Top-Antitop Threshold Production: NNLL QCD Uncertainties

Maximilian Stahlhofen

In collaboration with André Hoang

[arXiv: 1111.4486]

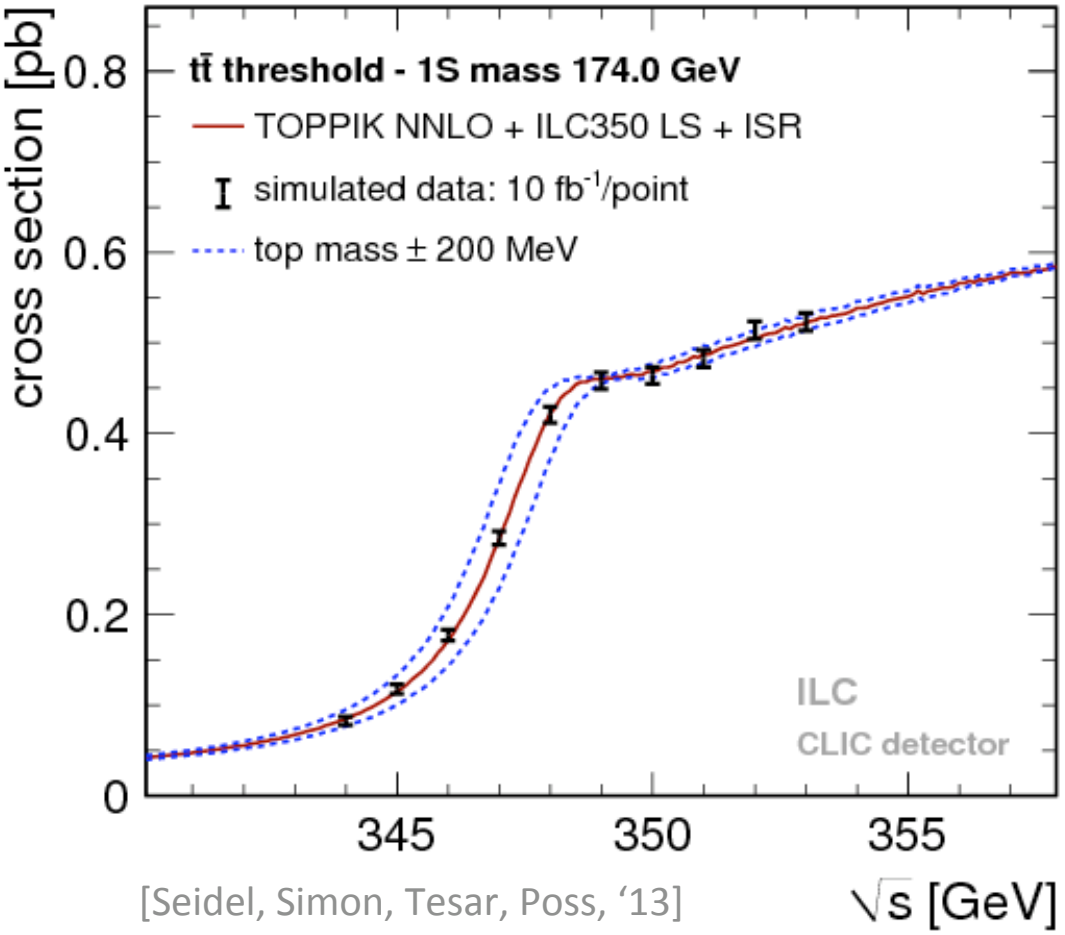
Outline

$$e^+e^- \longrightarrow t\bar{t}$$

- Top-antitop threshold @ lepton colliders
- Theory: NRQCD
- Total cross section at NNLL
- Theory error from scale uncertainties
- Summary

Top-antitop threshold @ lepton colliders

$t\bar{t}$ threshold scan:



[Seidel, Simon, Tesar, Poss, '13]

$\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{QCD}$

- Nonpert. effects suppressed
  [Fadin, Khoze, '87]
- No sharp resonance peaks

$v \ll 1$

- Nonrelativistic regime
- Multiple scales

Top-antitop threshold @ lepton colliders

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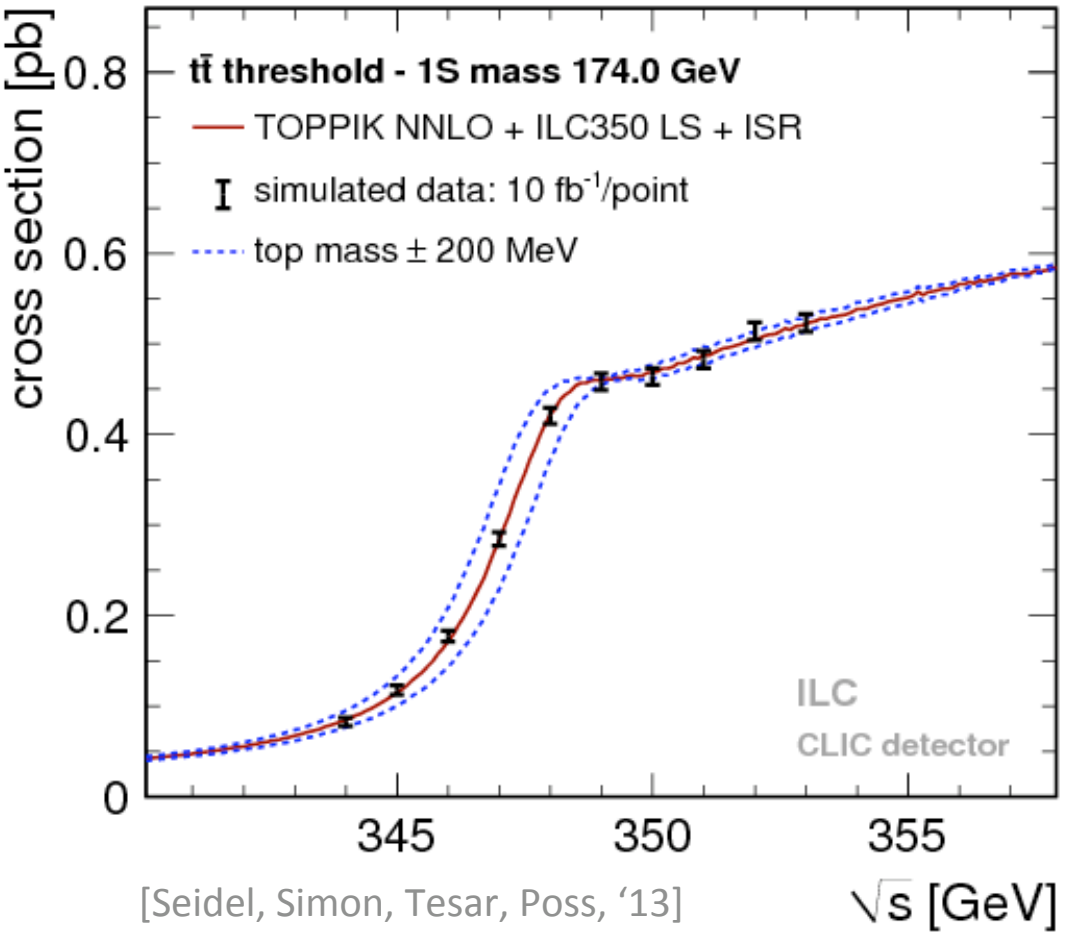


[Seidel, Simon, Tesar, Poss, '13]

Experiment (simulation):

$$\Delta m_t < 100\,\text{MeV}$$

$$\Delta \Gamma_t \sim 30\,\text{MeV}$$

$$\Delta \alpha_s \sim 0.001$$

$$\Delta y_t / y_t \sim 35\%$$

[Martinez, Miquel, '02]
[Seidel, Simon, Tesar, Poss '13]

Theory goal:

$$\Delta \sigma_{tot} / \sigma_{tot} \lesssim 3\%$$

Theory: NRQCD

QCD near $t\bar{t}$ threshold: $v \sim \alpha_s \ll 1$ "nonrelativistic bound state"

multiscale problem:

$$m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2 \quad (\gg \Lambda_{QCD})$$

hard soft ultrasoft

- "Coulomb singularities" $\sim (\alpha_s/v)^n$
- Large logarithms $\sim [\alpha_s \ln(v)]^n$

⇒ Resummation using Effective Field Theory

Theory: NRQCD

EW effects:

- LO: top decay ($t \to W^+ b$)

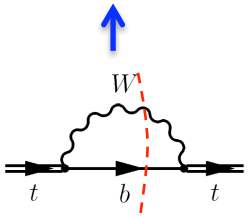
$$\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}}$$

$$v_{\text{eff}} \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \to \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}}\ ;$$

"IR cutoff"

$$|v_{\text{eff}}| \gtrsim 0.1$$

[Fadin, Khoze, '87]

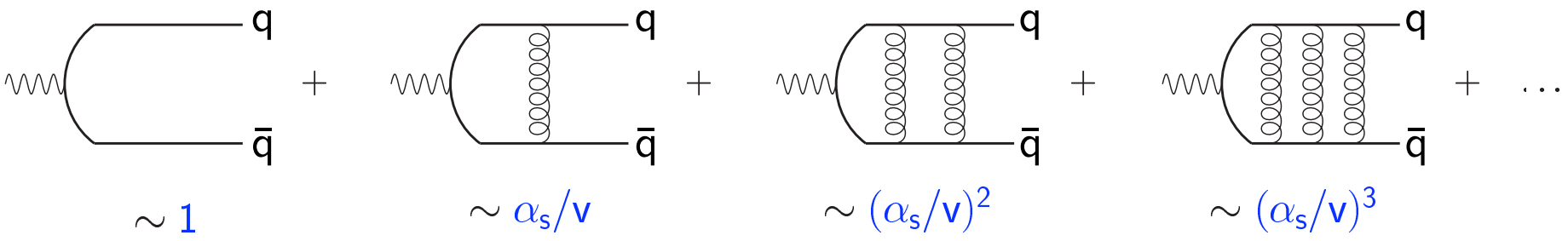


- Higher orders: known to NNLL → talk by Ruiz-Femenia

[Hoang, Reisser, Ruiz-Femenia, '10]
[Beneke, Jantzen Ruiz-Femenia, '10]

Theory: NRQCD

Problem of Coulomb singularities:



~ 1 $\sim \alpha_s/v$ $\sim (\alpha_s/v)^2$ $\sim (\alpha_s/v)^3$

@Threshold: $v \sim \alpha_s \ll 1$ ⇒ breakdown of perturbation theory

Solution:

Nonrelativistic EFT: (v)NRQCD

→ Use Schrödinger Equation to resum $(\alpha_s/v)^n$ terms !

Theory: NRQCD

Problem of large logarithms:

$$m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2$$

hard soft ultrasoft

$$\Rightarrow \quad \alpha_s \ln(E^2/m^2),\ \alpha_s \ln(\mathbf{p}^2/m^2),\ \alpha_s \ln(E^2/\mathbf{p}^2) \ \sim\ \alpha_s \ln v \ \sim\ 1$$

Solution:

Two renormalization scales: $\mu_s = m\nu,\ \mu_u = m\nu^2$ → "v"NRQCD

ν "subtraction velocity"

→ RGE's resum $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n$ ... terms

LL NLL NNLL

Theory: NRQCD

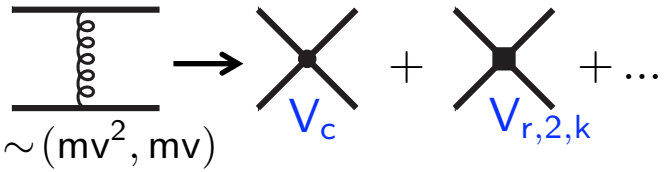
vNRQCD

[Luke, Manohar, Rothstein, '00]

nonrel. quark: $k^\mu \sim (mv^2, mv)$

soft gluon: $k^\mu \sim mv$

ultrasoft gluon: $k^\mu \sim mv^2$

potentials: 

effective production/annihilation current: 

power counting in $v \sim \alpha_s$

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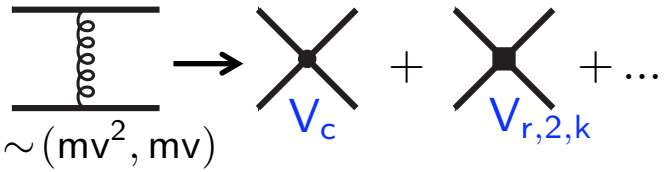
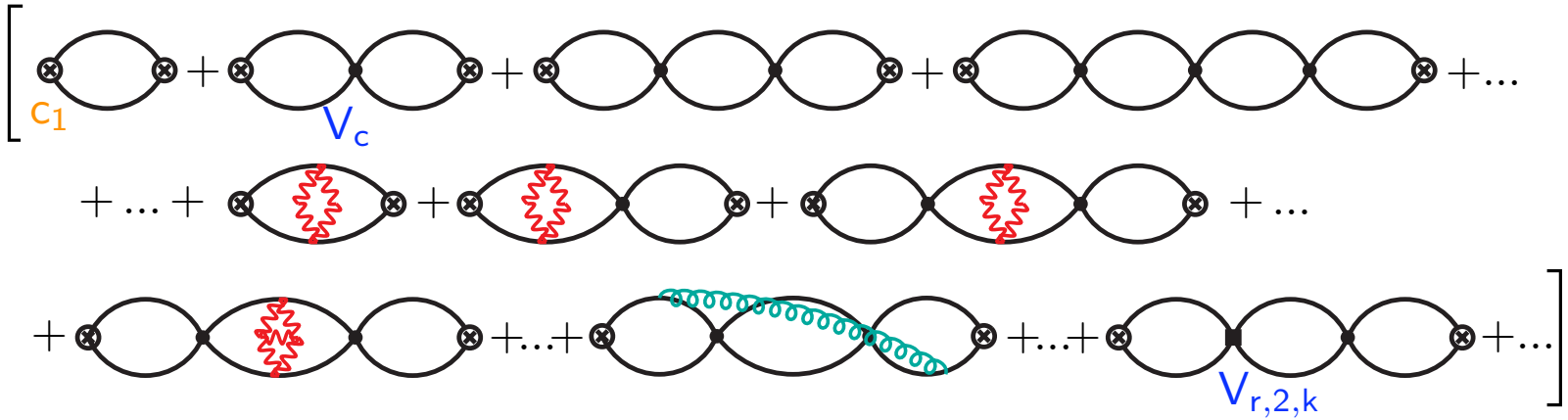
potentials: 

effective production/annihilation current: 

power counting in $v \sim \alpha_s$

$\sigma_{tot} \sim \mathrm{Im}$ 

Total cross section at NNLL

$$\sigma_{\text{tot}}(s) \sim \text{Im}\left[c_1(\nu)^2 \cdot G(0,0,E,\nu) + \dots \right]$$

NNLL known ✓ [Hoang, Manohar, Stewart, Teubner; 2002]
[Pineda, Signer; 2006]

N^3LO known ✓ [Beneke, Kiyo, Schuller; 2007]

Total cross section at NNLL

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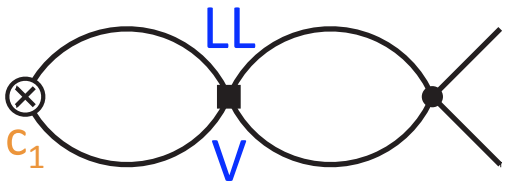
current renormalization

NLL:



[Luke, Manohar, Rothstein; 2000]
[Pineda; 2002] [Hoang, Stewart; 2003]

Total cross section at NNLL

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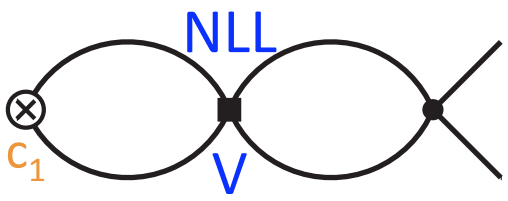
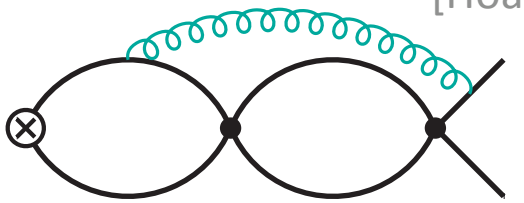
current renormalization

NNLL:

"mixing"



"nonmixing"



[Hoang, '03]

Total cross section at NNLL

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current renormalization

NNLL:

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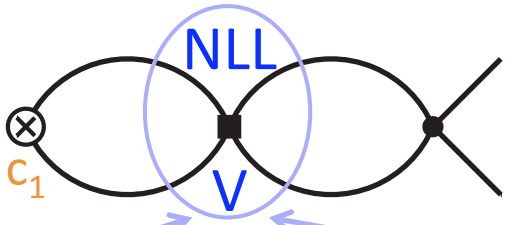
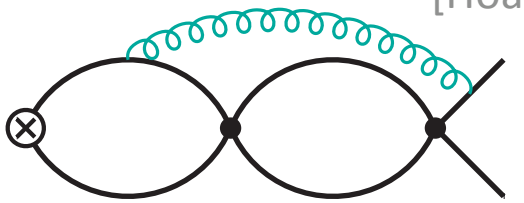
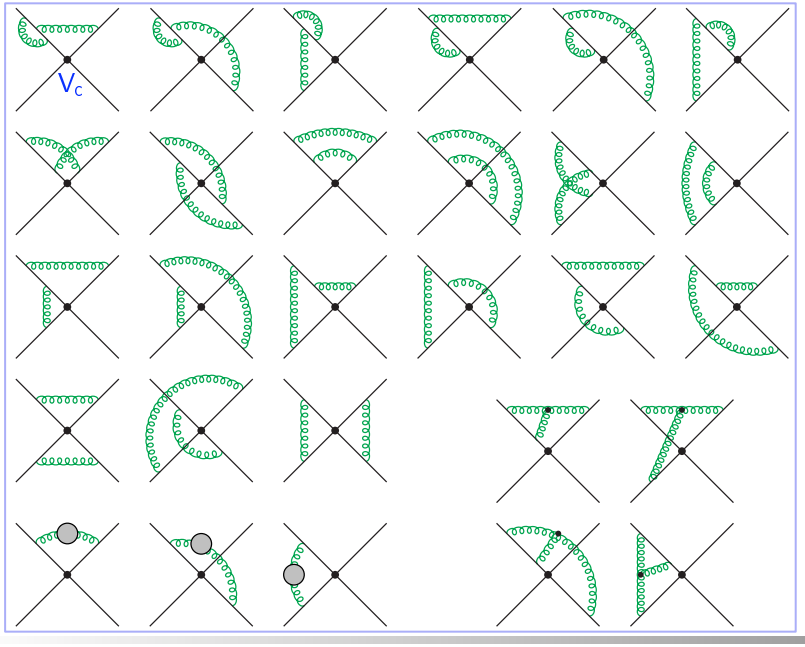


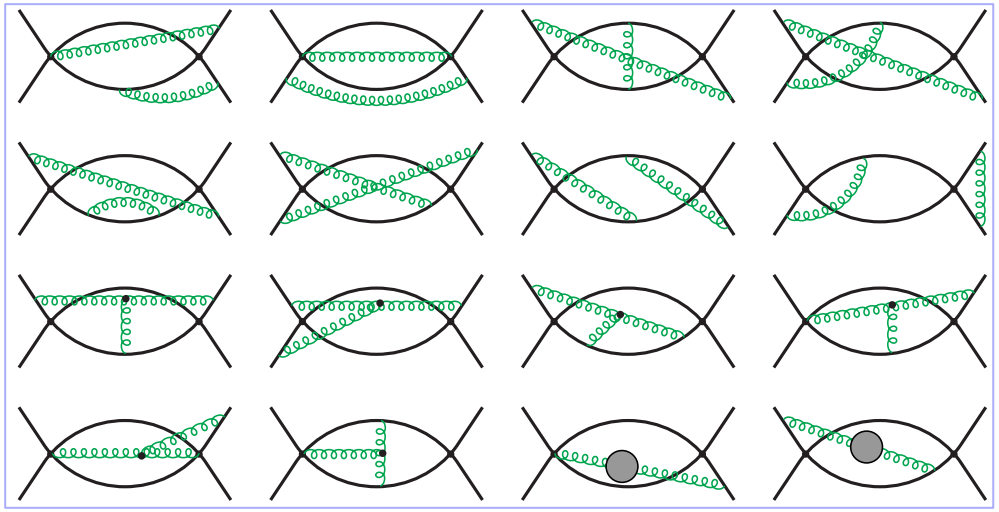
renormalize

"nonmixing"



[Hoang, '03]





[Hoang, MS, 2006/2011]
[Pineda, 2011]

Total cross section at NNLL

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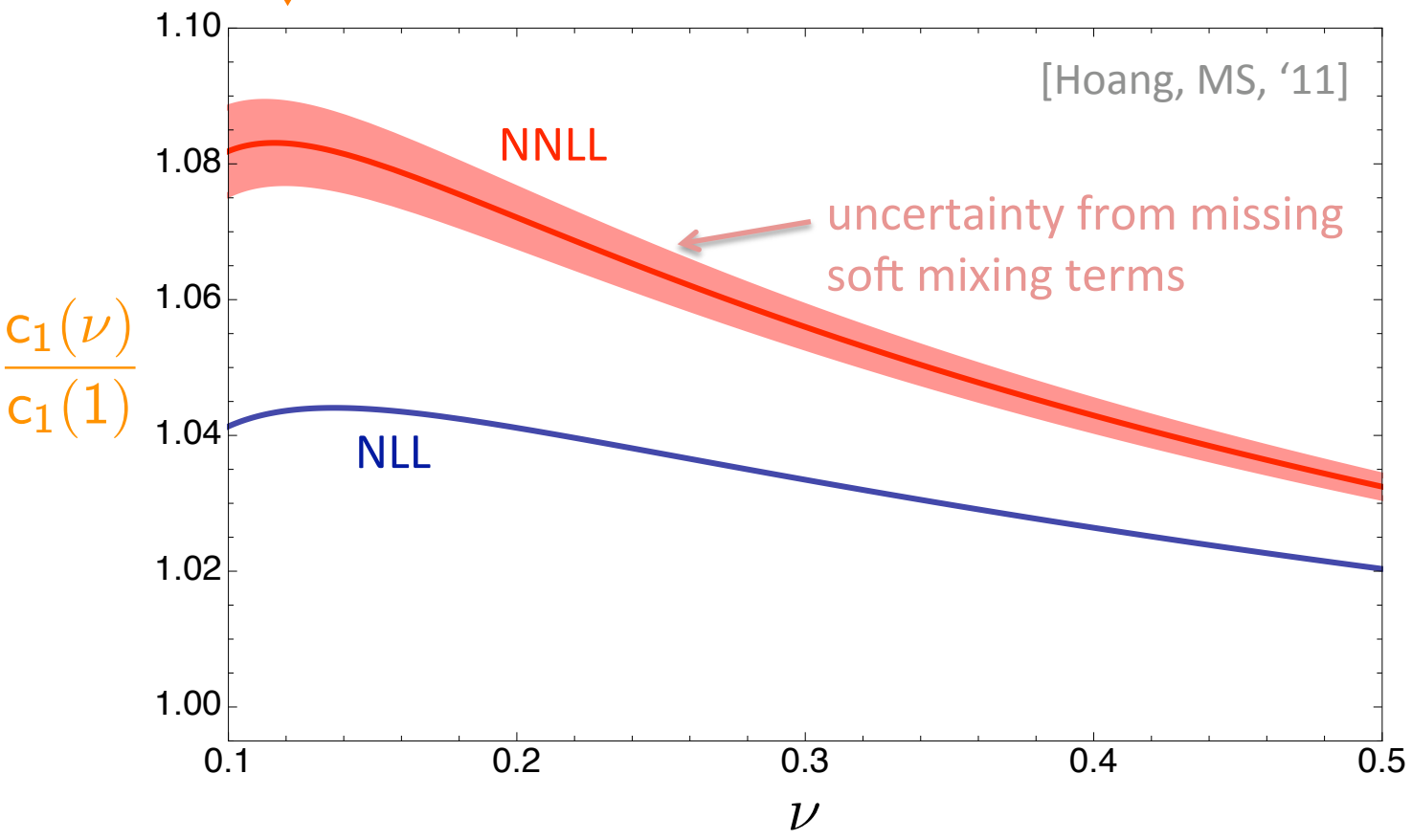
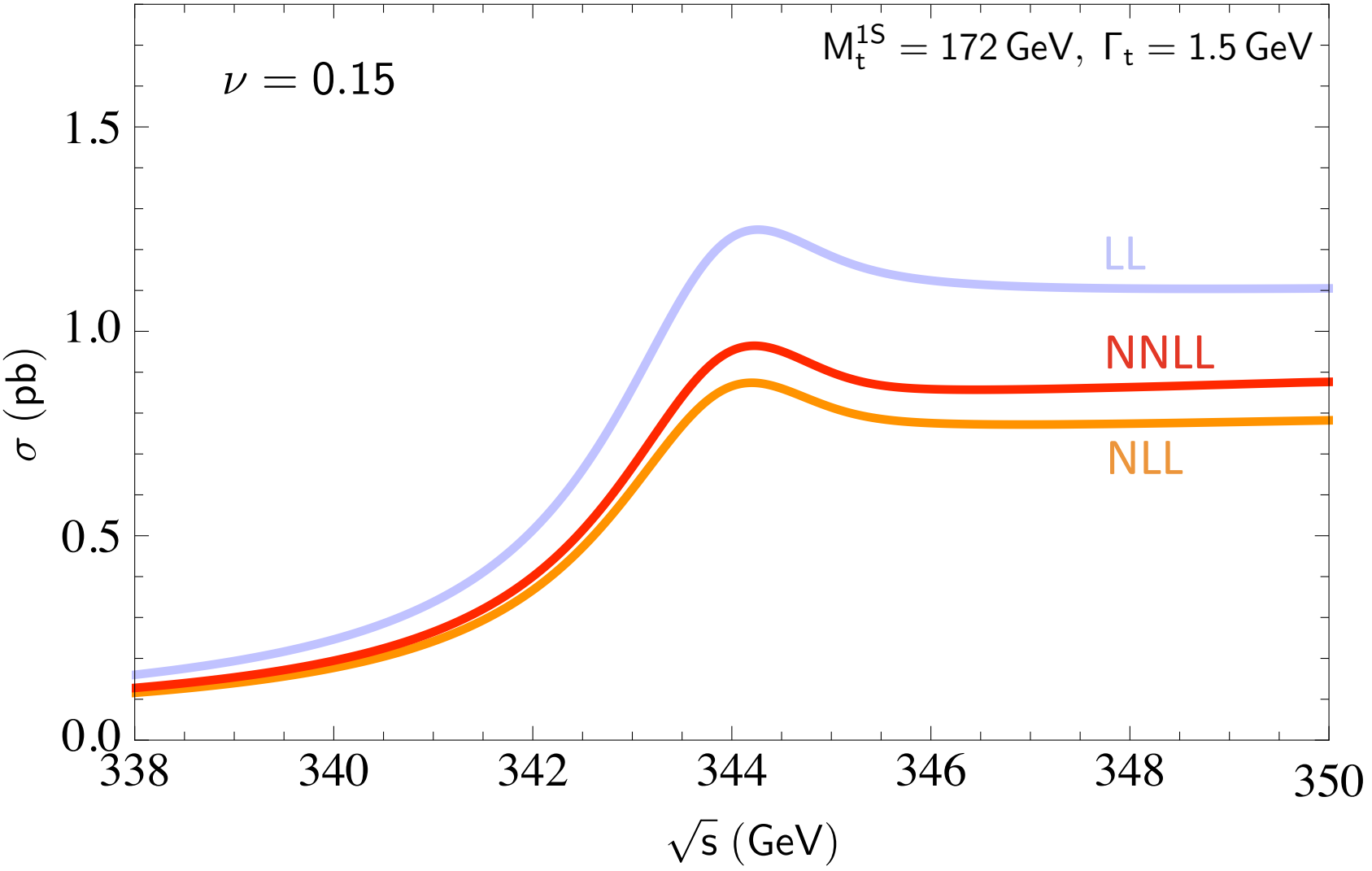


Total cross section at NNLL



Theory error from scale uncertainties

"Unphysical" scales:

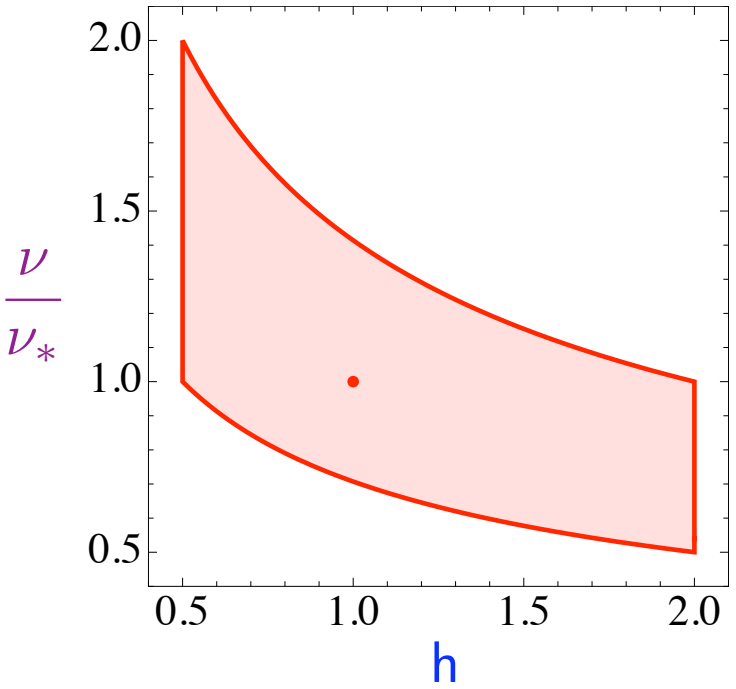
matching scale:

$$\mu_{\text{hard}} = h m$$

renormalization scales:

$$\mu_{\text{soft}} = h m \nu$$

$$\mu_{\text{usoft}} = h m \nu^2$$



default choice:

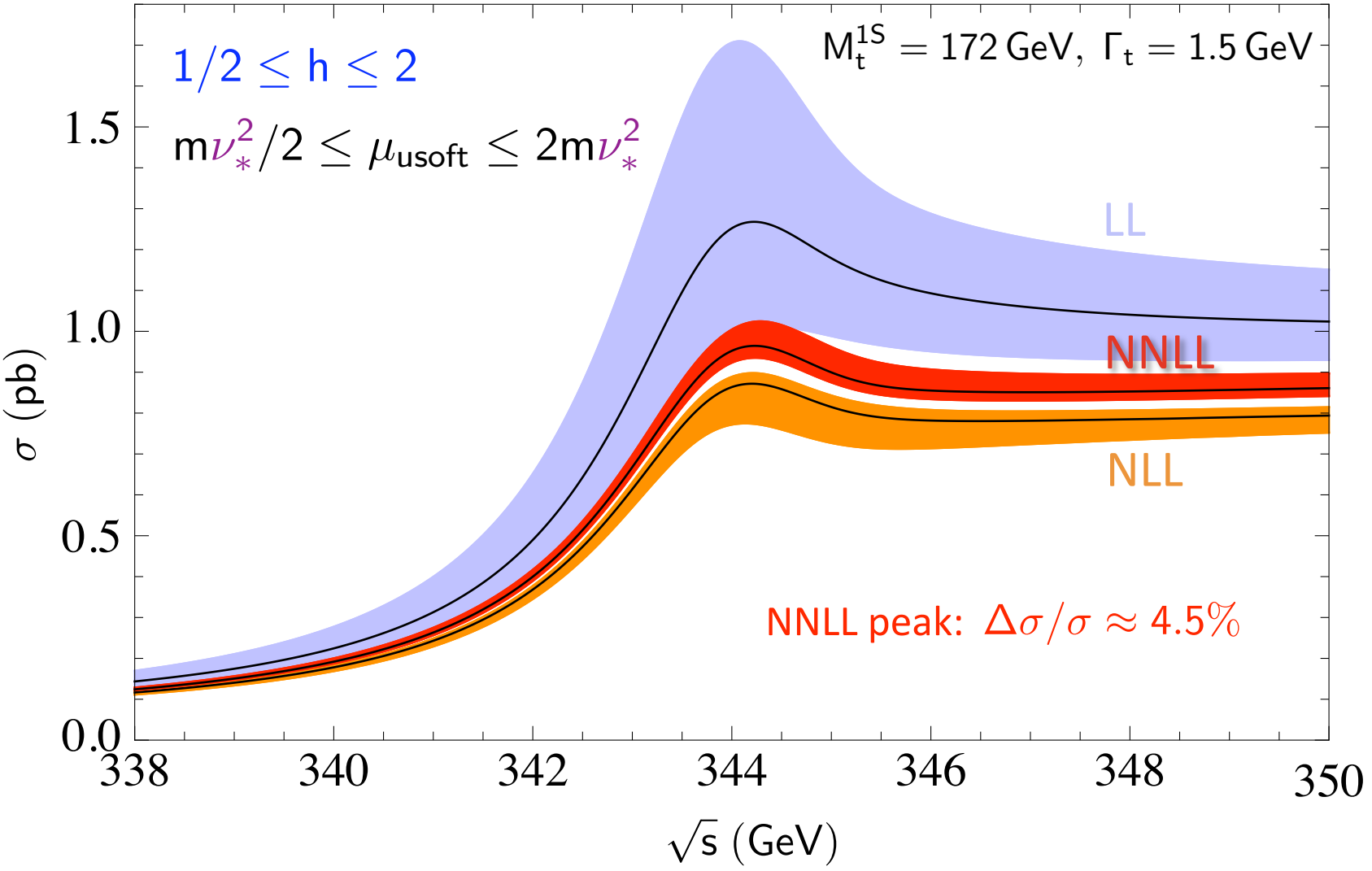
$$h = 1 \qquad \nu_* = 0.05 + |v_{\text{eff}}| \qquad v_{\text{eff}} = \sqrt{\frac{\sqrt{s} - 2m - i\Gamma_t}{m}}$$

scale variation:

$$1/2 \leq h \leq 2$$

$$m\nu_*^2/2 \leq \mu_{\text{usoft}} \leq 2m\nu_*^2$$

Theory error from scale uncertainties



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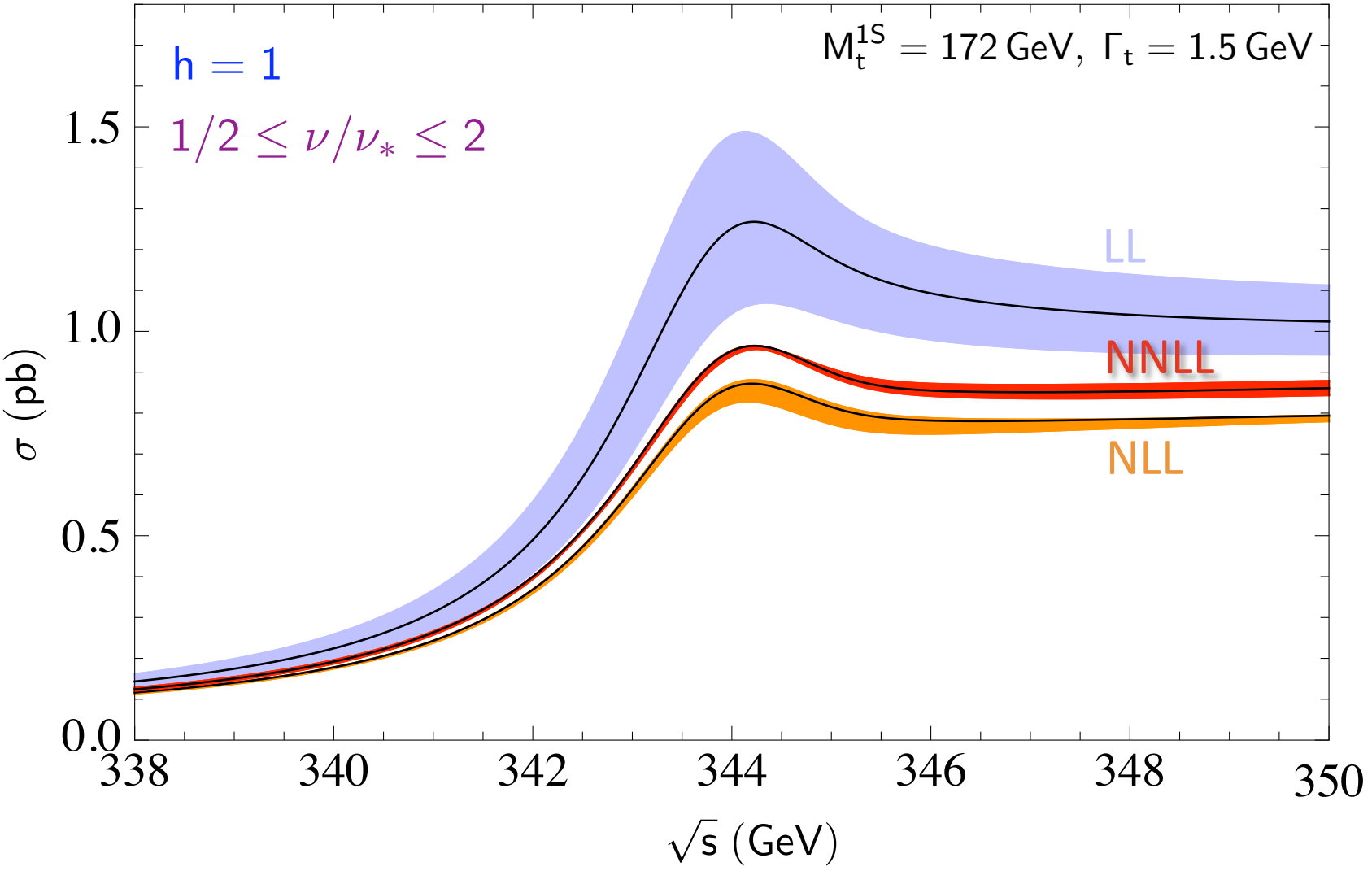

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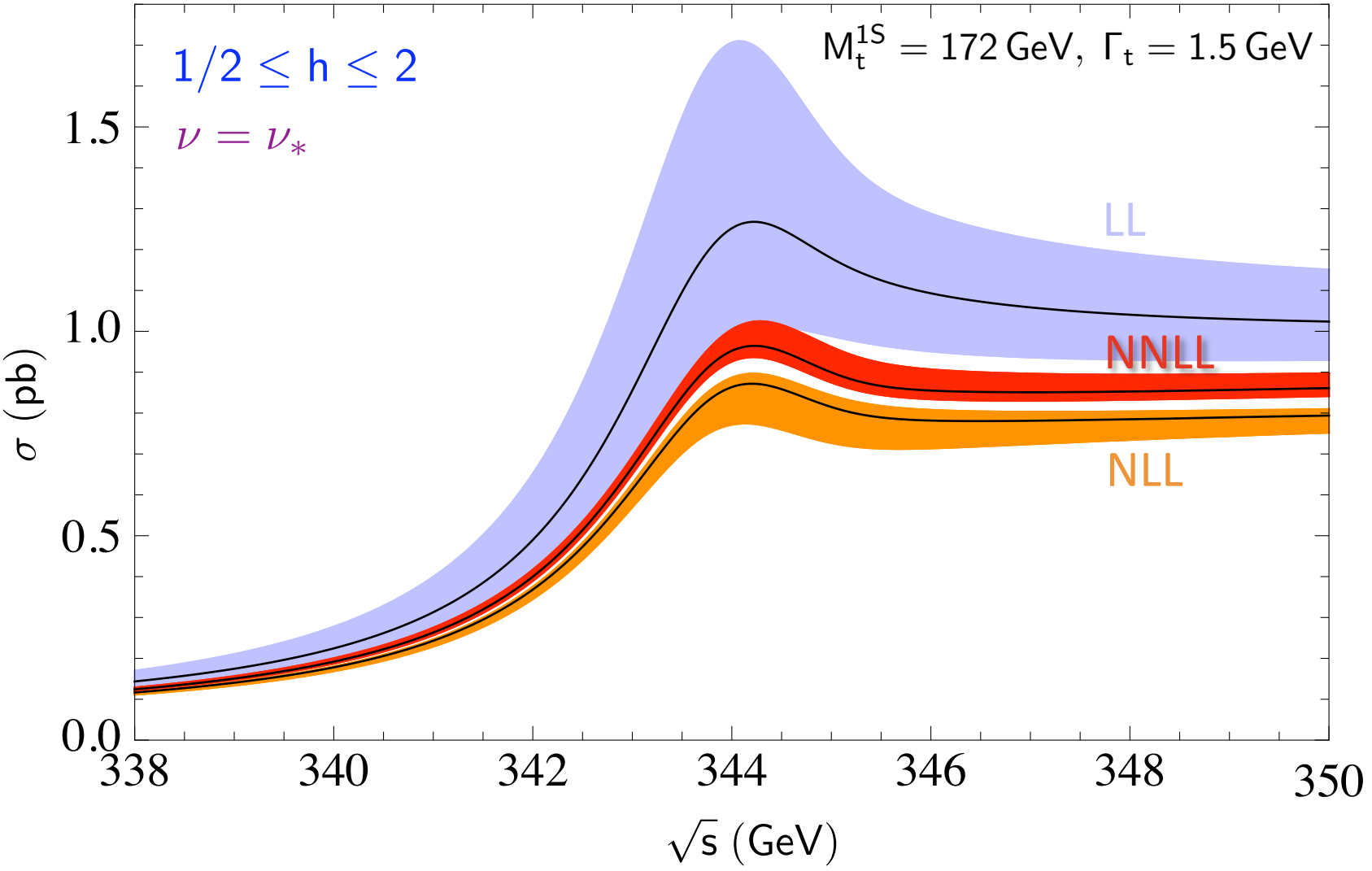
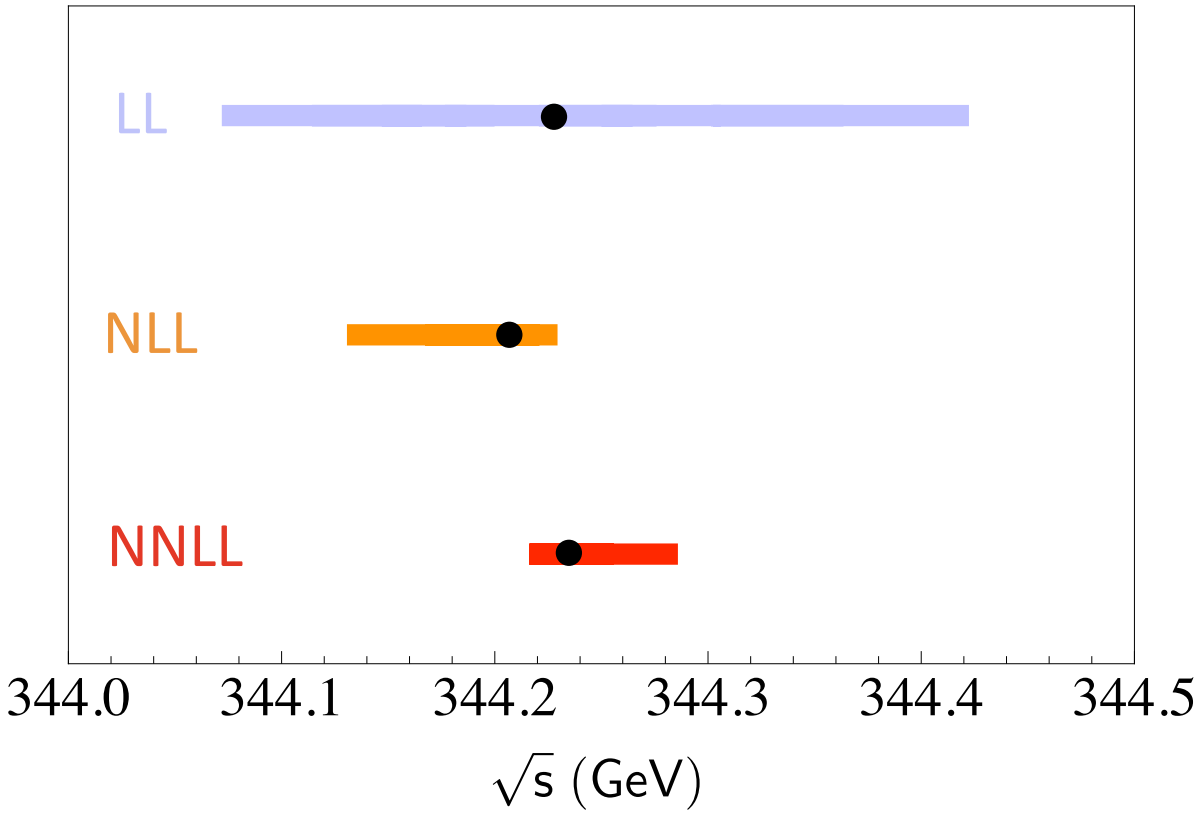


Theory error from scale uncertainties

Uncertainty for peak position:

$M_t^{1S} = 172\,\text{GeV},\ \Gamma_t = 1.5\,\text{GeV}$

$1/2 \leq h \leq 2$ $\qquad$ $m\nu_*^2/2 \leq \mu_{usoft} \leq 2m\nu_*^2$



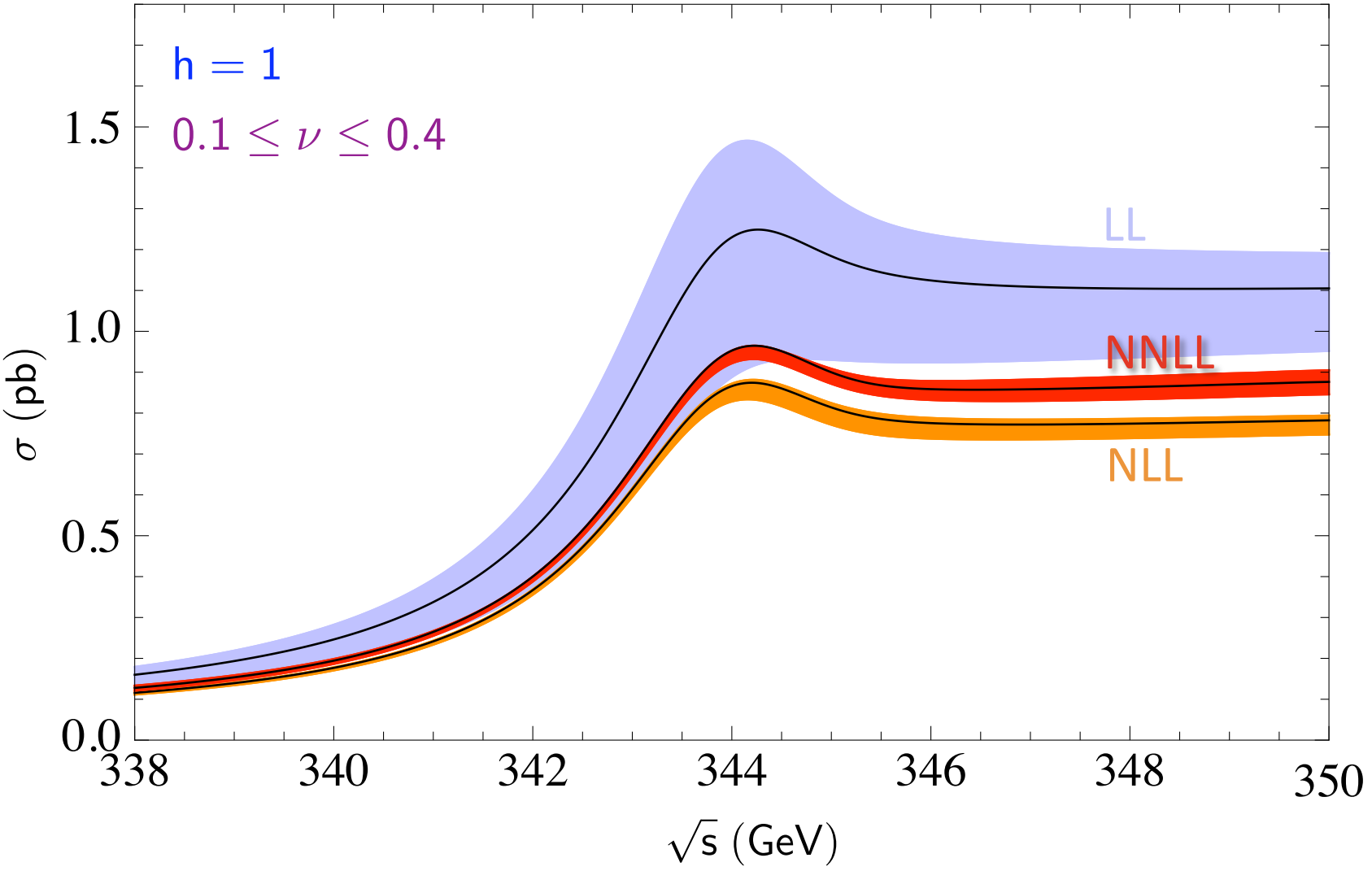
renormalon cancelation at work!

Summary/Outlook

- precise $m_t, y_t, \alpha_s, \Gamma_t$ from $t\bar{t}$ threshold @ LC
- $\sigma_{\text{tot}} \sim \text{Im}\left[c_1(\nu)^2 \cdot G(0,0,E,\nu) \right] + \dots$
- $G(0,0,E,\nu)$ known up to NNLL ✔
- New $c_1(\nu)$ at NNLL (good approximation) ✔
- EW conributions up to NNLL ✔
- $\Delta\sigma/\sigma \approx 4.5\%$ (NNLL peak)
- Stable peak position: $\Delta M_t^{1S,\text{QCD}} \sim 20\,\text{MeV}$
- More detailed error analysis soon! W.I.P

Backup

Theory error from scale uncertainties



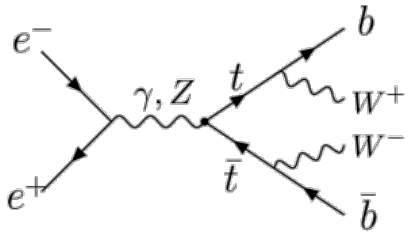
$$e^+e^- \longrightarrow t\bar{t}$$

Top-antitop threshold: EW effects

Top-antitop threshold: EW effects

- Power counting: $\Gamma_t/m_t \sim \alpha_{EW} \sim \alpha_s^2 \sim v^2 \ll 1$

- Physical final state: $e^+e^- \to W^+W^-b\bar{b}$

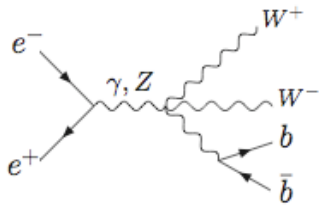


- Apply loose invariant mass cuts on reconstructed tops/antitops:

$$p_{t,\bar{t}}^2 = (m_t \pm \Delta M_t)^2 = m_t^2 + \Lambda^2 \qquad m_t\Gamma_t \ll \Lambda^2 \lesssim m_t^2$$

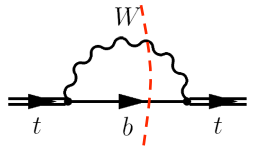
→ no effect on resonant contributions!

→ non-resonant background suppressed:



LO: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ (replacement rule)

unstable top propagator: $\dfrac{i}{E/2 + p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$

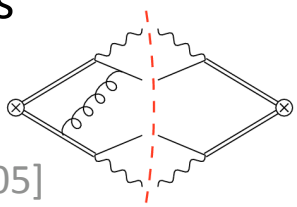


[Fadin, Khoze, '87]

Top-antitop threshold: EW effects

Beyond LO:

- QED: "Coulomb photon" → trivial extension of QCD corrections
- Gluon exchange with final state → negligible at NLO and NNLO



[Fadin, Khoze, Martin '94] [Hoang, Reisser '05]
[Melnikov, Yakovlev '94] [Beneke, Jantzen, Ruiz-Femenia '10]

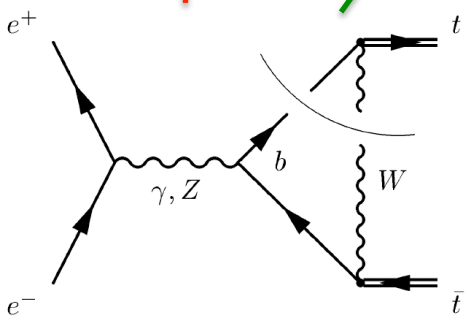
- Corrections to current matching:

$$c_1(1) = c_{1,\mathrm{LL}}^{\mathrm{born}} + c_{1,\mathrm{NLL}}^{\mathrm{QCD}} + c_{1,\mathrm{NNLL}}^{\mathrm{QCD}} + \mathrm{i}\, c_{1,\mathrm{NNLL}}^{\mathrm{bW,abs}} + c_{1,\mathrm{NNLL}}^{\mathrm{EW}} + \dots$$

[Kuhn, Guth '92]

complex matching

[Hoang, Reisser '06]



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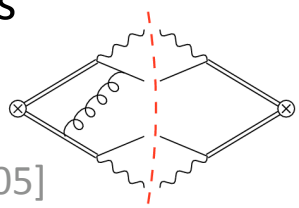

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$$\Rightarrow \sigma_{\mathrm{tot}} \sim \mathrm{Im}\left[c_1(\nu)^2\, G(0,0,E+\mathrm{i}\Gamma_t,\nu)\right] \sim \frac{\alpha_s \Gamma_t}{\epsilon} + \text{finite}$$

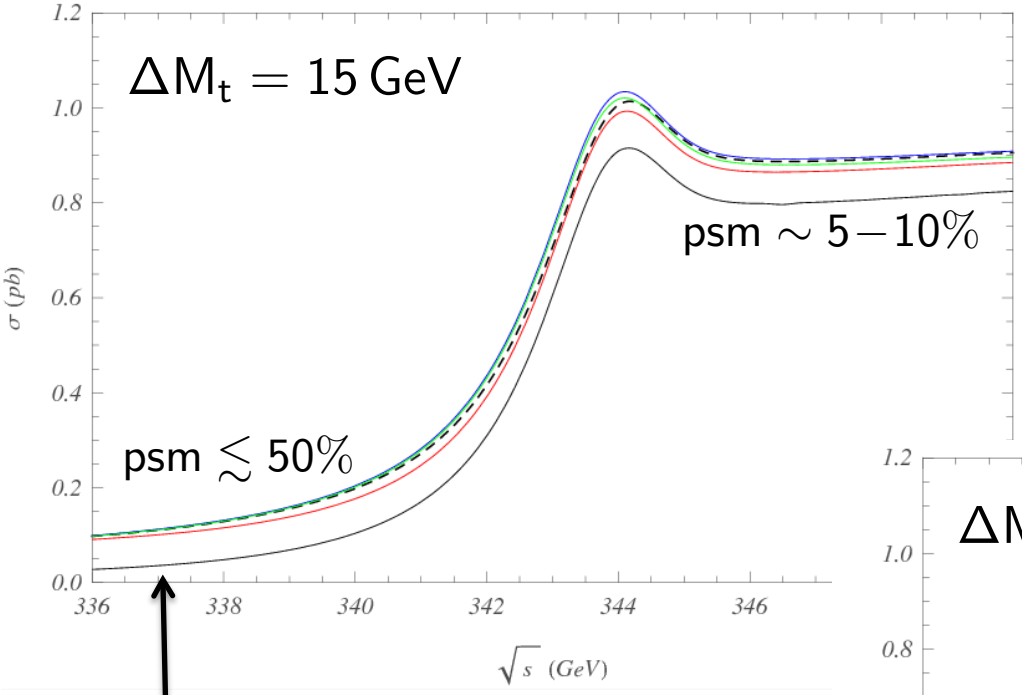
phase space divergence ↓ sums phase space logs in $C(\nu)$ at NLL ✔

$$\mathrm{i}\, C(\nu) \cdot$$

[Hoang, Reisser, Ruiz-Femenia '10]

- "Phase space matching" for $C(\nu)$ to allow for Λ cuts: NLO, NNLO, N^{3}LO ✔

Top-antitop threshold: EW effects



large psm correction due to unphysical phase space in pure QCD prediction

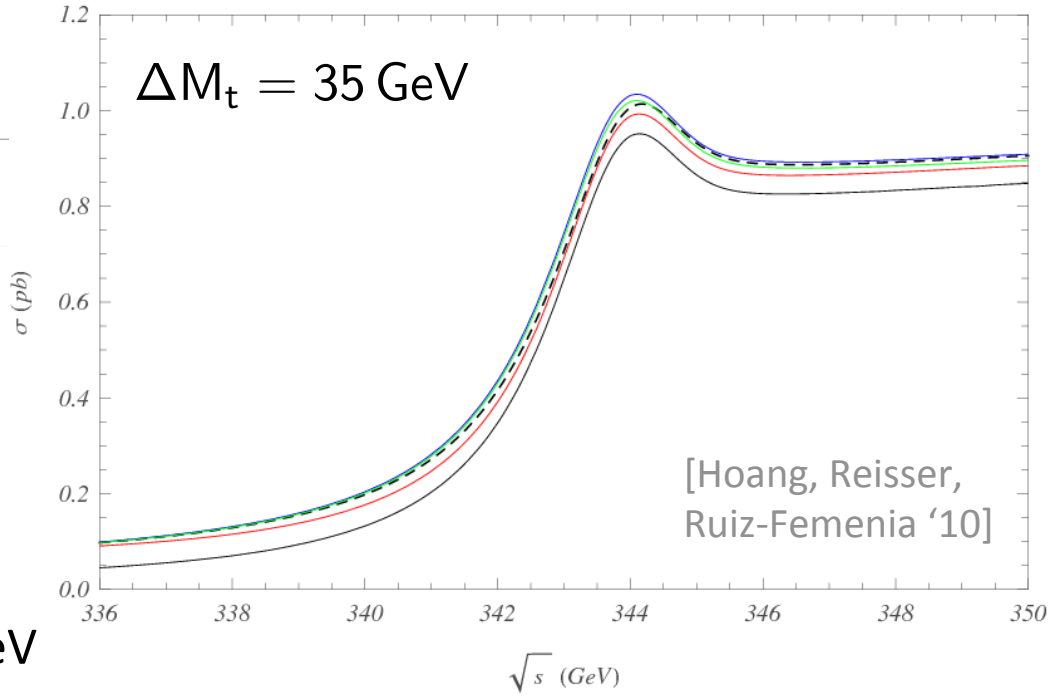
dashed line: NNLL pure QCD prediction (add step by step)

+ NNLL QED effects

+ NNLL EW current matching (real)

+ NNLL EW current matching (absorbtive)

+ NLL+NNLL+N^{3}LL phase space matching contributions (psm)

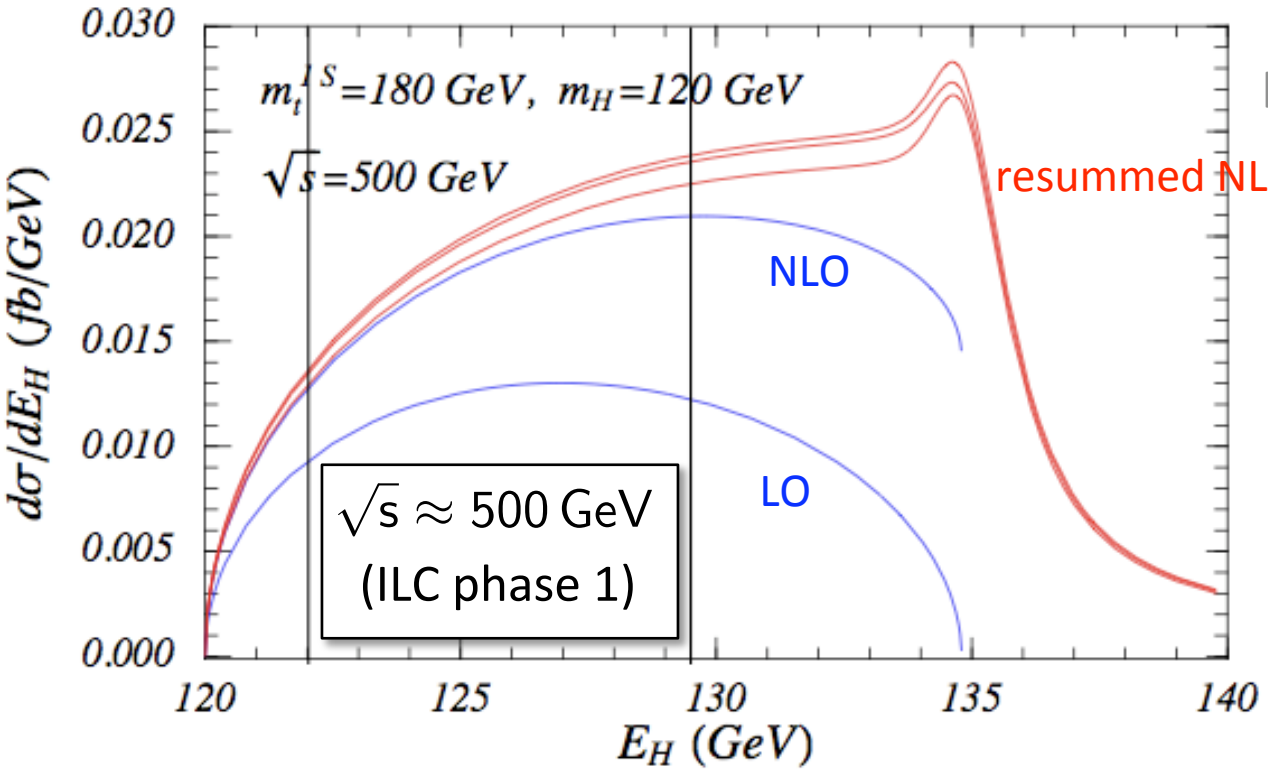


→ Shift in peak position: 30-50 MeV

$$e^+e^- \rightarrow t\bar{t}H$$

Associated Higgs production

Associated Higgs production: $e^+e^- \to t\bar{t}H$

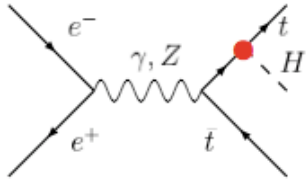


[Farrell, Hoang; 2006]

- For light Higgs ($m_H \approx 120$ GeV): full $t\bar{t}$ phase space nonrelativistic!
  → must sum $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ terms → recycle $t\bar{t}$ results (vNRQCD)
  → factor 2 enhancement over tree level (+ factor 2 from polarized beams)
- realistic studies: $(\delta y_t/y_t)^{\text{ILC}}_{500\text{GeV}} \sim 30\% \to 10-15\%$? [Juste, '02,'06]

Associated Higgs production: $e^+e^- \to t\bar{t}H$

- Dominant contributions from Higgs radiating of the top/antitop
- precise extraction of top Yukawa coupling possible
- At large E_H endpoint: $t\bar{t}$ dynamics nonrelativistic
- For $\sqrt{s} \lesssim 500\,\text{GeV}$ (ILC phase 1) and $m_H \approx 120\,\text{GeV}$:
  full $t\bar{t}$ phase space nonrelativistic !!! → must sum $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$!

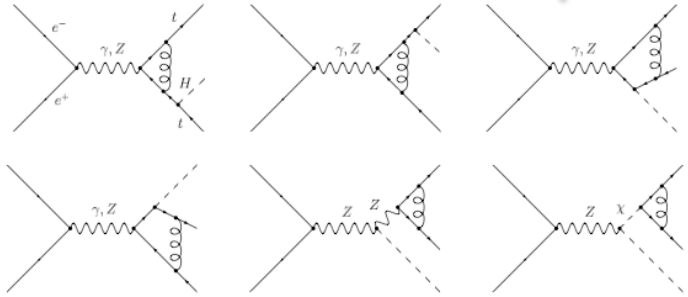


vNRQCD Recycle $t\bar{t}$ results: $c_1(\nu)/c_1(1)$, $G(0,0,v,\nu)$

$$\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_H^{max}} \sim \left[c_{1,0}^2(\nu, \sqrt{s}, m_t, m_H) + c_{1,1}^2(\nu, \sqrt{s}, m_t, m_H)\right] \times \text{Im}[G(0,0,v,\nu)]$$

[Farrell, Hoang; 2005]

NLO matching



[Denner, Dittmaier, Roth, Weber; 2004]