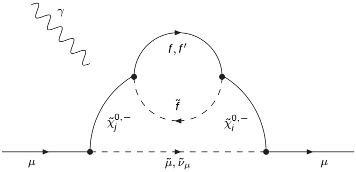
$(g-2)_\mu$ at the two-loop level — large contributions from heavy squarks

Dominik Stöckinger

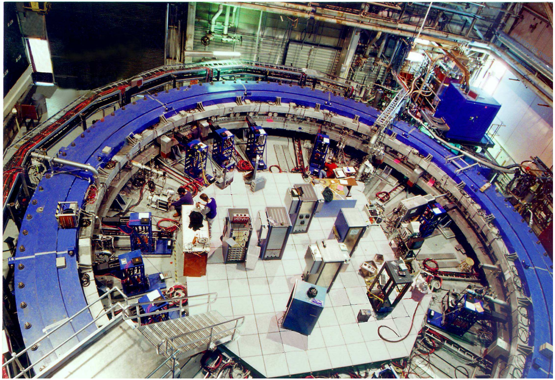
with H. Fargnoli, C. Gnendiger, S. Passehr, H. Stöckinger-Kim

TU Dresden



ECFA LC 2013, Hamburg

The opportunity



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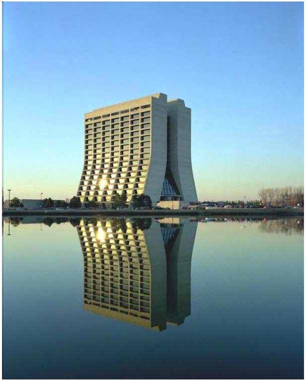
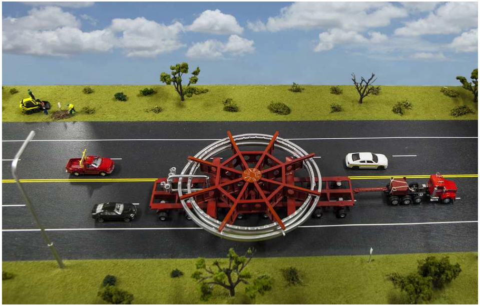
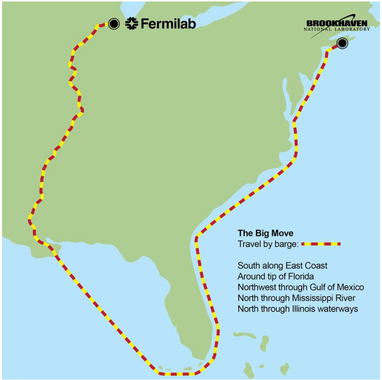


becomes reality





Data in 2016:

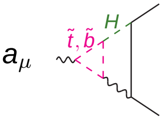
$$a_\mu^{\mathrm{Exp-SM}} = 28(8) \times 10^{-10}$$

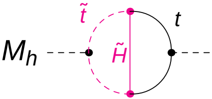
$$\longrightarrow a_\mu^{\mathrm{Exp-SM}} = ???(1.6)_{\mathrm{Exp}}(3)_{\mathrm{SM}} \times 10^{-10}$$

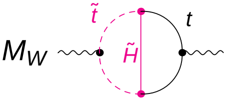
Outline

1. Motivation
   - $g-2$ is complementary to LC
   - $g-2$ is still important and can be explained by low-energy SUSY
2. Evaluation of $f\tilde{f}$-loop contributions
3. Large numerical effects
4. Constraints on new physics and complementarity

Complementarity $g-2$ – LC precision observables



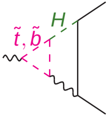




- EWPO, M_h will be measured more precisely at the LC $\rightarrow$ test of quantum structure of SUSY models
- a_μ motivates light SUSY now — more precise measurement soon

Complementarity $g-2$ – LC precision observables

a_μ

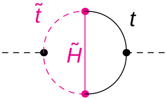


$\mathcal{O}(\mathrm{SM}\times\mathrm{SUSY})$

[Heinemeyer, DS, Weiglein ’03,’04]

Aim: full 2-loop

M_h

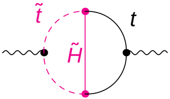


$\mathcal{O}(\alpha_t\alpha_{s,t})$

[FeynHiggs ’98–. . .]

[many others]

M_W



$\mathcal{O}(\alpha_t^2)$

[Haestier, Heinemeyer, DS, Weiglein ’05]

[Heinemeyer, Hollik, DS, Weber, Weiglein ’06]

- EWPO, M_h will be measured more precisely at the LC $\rightarrow$ test of quantum structure of SUSY models
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The tension is increasing

LHC: $m_{\tilde{q},\tilde{g}} >\sim$ 1TeV	a_μ $m_{\tilde{\mu},\chi} <\sim$ 700GeV
$m_h = 126$ GeV $m_{\tilde{t}} >\sim$ 1TeV	finetuning $m_{\tilde{t}}, \mu$ small

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Tension motivates non-traditional models: [Endo, Hamaguchi, Ibe, Yanagida, D.P. Roy, et al]

sleptons$\ll$squarks [1303.4256,1210.3122]
2nd gen$\ll$3rd gen [1303.6995]
non-universal gauginos [1303.5830]
more generic gauge mediation [1201.2611]
new extra matter or U(1)' (e.g. $\rightsquigarrow M_2 \ll \mu$) [1108.3071,1112.6412]

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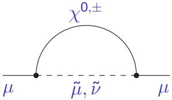
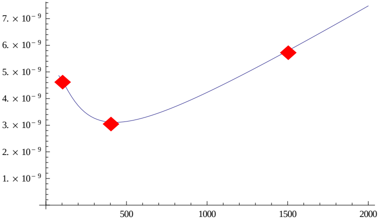
$\Rightarrow$ split/hierarchical spectra

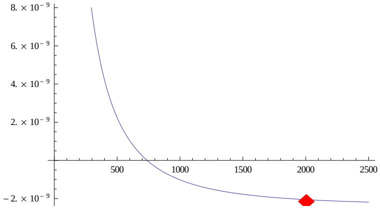
SUSY one-loop contributions



small/large μ



small/large $M_{\tilde{\mu}_L}$



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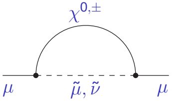

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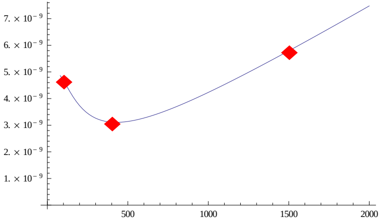

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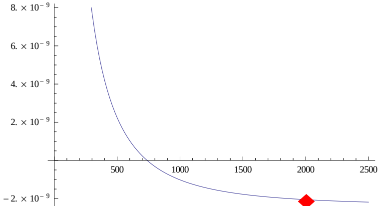
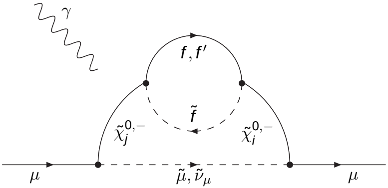


- Current deviation $a_\mu^{\mathrm{Exp-SM}} \approx 28(8) \times 10^{-10}$ important constraint on SUSY
- Motivates light SUSY particles
- Motivates split/hierarchical spectra

The new contributions

New

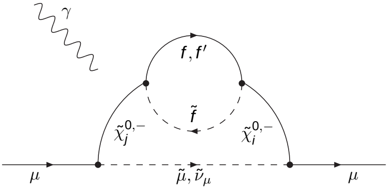


Motivation 1: Split spectra

- Questions:
  - influence of light/heavy stop/sbottom masses and mixings?
  - Enhancements possible?

The new contributions

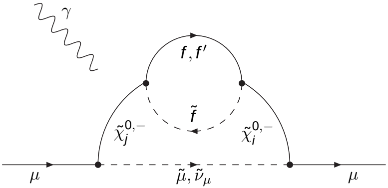
New



Motivation 2: Big step towards full two-loop calculation!

The new contributions

New



Motivation 2: Big step towards full two-loop calculation! Status before:

- known:
  - SUSY corrections to SM 1L diagrams [Heinemeyer, DS, Weiglein '03,'04]
  - $\tan^2\beta$-corrections to SUSY 1L diagrams [Marchetti, Mertens, Nierste, DS '08]
  - photonic corrections to SUSY 1L diagrams [v. Weitershausen, Schäfer, Stöckinger-Kim, DS '10]
- unknown: remaining corrections to SUSY 1L diagrams
- resulting theory error $\approx 3 \times 10^{-10}$ [DS '06]

The new contributions

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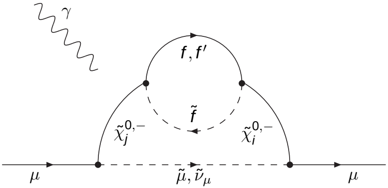
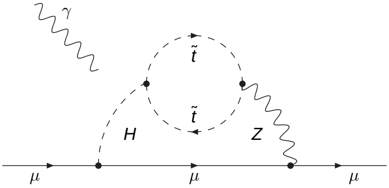


Old



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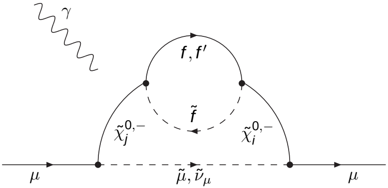
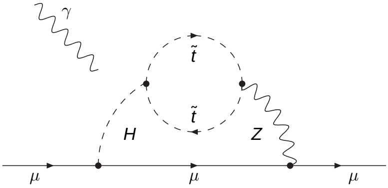


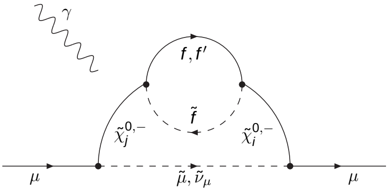
Old



Properties:

- subclass of remaining corrections to SUSY 1L diagrams
- only class with dependence on squarks
- maximum complexity: 5 different heavy masses + 2 light scales
- computed exactly, including required renormalization

Computation



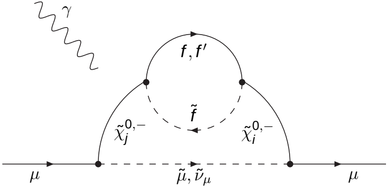
Computation 1

- "standard"
- FeynArts, TwoCalc
- projection operator
- Large mass expansion
- IBP reduction to master integrals

Computation 2

- "Barr-Zee", "by hand"
- inner loop as integral representation
- insert into outer loop
- elegant full result

Computation

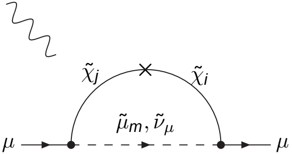


$$a_\mu^{ns}(i,j,\tilde{f}_k) = \int_0^1 \mathrm{d}w \Big\{ \mathcal{A}^{n+}_{ji\tilde{\mu}m} \left(\widetilde{\mathcal{A}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{ns+}_{AA} + \widetilde{\mathcal{B}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{ns+}_{AB} \right) + \mathcal{B}^{n+}_{ji\tilde{\mu}m} \left(\widetilde{\mathcal{A}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{ns+}_{BA} + \widetilde{\mathcal{B}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{ns+}_{BB} \right)$$
$$+ \mathcal{A}^{n-}_{ji\tilde{\mu}m} \left(\widetilde{\mathcal{A}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{ns-}_{AA} + \widetilde{\mathcal{B}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{ns-}_{AB} \right) + \mathcal{B}^{n-}_{ji\tilde{\mu}m} \left(\widetilde{\mathcal{A}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{ns-}_{BA} + \widetilde{\mathcal{B}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{ns-}_{BB} \right) \Big\}.$$

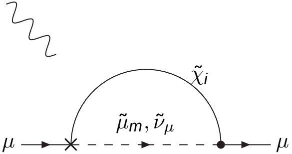
$$\mathcal{T}^{ns1}_{BB} = -\left(\frac{(2w-1)x_{\mathrm{BZ}} - x_{\mathrm{mf}} + x_{\mathrm{msf}}}{1-w} \frac{2}{x_{\mathrm{BZ}} - x_i} \right) \frac{F_2^N(x_{\mathrm{BZ}})}{24},$$
$$\mathcal{T}^{ns2}_{BB} = \Big[-4L(m_{\mathrm{MS}}) - l_i - 2l_{\mathrm{msf}} + \frac{1}{2x_i} + 2 + \frac{x_i}{2}$$
$$+ \left(\frac{(2w-1)x_i - x_{\mathrm{mf}} + x_{\mathrm{msf}}}{1-w} \frac{2}{x_{\mathrm{BZ}} - x_i} \right) \Big] \frac{F_2^N(x_i)}{24} + \frac{-1-x_i}{16x_i}.$$

Computation of counterterms

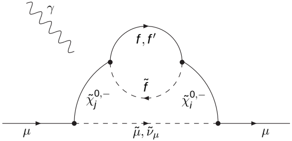
- at $\chi^{0,\pm}$ propagator
  cancel 2L divergences
  insertions to $\delta\mu, \delta M_{1,2}, \delta Z_\chi, \ldots$
  from $f\tilde{f}$-loops



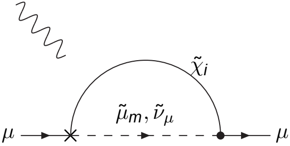
- at muon vertex
  no corresponding 2L diagrams⇒finite
  insertions to $\delta g_{1,2}, \delta y_\mu, \delta s_W, \ldots$
  from f or $\tilde{f}$-loops



Result contains large logs, $\Delta\rho$

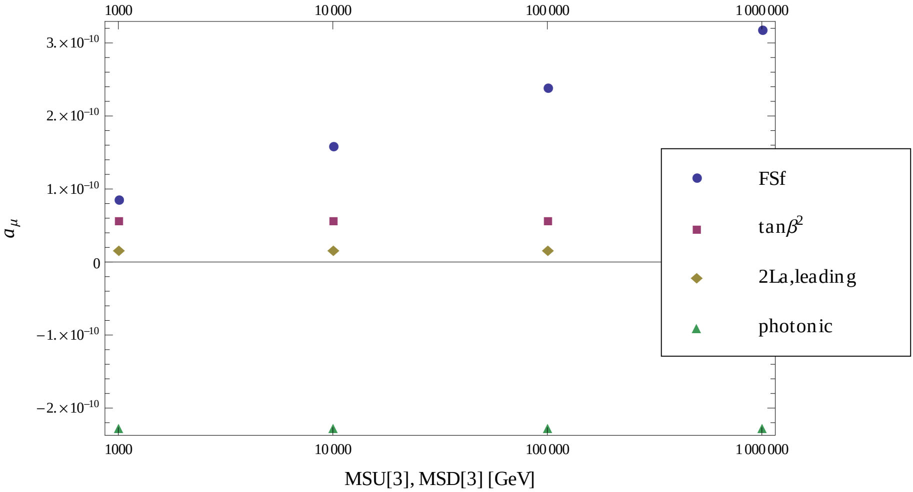
1 

$\rightarrow a_\mu^{1L} \times \log(m_{\tilde{f}})$

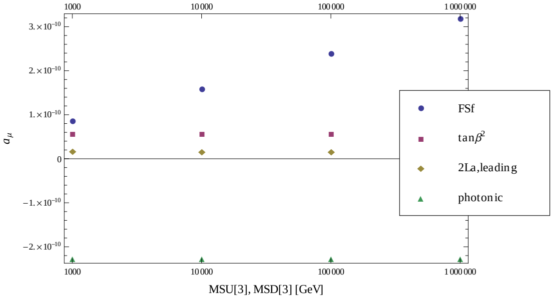
2 

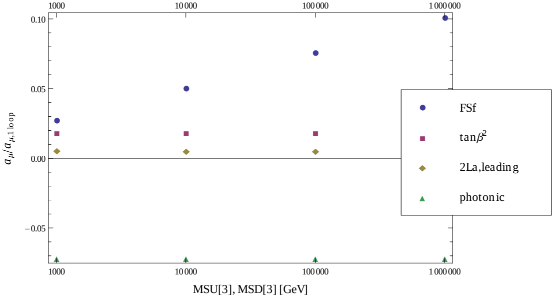
$\rightarrow a_\mu^{1L} \times \Delta\rho$

Large contributions from heavy squarks



Large contributions from heavy squarks



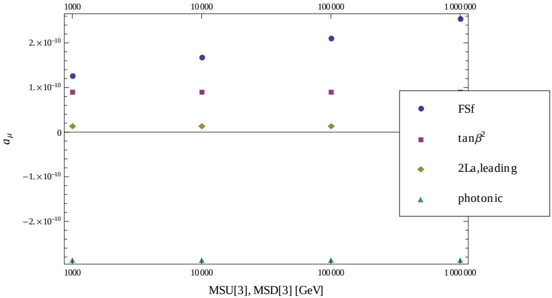


SPS1a + heavy squarks

competes with photonic, $\tan^2\beta$-corrections,

can be largest 2L contribution $\mathcal{O}(10\%)$ (for very heavy squarks)

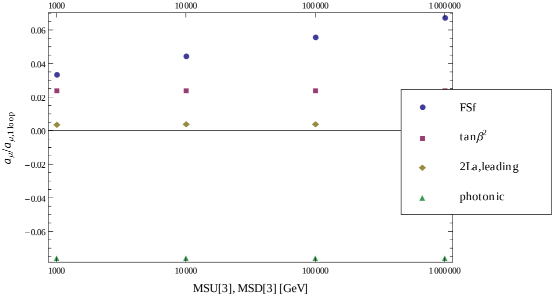
Large contributions from heavy squarks



Point 4 from

[Endo,Hamaguchi,Iwamoto,Yoshinaga '13]

(large μ; $M_2 = M_L = 300$)
+ heavy squarks



competes with photonic,
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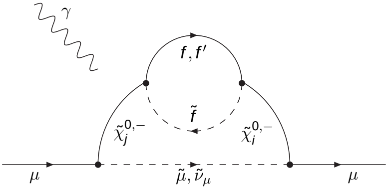
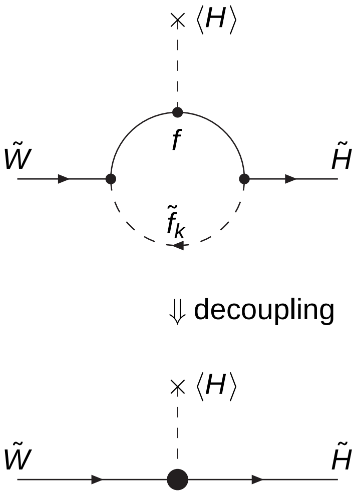
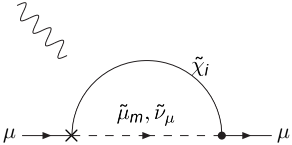
Where do these logs come from?





renormalizable but non-SUSY term in EFT

Contributions involving $\Delta\rho$

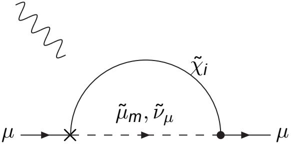


$$= a_\mu^{1L} \times \left(\ldots + \frac{\delta(e^2/s_W^2)}{e^2/s_W^2} \right)$$
$$= a_\mu^{1L} \times \left(\Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho + \ldots \right)_{f,\tilde{f}\text{-loops}}$$

One-loop ambiguity

Fixed by full $2Lf\tilde{f}$ calculation

Contributions involving $\Delta\rho$



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One-loop ambiguity

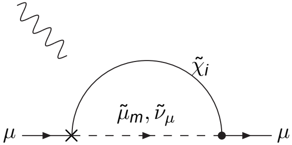
$$\left.\begin{array}{lll} a_\mu^{1L} & = \alpha(0)\ldots & = 29.4 \\ a_\mu^{1L} & = \alpha(M_Z)\ldots & = 31.6 \\ a_\mu^{1L} & = \alpha(G_F)\ldots & = 30.5 \end{array}\right\}$$

Fixed by full $2Lf\tilde{f}$ calculation

differ by $\Delta\alpha$, $\Delta\rho$: 2L$f\tilde{f}$-terms

(for SPS1a, unit: 10^{-10})

Contributions involving $\Delta\rho$



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differ by $\Delta\alpha$, $\Delta\rho$: 2L$f\tilde{f}$-terms

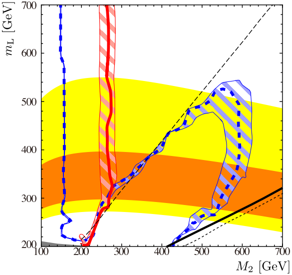
Fixed by full $2Lf\tilde{f}$ calculation

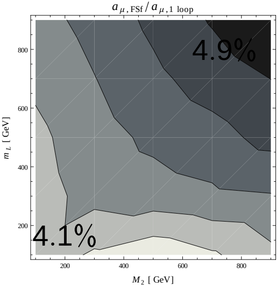
$$a_\mu^{1L+2Lf\tilde{f}} = 32.2$$

(for SPS1a, unit: 10^{-10})

Further numerical examples

Point 4 from [Endo,Hamaguchi,Iwamoto,Yoshinaga '13]





- 2L$f\tilde{f}$ contributions under control, two very different calculations
- decreases theory uncertainty
- numerically significant particularly for split spectra

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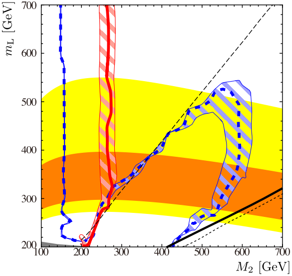

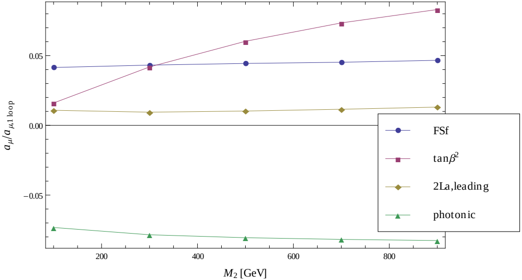

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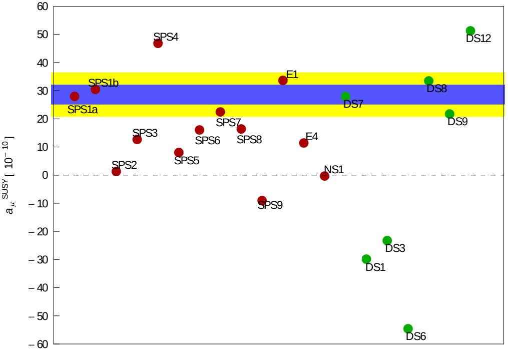
- numerically significant particularly for split spectra

a_μ central complement for SUSY parameter analyses

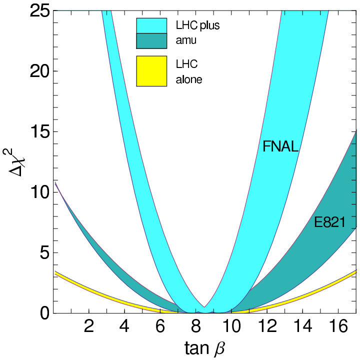


SPS benchmark points

LHC Inverse Problem ($300fb^{-1}$) can't be distinguished at LHC [Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas '10]

- a_μ sharply distinguishes SUSY models
- breaks LHC degeneracies (before Linear Collider!)

a_μ central complement for SUSY parameter analyses



[Hertzog, Miller, de Rafael, Roberts, DS '07]

$\tan\beta = \frac{v_2}{v_1}$
central for understanding EWSB

LHC: $(\tan\beta)^{\text{LHC,masses}} = 10 \pm 4.5$ bad
[Sfitter: Lafaye, Plehn, Rauch, Zerwas '08, assume SPS1a]

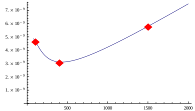
a_μ improves $\tan\beta$ considerably
Also complementary to LC!

vision: test universality of $\tan\beta$, like for $\cos\theta_W = \frac{M_W}{M_Z}$ in the SM:

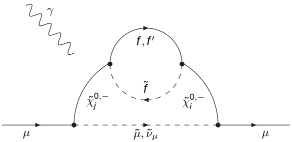
$$(t_\beta)^{a_\mu} = (t_\beta)^{\text{masses}} = (t_\beta)^H = (t_\beta)^b?$$

Summary

- a_μ still viable, complementary constraint on SUSY
  - motivates split scenarios



- $a_\mu^{2\mathrm{L}f\tilde{f}}$ computed
  - first full calculation of a_μ^{SUSY} 2L 5-scale diagrams
  - elegant results



- New contributions are relevant particularly for heavy squarks
  - fix 1L ambiguity $\alpha(0) \leftrightarrow \alpha(M_Z) \leftrightarrow \alpha(G_F)$
  - $\log(m_{\tilde{f}})$-enhanced
  - up to $\mathcal{O}(10\%)$