The Higgs boson mixes with an SU(2) septet

NAGOYA UNIVERSITY

PRESENTATION

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The Higgs boson mixes with an SU(2) septet
J. Hisano, K. Tsumura
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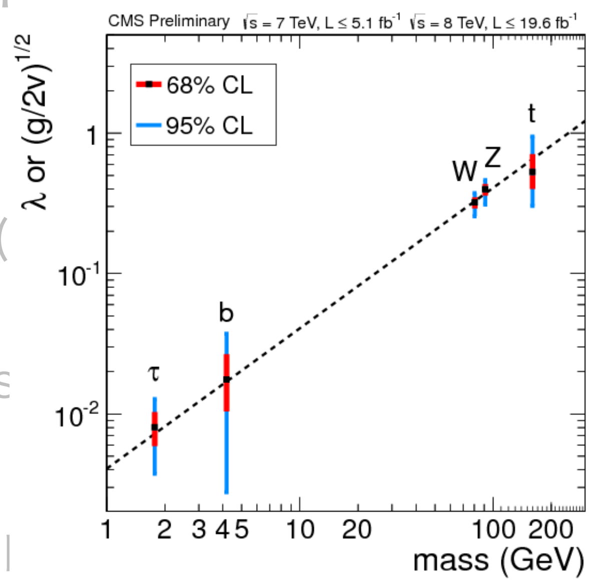
The Higgs boson

"the" = Higgs boson (h) in the SM

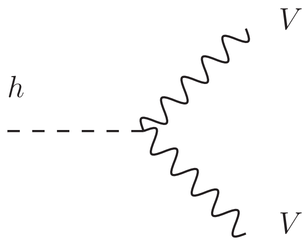
- ## Minima for the Higgs boson
  - The vacuum expectation value [VEV] (... B]
    the Higgs boson triggers electroweak
    → generate weak gauge boson mass

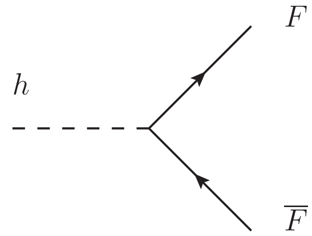


  - Fermion masses are generated via Yul

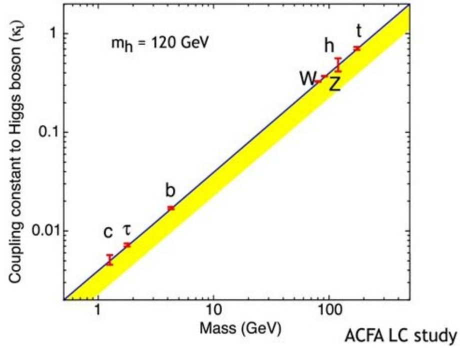
$$m_F = \frac{Y_F}{\sqrt{2}} v$$

  - "Mass" and "Coupling" relation



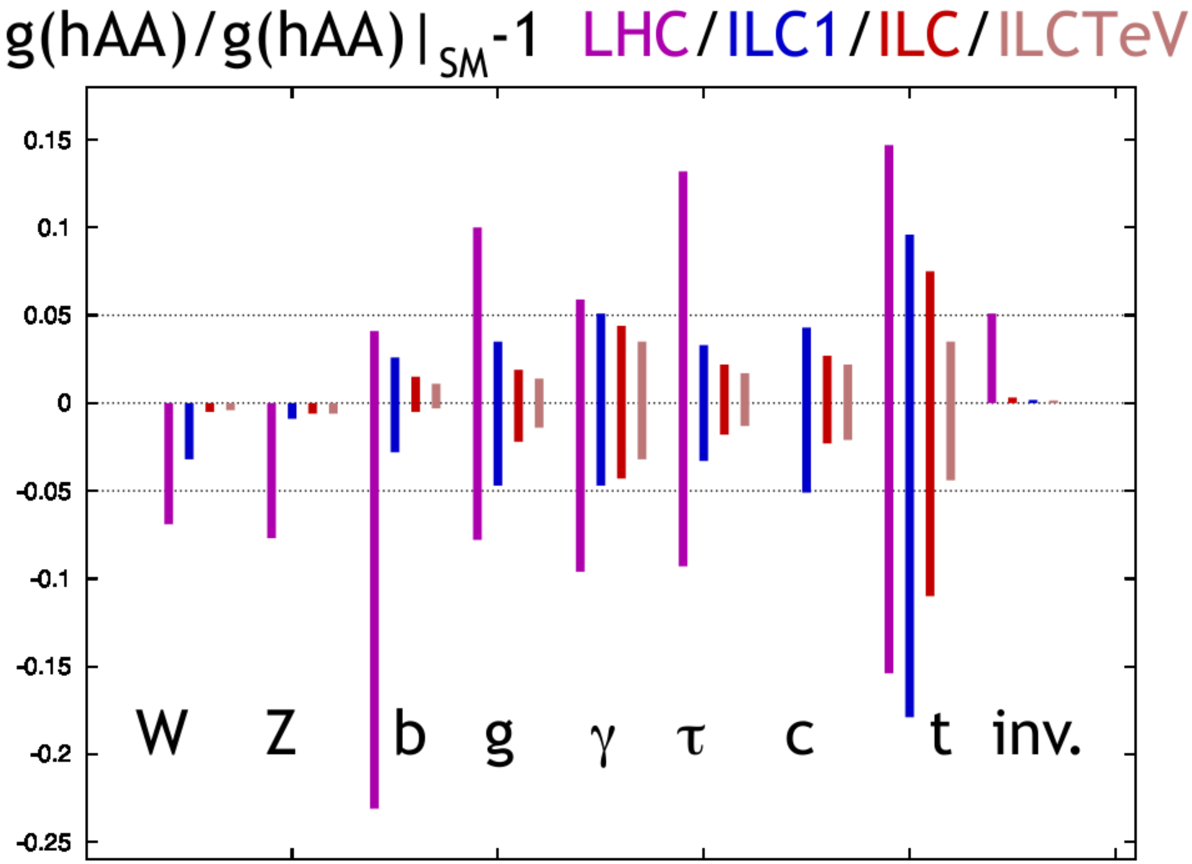


$$\lambda_{hVV} = 2m_V^2/v \qquad \lambda_{hF\overline{F}} = m_F/v$$



ACFA LC study

Precision coupling measurement



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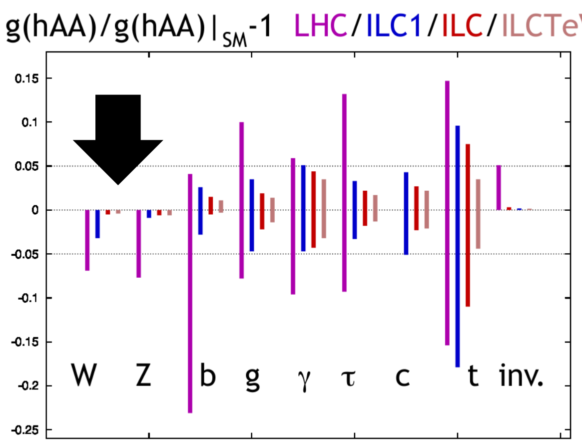
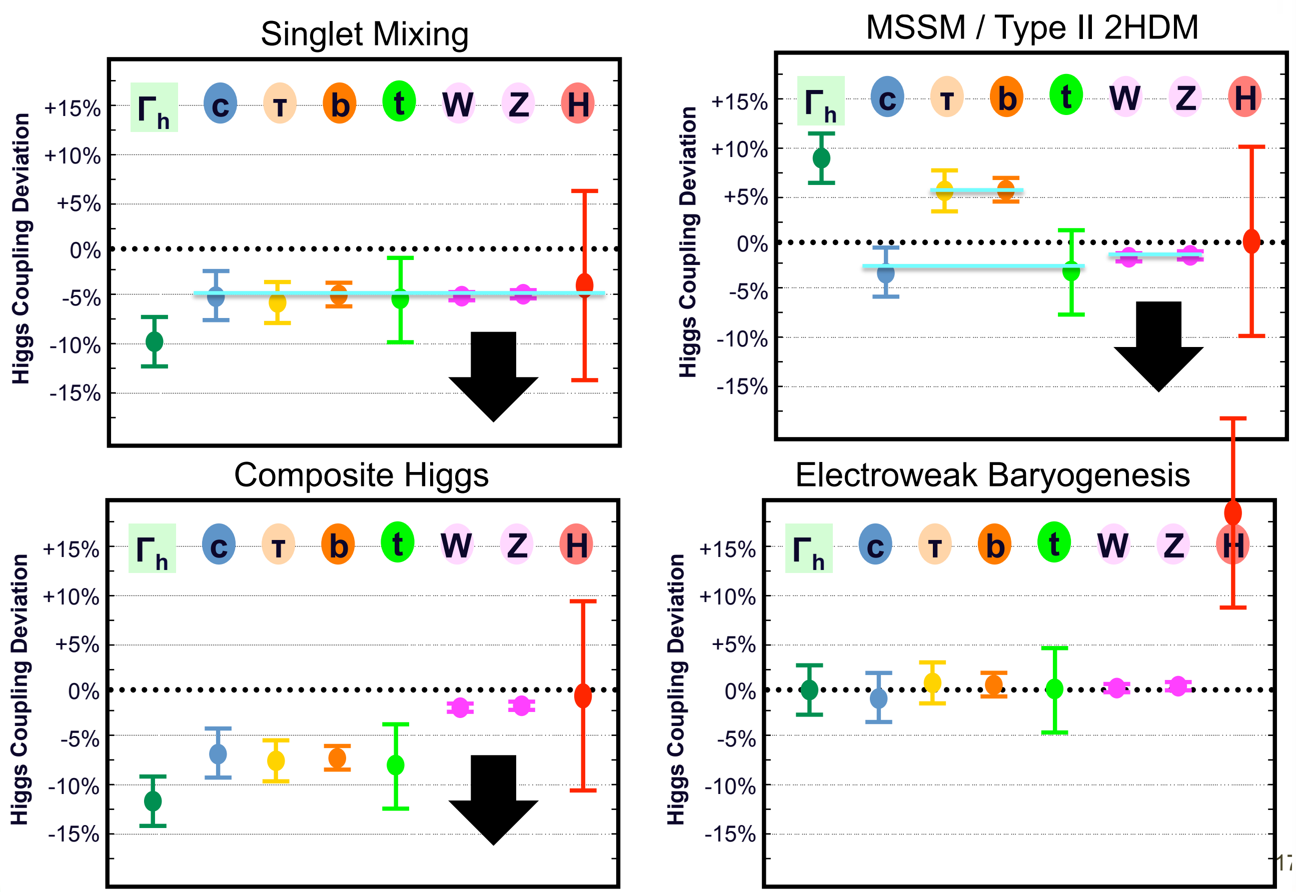


Is the deviation always negative?



How can we extend the SM?

- Electroweak (EW) ρ parameter

$$\rho_{\text{tree}} = \frac{\sum_\alpha [I_\alpha(I_\alpha+1) - Y_\alpha^2] v_\alpha^2}{\sum_\beta 2Y_\beta^2 v_\beta}$$

  - For an arbitrary number of Higgs field with an isospin (I_α), a $U(1)_Y$ hypercharge (Y_α) and a VEV (v_α)

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$$\rho_{\text{tree}}^{\text{SM}} = \frac{m_W^2}{c_W^2\, m_Z^2} = 1 \quad \text{for} \quad I_\alpha = 1/2, Y_\alpha = 1/2, v_\alpha = (\sqrt{2}G_F)^{-1/2}$$

SU(2)$_L$ doublet **in the SM**

Fermi constant

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

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  - Very accurately measured & consistent with the SM

$$\rho_0 = (\rho/\rho_{\rm SM}) = 1.0004^{+0.0003}_{-0.0004}$$

Most important test of the SM [$SU(2)_L$ x $U(1)_Y$ structure]

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$$\rho_{\rm tree}^{\rm triplet} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1}{2} \quad \text{for} \quad I_\alpha = 1, Y_\alpha = 1$$

obviously different from unity

SU(2)$_L$ triplet **in a triplet model** w/o the doublet

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

How can we extend the SM?

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↖ $SU(2)_L$ triplet **in a triplet model** w/o the doublet

$$\rho_{\rm tree}^{\rm HTM} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1+2x^2}{1+4x^2} \approx 1 - 2x^2 \quad \text{with} \quad x = \frac{\langle \Delta^0 \rangle}{\langle \phi^0 \rangle}$$

the SM doublet w/ a $SU(2)_L$ triplet **(Higgs triplet model [HTM])**

$\langle\Delta^0\rangle$ has to be very small

(less contributions to EWSB)

How can we extend the SM?

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$$\rho_{\rm tree} = \frac{\sum_\alpha [I_\alpha(I_\alpha+1) - Y_\alpha^2] v_\alpha^2}{\sum_\beta 2Y_\beta^2 v_\beta}$$

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Most important test of the SM [SU(2)$_L$ x U(1)$_Y$ structure]

ρ=1 seems to be a good guideline to construct Beyond the SM

Why Higgs septet?

- ρ=1 leads Pell's equation (in Number theory)

  with $x=(2I+1), y=2Y, n=3$

$$x^2 - ny^2 = 1$$

  - Trivial solution: (x,y)=(1,0) for arbitrary n

The SM singlet real scalar

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  - Fundamental sol.: $(x_1,y_1)=(2,1)$ for n=3 ["the" Higgs field in the SM]

    SU(2) doublet w/ Y=1/2

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    SU(2) doublet w/ Y=1/2

  - General sol.: $$x_k = \frac{1}{2}[(x_1 + y_1\sqrt{n})^k + (x_1 - y_1\sqrt{n})^k]$$ Bhaskara II (1150)

$$y_k = \frac{1}{2\sqrt{n}}[(x_1 + y_1\sqrt{n})^k - (x_1 - y_1\sqrt{n})^k]$$

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    $$y_k = \frac{1}{2\sqrt{n}}[(x_1 + y_1\sqrt{n})^k - (x_1 - y_1\sqrt{n})^k]$$

→ <u>Next minimal sol.</u>: $(x_2,y_2)=(7,2)$ SU(2) septet w/ Y=1

Septet can have sizable VEV & give significant contributions to EWSB!!

Why Higgs septet?

- Models with $\rho_{tree}=1$

At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

  - SM [1 Higgs doublet]

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At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

- SM [1 Higgs doublet]
- 2HDM [2 Higgs doublet]
  - ✓ MSSM (Minimal Supersymmetric SM)

    an even number of Higgs doublets is required by the theory

    (holomorphy of superpotential, anomaly cancellation, mass generation of up & down)
  - ✓ Zee model (a radiative seesaw model for neutrino mass generation)

    at least 2 Higgs doublets is required to have lepton number violation
  - ✓ etc.

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- **New [1 Higgs doublet + 1 Higgs septet]**
- etc. (usually VEV alignment is required)

Physical Higgs bosons

- Models with $\rho_{tree}=1$

At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

  - SM [1 Higgs doublet] h
  - 2HDM [2 Higgs doublet] h, H, A, $H^{\pm}$
    - h, H: CP even Higgs bosons
    - A: A CP odd Higgs boson
    - $H^{\pm}$: A pair of charged Higgs bosons
    - ✓ MSSM (Minimal Supersymmetric SM)
      - an even number of Higgs doublets is required by the theory (holomorphy of superpotential, anomaly cancellation, mass generation of up & down)
    - ✓ Zee model (a radiative seesaw model for neutrino mass generation)
      - at least 2 Higgs doublets is required to have lepton number violation
    - ✓ etc.
  - **New [1 Higgs doublet + 1 Higgs septet]** h, H, A, $H_1^{\pm}$, $H_2^{\pm}$, $H^{2\pm}$, $H^{3\pm}$, $H^{4\pm}$, $H^{5\pm}$
    - $H_1^{\pm}$, $H_2^{\pm}$: 2 pairs of charged Higgs bosons
    - $H^{2\pm}$, $H^{3\pm}$, $H^{4\pm}$, $H^{5\pm}$: Multiply charged Higgs bosons
  - etc. (usually VEV alignment is required)

Difficulty of the model

- An accidental global U(1) symmetry in the Higgs potential

$$\mathcal{V} = -\mu_2^2\Phi^\dagger\Phi + M_7^2\chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 + \sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

Φ(doublet) and χ(septet) are invariant under the separate U(1)

→ Exact Massless NG boson (experimentally disfavored)

$$(\Phi^\dagger\Phi\chi^\dagger\chi)_1 = \phi^{*i}\phi_i\chi^{*abcdef}\chi_{abcdef}$$
$$(\Phi^\dagger\Phi\chi^\dagger\chi)_2 = \phi^{*i}\phi_j\chi^{*jabcde}\chi_{iabcde}$$
$$(\chi^\dagger\chi\chi^\dagger\chi)_1 = \chi^{*ijklmn}\chi_{ijklmn}\chi^{*abcdef}\chi_{abcdef}$$
$$(\chi^\dagger\chi\chi^\dagger\chi)_2 = \chi^{*ijklmn}\chi_{ijklmf}\chi^{*abcdef}\chi_{abcden}$$
$$(\chi^\dagger\chi\chi^\dagger\chi)_3 = \chi^{*ijklmn}\chi_{ijklef}\chi^{*abcdef}\chi_{abcdmn}$$
$$(\chi^\dagger\chi\chi^\dagger\chi)_4 = \chi^{*ijklmn}\chi_{ijkdef}\chi^{*abcdef}\chi_{abclmn}$$

$$\begin{cases} \Phi_1 = \omega_2^+ \\ \Phi_2 = (v_2 + h_2 + i\,z_2)/\sqrt{2} \end{cases}$$

$$\begin{cases} \chi_{111111} = \chi_3 \\ \chi_{111112} = \chi_2/\sqrt{6} \\ \chi_{111122} = \chi_1/\sqrt{15} \\ \chi_{111222} = \chi_0/\sqrt{20} \\ \chi_{112222} = \chi_{-1}/\sqrt{15} \\ \chi_{122222} = \chi_{-2}/\sqrt{6} \\ \chi_{222222} = \chi_{-3} \end{cases}$$

with

$$\chi_{-2} = (v_7 + h_7 + i\,z_7)/\sqrt{2}$$
$$\chi_3 = H^{5+}, \chi_2 = H^{4+}, \chi_1 = H^{3+}, \chi_0 = H^{2+}$$

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$$\mathcal{V} = -\mu_2^2\Phi^\dagger\Phi + M_7^2\chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \mathrm{H.c.}\}$$
$$+ \sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

U(1) breaking term

~~Φ(doublet) and χ(septet) are invariant under the separate U(1)~~

~~→ Exact Massless NG boson (experimentally disfavored)~~

$$(\chi^*\Phi^5\Phi^*) = \chi^{*abcdef}\Phi_a\Phi_b\Phi_c\Phi_d\Phi_e\Phi^{*g}\epsilon_{fg}$$

$$(\Phi^\dagger\Phi\chi^\dagger\chi)_1 = \phi^{*i}\phi_i\chi^{*abcdef}\chi_{abcdef}$$
$$(\Phi^\dagger\Phi\chi^\dagger\chi)_2 = \phi^{*i}\phi_j\chi^{*jabcde}\chi_{iabcde}$$
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Physical basis

- **For simplicity**

An accidental global U(1) symmetry in the Higgs potential

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger \Phi + M_7^2 \chi^\dagger \chi + \lambda (\Phi^\dagger \Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^* \Phi^5 \Phi^*) + \text{H.c.}\}$$

~~$$+ \sum_{A=1}^{4} \lambda_A (\chi^\dagger \chi \chi^\dagger \chi)_A + \sum_{B=1}^{2} \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B$$~~

$$\begin{pmatrix} h_7 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\begin{pmatrix} z_7 \\ z_2 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}$$

$$\begin{pmatrix} \chi_{-1} \\ \chi_{-3}^* \\ \omega_2^+ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}}{4} & \frac{\sqrt{6}}{4} & 0 \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{10}}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\beta & 0 & -s_\beta \\ 0 & 1 & 0 \\ s_\beta & 0 & c_\beta \end{pmatrix} \begin{pmatrix} \omega^+ \\ H_2^+ \\ H_1^+ \end{pmatrix}$$

$z, \omega^\pm$:EW NG bosons which are absorbed by Z, W± bosons

$$\begin{cases} \chi_{111111} = \chi_3 \\ \chi_{111112} = \chi_2/\sqrt{6} \\ \chi_{111122} = \chi_1/\sqrt{15} \\ \chi_{111222} = \chi_0/\sqrt{20} \\ \chi_{112222} = \chi_{-1}/\sqrt{15} \\ \chi_{122222} = \chi_{-2}/\sqrt{6} \\ \chi_{222222} = \chi_{-3} \end{cases}$$

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Parameters

❖ An accidental global U(1) symmetry in the Higgs potential

For simplicity   **Mass of Septet**

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger\Phi + M_7^2 \chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^* \Phi^5 \Phi^*) + \text{H.c.}\}$$

$$+ \sum_{A=1}^{4} \lambda_A (\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2} \kappa_B (\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

CP even Higgs mixing

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$$\begin{pmatrix} \chi_{-1} \\ \chi_{-3}^* \\ \omega_2^+ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}}{4} & \frac{\sqrt{6}}{4} & 0 \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{10}}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\beta & 0 & -s_\beta \\ 0 & 1 & 0 \\ s_\beta & 0 & c_\beta \end{pmatrix} \begin{pmatrix} \omega^+ \\ H_2^+ \\ H_1^+ \end{pmatrix}$$

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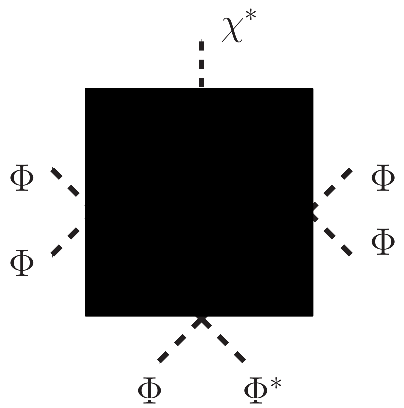
Ratio of VEV

$$\tan\beta = \frac{v_2}{4 v_7}$$

A model

A renormalizable model with Higgs septet

Don't introduce VEV of exotic multiplets other than those of doublet and septet



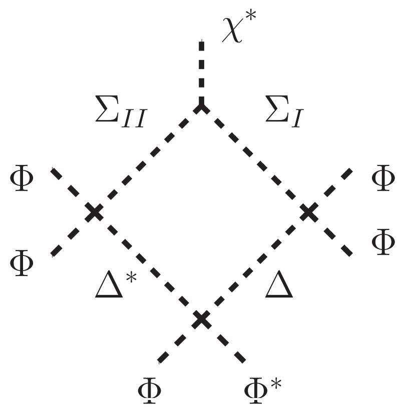
$$-\frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \text{H.c.}\}$$

Non-renormalizable term

External fields are fixed by dim.7 operator

A renormalizable model with Higgs septet

<u>Don't introduce VEV of exotic multiplets</u> other than those of doublet and septet



$$-\frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*)+\text{H.c.}\}$$

~~Non-renormalizable term~~

External fields are fixed by dim.7 operator

Decompose the diagram

- 2 quintuplets ($\Sigma_{I,II}$) and 1 triplet (Δ) with exact Z_2 parity

Forbid VEV of extra multiplet
(Bonus: Dark Matter candidate)

$$\mathcal{L}_{\not U(1)} = \mu\,\chi_{abcdef}\Sigma_I^{*abci}\Sigma_{II}^{*defj}\epsilon_{ij} + \Phi_i\Phi_j(c_I\,\Sigma_I^{*ijkl} + c_{II}\,\Sigma_{II}^{*ijkl})\Delta_{kl} + f\,\Phi_a\Phi^{*b}\Delta^{*ac}\Delta_{bc} + \text{H.c.}$$

Soft breaking term of the global U(1)

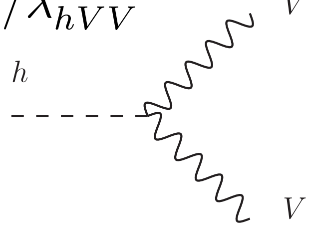
We obtain correct dim.7 operator from a renormalizable theory!!

Model predictions

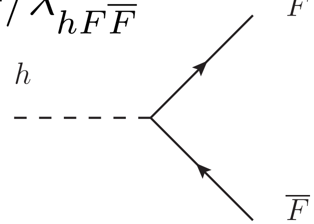
Model predictions

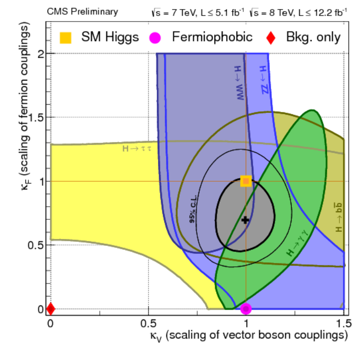
Modified Gauge/Yukawa coupling

$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$



$\kappa_F = \lambda_{hF\overline{F}}/\lambda_{hF\overline{F}}^{\rm SM}$





- ✧ SM: $\kappa_V^{\rm SM} = 1, \kappa_F^{\rm SM} = 1$

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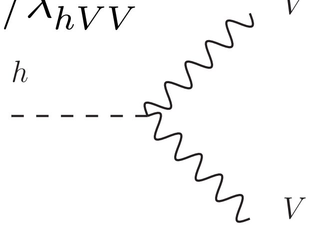
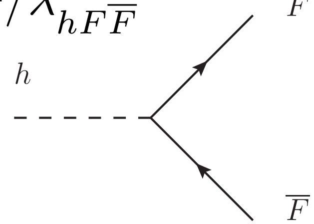
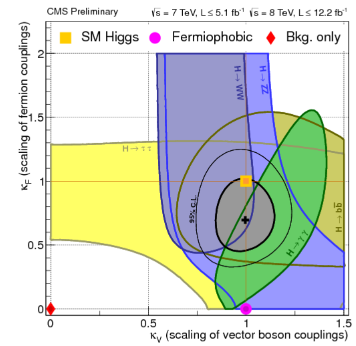


$\kappa_F = \lambda_{hF\overline{F}}/\lambda_{hF\overline{F}}^{\rm SM}$





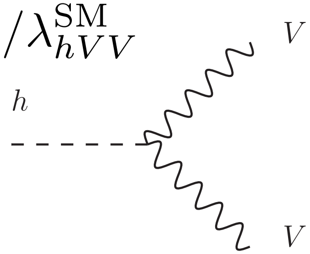
- ✧ SM: $\kappa_V^{\rm SM} = 1, \kappa_F^{\rm SM} = 1$
- ✧ 2HDM: $\kappa_V^{\rm 2HDM} = \sin(\beta - \alpha), \kappa_F^{\rm 2HDM(\text{-}I)} = \cos\alpha / \sin\beta$

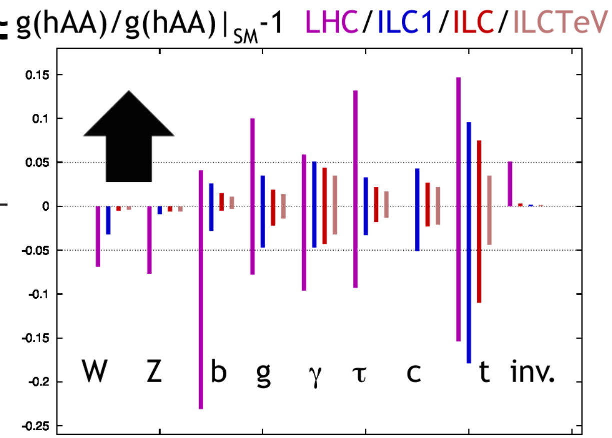
$$\kappa_V^{\rm 2HDM} \leq 1$$

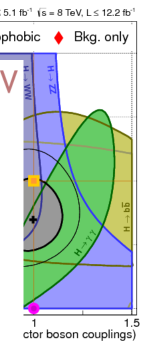
Model predictions

Modified Gauge/Yukawa coupling

$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$ $\kappa_F = \lambda_{hF\overline{F}}/\lambda_{hF\overline{F}}^{\rm SM}$







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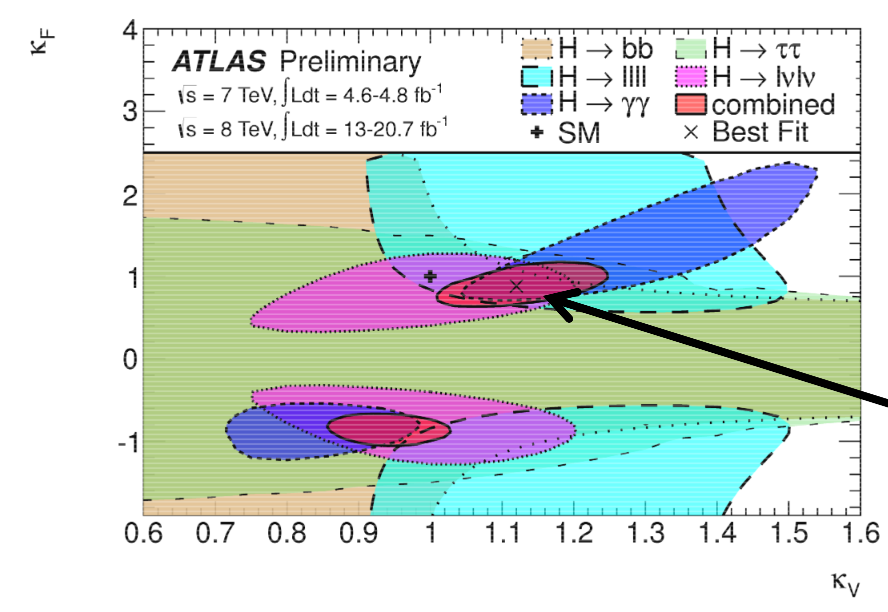
- ✧ 2HDM: $\kappa_V^{\rm 2HDM} = \sin(\beta - \alpha), \kappa_F^{\rm 2HDM(\text{-}I)} = \cos\alpha/\sin\beta$

$$\kappa_V^{\rm 2HDM} \leq 1 \qquad \kappa_{\mathbf{V}}^{\rm septet} \geq 1$$

- ✧ **Septet:** $\kappa_V^{\rm septet} = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha, \kappa_F^{\rm septet} = \cos\alpha/\sin\beta$

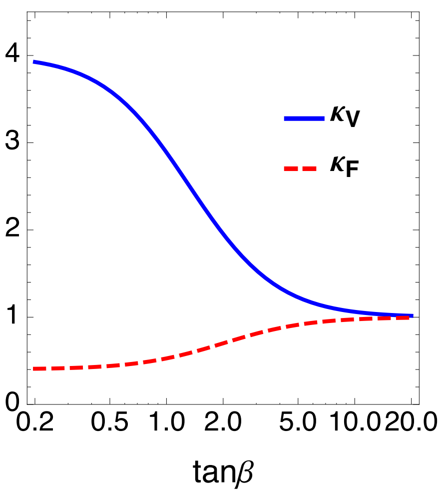
κ_V can be larger than one!!

Model predictions



ATLAS (Moriond QCD 2013)

$\kappa_V > 1$ is favored



✧ **Septet:** $\kappa_V^{\text{septet}} = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha, \kappa_F^{\text{septet}} = \cos\alpha/\sin\beta$

κ_V can be larger than one!!

Distinctive feature of the septet model

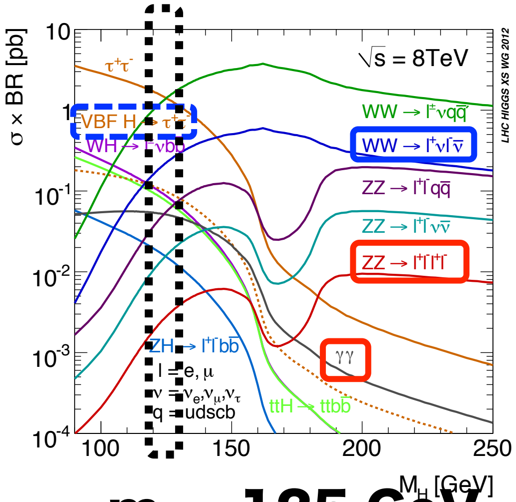
Signal strength of the Higgs boson

Signal strength for "xx" channel

$$\mu_{xx} = \frac{\sigma \cdot \mathcal{B}_{xx}}{\sigma^{\rm SM} \cdot \mathcal{B}_{xx}^{\rm SM}}$$

σ : Production cross section of the Higgs boson
B_{xx}: Decay Branching ratio of the Higgs boson into xx

Normalized by SM



m_H =125 GeV

Signal strength for "xx" channel

$$\mu_{xx} = \frac{\sigma \cdot \mathcal{B}_{xx}}{\sigma^{\rm SM} \cdot \mathcal{B}_{xx}^{\rm SM}}$$

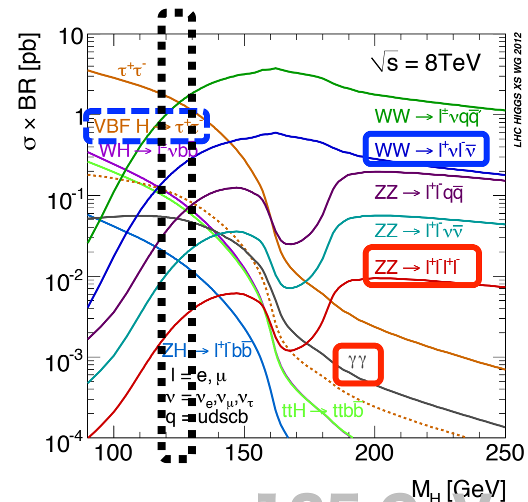
μ_{xx}=1 for SM

μ_{xx}≠1 for Beyond the SM

σ : Production cross section of the Higgs boson

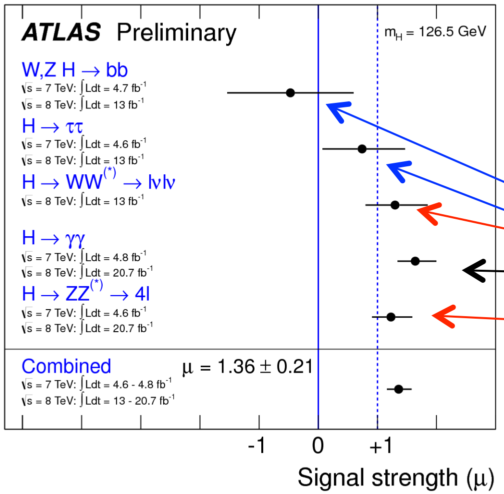
B_{xx}: Decay Branching ratio of the Higgs boson into xx

Normalized by SM



m_H =125 GeV

Results from LHC



Seems to be compatible with "ATLAS" results

γγ: Enhanced

VV: Slightly Enhanced

FF: Reduced

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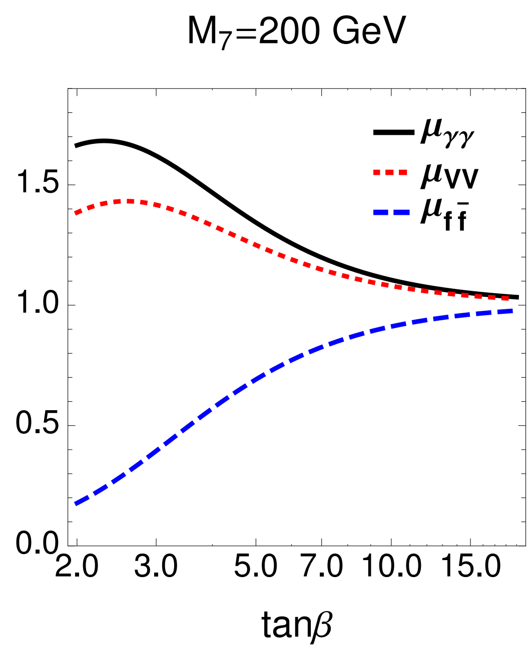
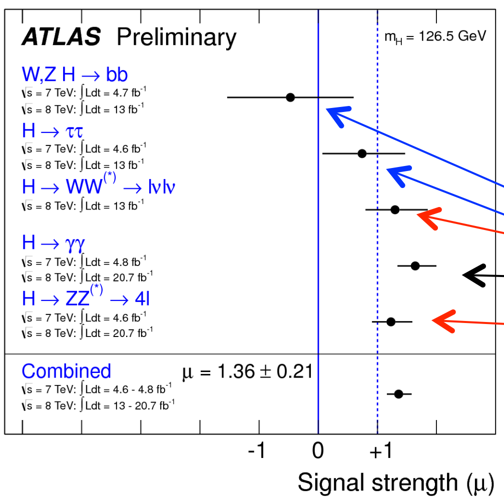


Results from LHC



Seems to be compatible with "ATLAS" results

γγ: Enhanced

VV: Slightly Enhanced

FF: Reduced

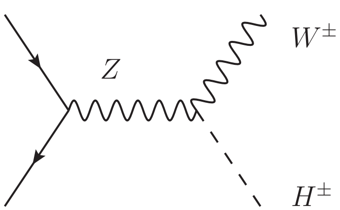
More phenomenology

$W^{\pm}ZH^{\pm}$ vertex

- Anomalous $W^{\pm}ZH^{\mp}$ coupling?
  - No $H^{\pm}$ in the SM
  - Forbidden also in the MSSM (2HDM)
  - Case with septet
    - → Septet naturally induces $W^{\pm}ZH^{\mp}$ vertex

$$\chi = \begin{pmatrix} H^{++++++} \\ H^{+++++} \\ H^{++++} \\ H^{+++} \\ H^{++} \\ H_1^{+} \\ (v_7 + h_7 + i\, z_7)/\sqrt{2} \\ H_2^{-} \end{pmatrix}$$

Charged Higgs strahlung @ ILC



- Counter measurement of Higgs strahlung ($e^+e^- \to Zh$)

Most important measurement of hVV coupling @ ILC

- Recoil method can be applied

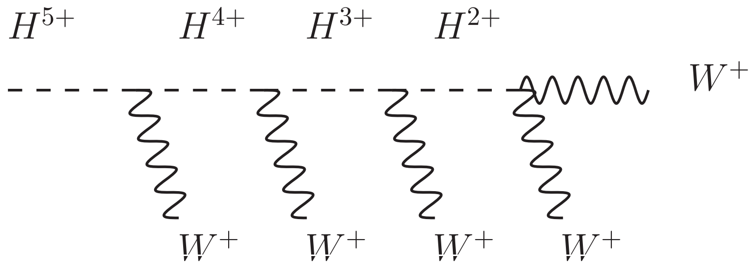
$W^{\pm}ZH^{\pm}$ vertex is tested without measuring $H^{\pm}$

Kanemura, Yagyu, Yanase, PRD83, 075018 (2011)

$v_7 \sim O(\text{GeV})$ can be tested!!

Multiply charged Higgs bosons

- Multiple W bosons



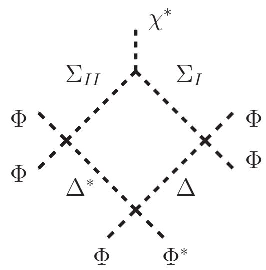
  - ✓ Long decay chain (Maybe long-lived)
  - ✓ Large cross section (Q=5)

Summary

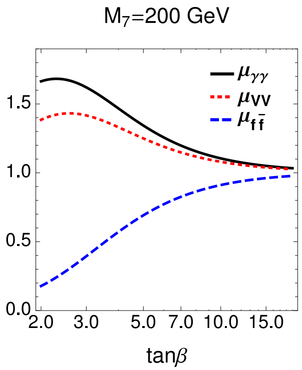
- Beyond the Higgs
  - ρ parameter and Beyond the SM → Septet (next minimal)

$$\rho_{\rm tree} = \frac{\sum_\alpha [I_\alpha(I_\alpha+1) - Y_\alpha^2] v_\alpha^2}{\sum_\beta 2Y_\beta^2 v_\beta}$$

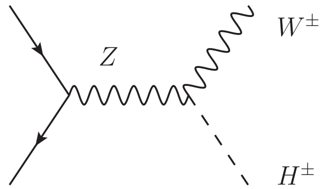
  - Model with septet



  - LHC Higgs signal vs Septet



  - Smoking gun of Septet @ILC



Thank you very much for your attention

Back up

Mass spectrum

- An accidental global U(1) symmetry in the Higgs potential

For simplicity

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger \Phi + M_7^2 \chi^\dagger \chi + \lambda (\Phi^\dagger \Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^* \Phi^5 \Phi^*) + \text{H.c.}\}$$

$$\sout{+ \sum_{A=1}^{4} \lambda_A (\chi^\dagger \chi \chi^\dagger \chi)_A + \sum_{B=1}^{2} \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B}$$

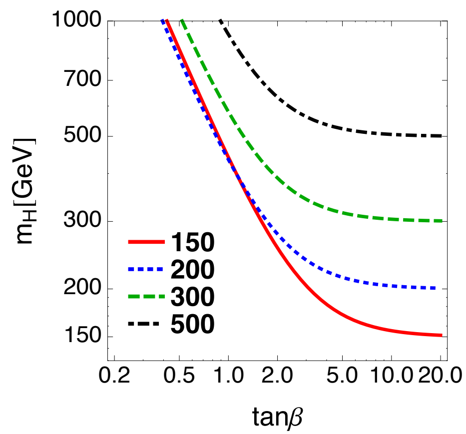
- Mass eigenvalues

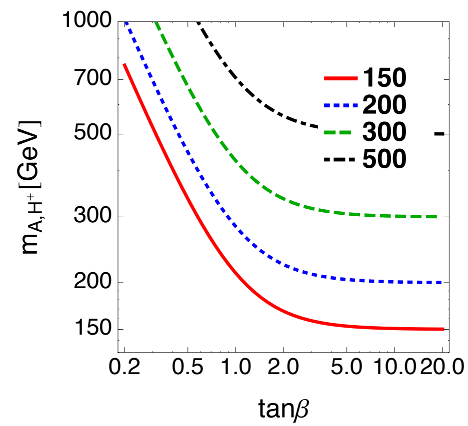
$$m_h^2 = \left(1 + \frac{3}{2} \frac{1}{t_\beta \, t_\alpha}\right) M_7^2$$

$$m_H^2 = \left(1 - \frac{3}{2} \frac{t_\alpha}{t_\beta}\right) M_7^2$$

$$m_A^2 = m_{H_1^\pm}^2 = M_7^2 / s_\beta^2$$

$$m_{H_2^\pm}^2 = m_{H^{2\pm}}^2 = m_{H^{3\pm}}^2 = m_{H^{4\pm}}^2 = m_{H^{5\pm}}^2 = M_7^2$$



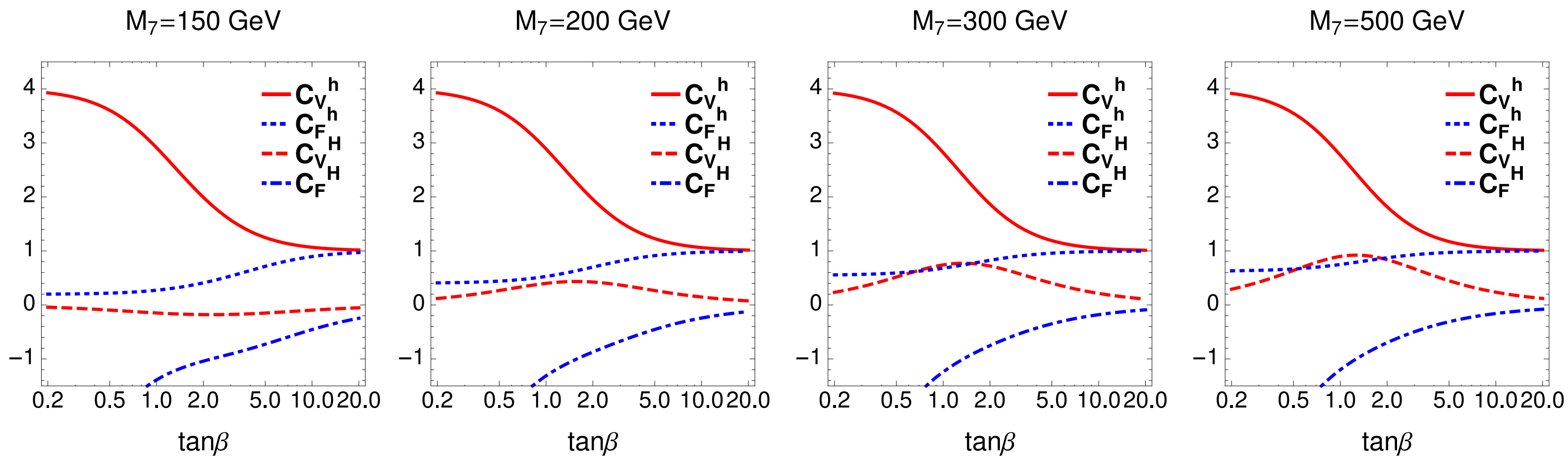


$$\tan\beta = \frac{v_2}{4 v_7}$$

More κ_V and κ_F

- 2 CP even Higgs bosons (h, H)

$$\kappa_V^h = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha, \kappa_F^h = \cos\alpha/\sin\beta$$
$$\kappa_V^H = \sin\beta\sin\alpha + 4\cos\beta\cos\alpha, \kappa_F^H = \sin\alpha/\sin\beta$$



Electroweak precision data

- An accidental global U(1) symmetry in the Higgs potential

For simplicity

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger\Phi + M_7^2 \chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \mathrm{H.c.}\}$$

$$+\sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

- Oblique parameters

$$S = \frac{1}{4\pi}\Big[(C_V^h)^2\, G^{hZ'} + (C_V^H)^2\, G^{HZ'} + 30\, c_\beta^2\, G^{H_2^\pm W'} + 30\, s_\beta^2\, F^{H_1^\pm H_2^\pm\,'}$$
$$- \tfrac{1}{3}\ln m_{H_1^\pm}^2 - 15 \ln m_{H_2^\pm}^2 + (4\, s_\alpha s_\beta - c_\alpha c_\beta)F^{hA'} + (4\, c_\alpha s_\beta + s_\alpha c_\beta)F^{HA'}\Big]$$

$$T = \frac{\sqrt{2}G_F}{\alpha_{\mathrm{EM}}(4\pi)^2}\Big[(C_V^h)^2\,\Delta G^h + (C_V^H)^2\,\Delta G^H - 15\, c_\beta^2\,\Delta G^{H_2^\pm}\Big]$$

$$\begin{cases}
F^{xy} = \dfrac{m_x^2+m_y^2}{2} - \dfrac{m_x^2 m_y^2}{m_x^2-m_y^2}\ln\dfrac{m_x^2}{m_y^2} \\
G^{xV} = F^{xV} + 4\, m_V^2\left(-1 + \dfrac{m_x^2\ln m_x^2 - m_V^2 \ln m_V^2}{m_x^2 - m_V^2}\right) \\
\Delta G^x = G^{xW} - G^{xZ} \\
F^{xy\prime} = -\dfrac{1}{3}\left(+\dfrac{4}{3} - \dfrac{m_x^2\ln m_x^2 - m_y^2\ln m_y^2}{m_x^2-m_y^2} - \dfrac{m_x^2+m_y^2}{(m_x^2-m_y^2)^2}F^{xy}\right) \\
G^{xV\prime} = F^{xV\prime} + 4\, m_V^2\left(-\dfrac{1}{(m_x^2-m_V^2)^2}F^{xV}\right)
\end{cases}$$

$$\tan\beta = \frac{v_2}{4v_7}$$

Electroweak precision data

- Best fit values

$$\Delta S = 0.04 \pm 0.09$$
$$\Delta T = 0.07 \pm 0.08$$
$$(\sigma_{ST} = 0.88)$$

- Oblique parameters

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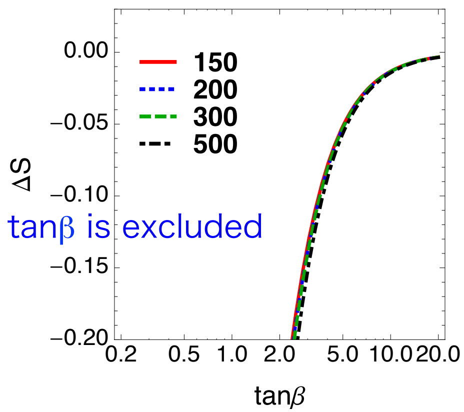

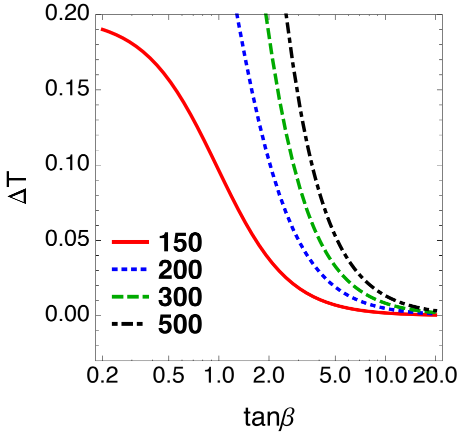

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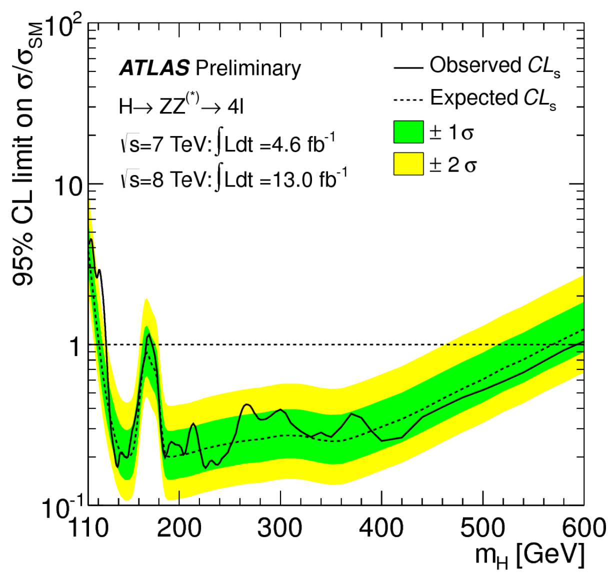
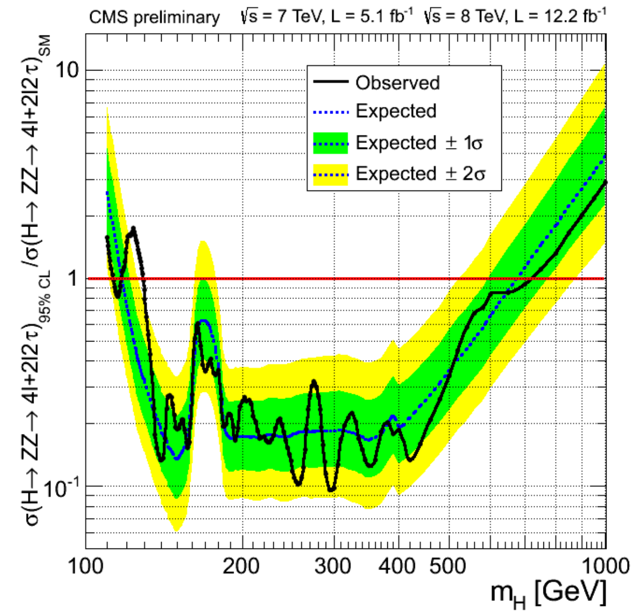
$(\Delta S, \Delta T)$	$\tan\beta = 3$	$\tan\beta = 5$	$\tan\beta = 10$	$\tan\beta = 20$
$M_7 = 150$ GeV	(-0.13, 0.019)	(-0.05, 0.007)	(-0.013, 0.002)	(-0.003, 0.)
$M_7 = 200$ GeV	(-0.14, 0.050)	(-0.05, 0.019)	(-0.014, 0.005)	(-0.003, 0.001)
$M_7 = 300$ GeV	(-0.14, 0.088)	(-0.05, 0.033)	(-0.013, 0.008)	(-0.003, 0.002)
$M_7 = 500$ GeV	(-0.15, 0.14)	(-0.06, 0.053)	(-0.014, 0.013)	(-0.004, 0.003)

$$\tan\beta = \frac{v_2}{4 v_7}$$

Signal strength of the extra Higgs boson

- Search for the SM Higgs boson can be interpret as a constraint on extra Higgs bosn (H)
- VV decay channel [μ^H_{VV} (gg→H→VV)] gives stronger limits for heavier mass region





- Results from the septet

$(m_H[\text{GeV}], \mu^H_{VV})$	$\tan\beta = 5$	$\tan\beta = 6$	$\tan\beta = 7$	$\tan\beta = 8$	$\tan\beta = 9$	$\tan\beta = 10$
$M_7 = 150$ GeV	(171., 0.44)	(165., 0.31)	(161., 0.20)	(159., 0.13)	(157., 0.081)	(156., 0.062)
$M_7 = 200$ GeV	(214., 0.21)	(210., 0.15)	(207., 0.11)	(206., 0.089)	(205., 0.071)	(204., 0.059)
$M_7 = 300$ GeV	(316., 0.12)	(311., 0.087)	(308., 0.065)	(306., 0.050)	(305., 0.040)	(304., 0.032)
$M_7 = 500$ GeV	(523., 0.12)	(516., 0.084)	(512., 0.063)	(509., 0.048)	(507., 0.038)	(503., 0.031)

Small tanβ/m_H is excluded

Extended Higgs sector

An example: supersymmetry (boson $\Leftrightarrow$ fermion)

- stabilize Quantum corrections to Higgs mass
- unify Gauge coupling precisely
- provide Dark Matter candidate

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→ The Higgs sector is required
to have even number of the SU(2) doublet

- Holomorphy of the superpotential
- Mass generation for up & down type quarks
- Anomaly cancellation

Extended Higgs sector

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- provide Dark Matter candidate

→ The Higgs sector is required
to have even number of the SU(2) doublet

- Holomorphy of the superpotential
- Mass generation for up & down type quarks
- Anomaly cancellation

So far, No signal of SUSY is observed @ LHC

Why Higgs septet?

- Models with $\rho_{tree}=1$

At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

  - SM [1 Higgs doublet]
  - 2HDM [2 Higgs doublet]
    - ✓ MSSM (Minimal Supersymmetric SM)

      a even number of Higgs doublets is required by the theory

      (holomorphy of superpotential, anomaly cancellation, mass generation of up & down)
    - ✓ Zee model (a radiative seesaw model for neutrino mass generation)

      at least 2 Higgs doublets is required to have lepton number violation
    - ✓ etc.
  - New [1 Higgs doublet + 1 Higgs septet]
  - etc. (usually VEV alignment is required)

    → **Georgi-Machacek Model**

Georgi-Machacek Model

- 1 doublet w/ Y=1/2 & 1 complex triplet w/ Y=1 + 1 real triplet w/ Y=0

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \end{pmatrix} \qquad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Higgs triplet model (Φ and Δ)

Georgi-Machacek Model

- 1 doublet w/ Y=1/2 & 1 complex triplet w/ Y=1 + 1 real triplet w/ Y=0

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \end{pmatrix} \qquad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Higgs triplet model (Φ and Δ)

- EW ρ parameter

$$\rho_{\rm tree}^{\rm GM} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1+2x_\Delta^2+2x_\xi^2}{1+4x_\Delta^2} \quad \text{with} \quad x_\Delta = \frac{\langle\Delta^0\rangle}{\langle\phi^0\rangle},\; x_\xi = \frac{\langle\xi^0\rangle}{\langle\phi^0\rangle}$$

→ VEV alignment ($\langle\Delta^0\rangle = \langle\xi^0\rangle$) leads $\rho_{\rm tree}$=1

Georgi-Machacek Model

- 1 doublet w/ Y=1/2 & 1 complex triplet w/ Y=1 + 1 real triplet w/ Y=0

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \end{pmatrix} \qquad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Higgs triplet model

- EW ρ parameter

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→ VEV alignment ($\langle \Delta^0 \rangle = \langle \xi^0 \rangle$) leads ρ_{tree}=1

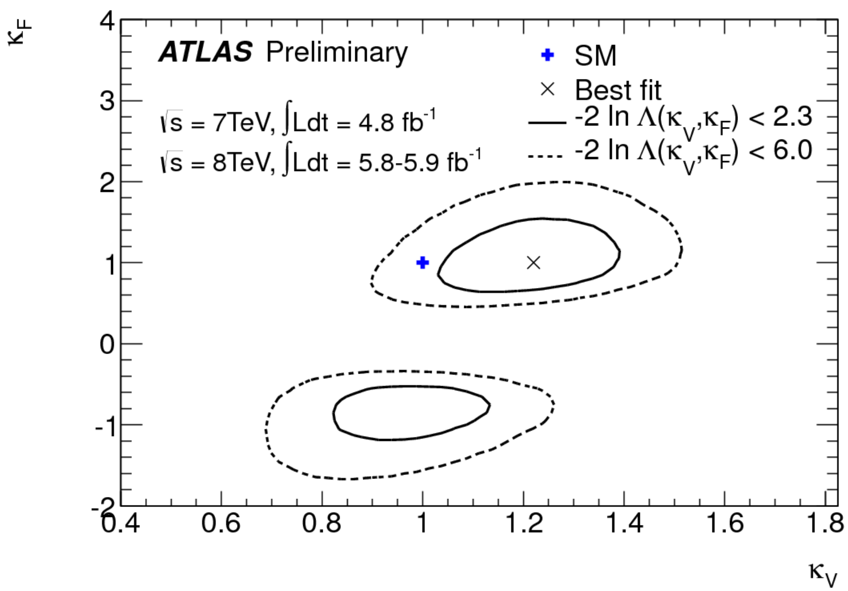
By constructing the global $SU(2)_L$ x $SU(2)_R$ bi-doublet/triplet and assuming the diagonal VEV, the custodial $SU(2)_V$ symmetry can be introduced in the Higgs potential (Classical Level)

$\mathcal{V}(M, X)$ is invariant under $M \to U_L M U_R^\dagger,\ X \to U_L X U_R^\dagger$

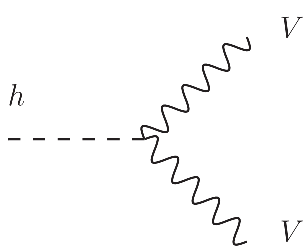
$$M = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \Delta^{0*} & \xi^+ & \Delta^{++} \\ \Delta^- & \xi^0 & \Delta^+ \\ \Delta^{--} & \xi^- & \Delta^0 \end{pmatrix}$$

Is it SM-like?

- κ_V vs κ_F



Production	Decay	LO SM	
VH	$H \to bb$	$\sim \frac{C_V^2 \times C_F^2}{C_F^2}$	$\sim C_V^2$
ttH	$H \to bb$	$\sim \frac{C_F^2 \times C_F^2}{C_F^2}$	$\sim C_F^2$
VBF/VH	$H \to \tau\tau$	$\sim \frac{C_V^2 \times C_F^2}{C_F^2}$	$\sim C_V^2$
ggH	$H \to \tau\tau$	$\sim \frac{C_F^2 \times C_F^2}{C_F^2}$	$\sim C_F^2$
ggH	$H \to ZZ$	$\sim \frac{C_F^2 \times C_V^2}{C_F^2}$	$\sim C_V^2$
ggH	$H \to WW$	$\sim \frac{C_F^2 \times C_V^2}{C_F^2}$	$\sim C_V^2$
VBF/VH	$H \to WW$	$\sim \frac{C_V^2 \times C_V^2}{C_F^2}$	$\sim C_V^4/C_F^2$
ggH	$H \to \gamma\gamma$	$\sim \frac{C_F^2 \times (8.6C_V - 1.8C_F)^2}{C_F^2}$	$\sim C_V^2$
VBF	$H \to \gamma\gamma$	$\sim \frac{C_V^2 \times (8.6C_V - 1.8C_F)^2}{C_F^2}$	$\sim C_V^4/C_F^2$



$$\kappa_V = \lambda_{hVV} / \lambda_{hVV}^{SM}$$

κ_V can be different from unity