributions (dotted-black)