Calibration of Pion and Positron Showers with Longitudinal Weights

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Motivation and Method

Goal: Improve pion/positron energy resolution.

Method: Identify the interaction layer IL. Apply weights to hits in each layer following the IL, such that the resolution of E^{w}_{rec} is minimized.

Similar method previously proposed for the ATLAS and D0 ECALs. However only tested at one energy point.

Our method developed for the 6 – 60 (6 – 32) GeV energy range for pions (positrons)

Definitions

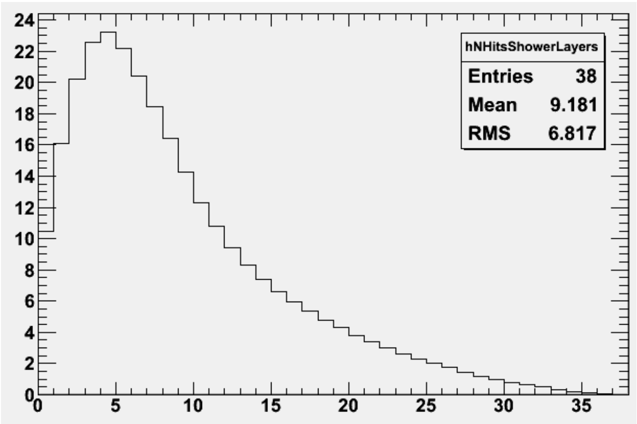
Weighted reconstructed energy $\mathbf{E^w_{rec}}$ – eq. (1), where H_i is the sum of hits in layer i corrected with the specific efficiency and multiplicity of this layer

$$(1) \quad E^w_{rec} = \sum_{i=0}^{IL-1} H_i + \sum_{i=IL}^{n} w_i H_i$$

$$H_i = c_i H'_i, \text{ where } c_i = \frac{\varepsilon_0 \mu_0}{\varepsilon_i \mu_i}, \text{ and } H'_i = \sum_{hits} 1$$

Average number of hits in layer i $\boldsymbol{\mu_i}$ – eq. (2)

$$(2) \quad \mu_i = \frac{\sum_k^{N_i} H_i^{(k)}}{N_i}$$



Longitudinal profile as function of shower layer

Covariance of number of hits in layer I $\mathbf{C_{i,j}}$ – eq. (3)

$$(3) \quad C_{i,j} = \frac{\sum_k^{N_{i,j}} \left(H_i^{(k)} - \mu_i\right)\left(H_j^{(k)} - \mu_j\right)}{N_{i,j} - 1}$$

for (2) and (3), i,j refers to a layer within the shower, so all physical layers are not involved in every event, hence N_i and $N_{i,j}$

Derivation of Weights

Mean and variance of E^{w}_{rec} are

$$\mu\left(E_{rec}^{w}\right) = \sum_{i} w_i \mu_i$$

$$\sigma^2\left(E_{rec}^{w}\right) = \sum_{i,j} w_i w_j C_{i,j}$$

Energy resolution in terms of weights (using a vector notation)

$$\frac{\sigma\left(E_{rec}^{w}\right)}{\mu\left(E_{rec}^{w}\right)} = \frac{\sqrt{\vec{w}^T C \vec{w}}}{\vec{w}^T \vec{\mu}}$$

Minimize resolution
with respect to weights

$$0 = \frac{\partial}{\partial \vec{w}^T}\left(\frac{\sqrt{\vec{w}^T C \vec{w}}}{\vec{w}^T \vec{\mu}}\right)$$

$$= \frac{1}{\left(\vec{w}^T \vec{\mu}\right)^2}\left[\frac{\left(C\vec{w}\right)\left(\vec{w}^T \vec{\mu}\right)}{\sqrt{\vec{w}^T C \vec{w}}} - \vec{\mu}\sqrt{\vec{w}^T C \vec{w}}\right]$$

$$= \frac{\left[C\vec{w}\vec{w}^T \vec{\mu} - \vec{\mu}\vec{w}^T C \vec{w}\right]}{\left(\vec{w}^T \vec{\mu}\right)^2 \sqrt{\vec{w}^T C \vec{w}}}$$

$$\Rightarrow \left(\vec{w}^T \vec{\mu}\right)\vec{w} = \left(\vec{w}^T C \vec{w}\right) C^{-1} \vec{\mu}$$

$$\vec{w} = \lambda\left(C^{-1}\vec{\mu}\right)$$

Vector notation

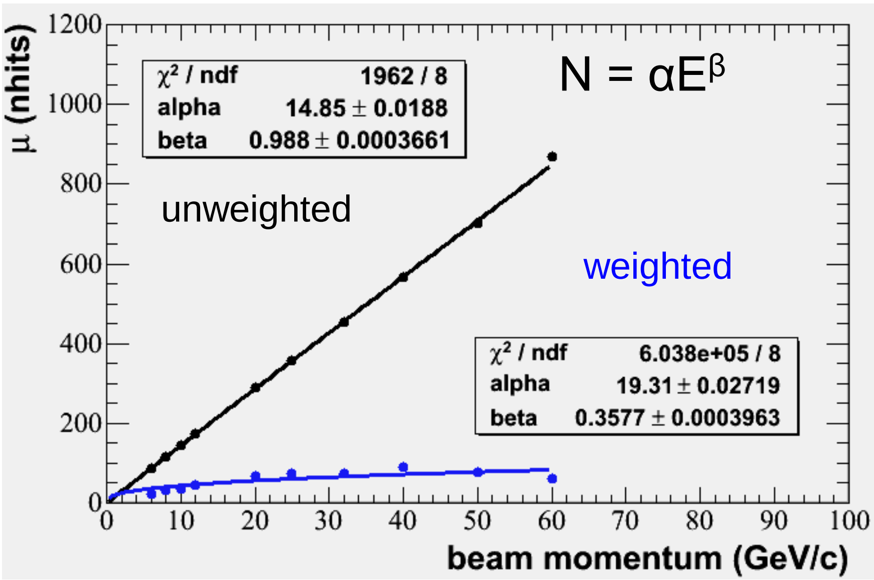
$$w_i = \lambda \sum_{j=1} C_{i,j}^{-1} \mu_i$$

or weight of layer i

λ is a normalization constant yet to be determined
(hint: sets the energy scale of E^{w}_{rec})

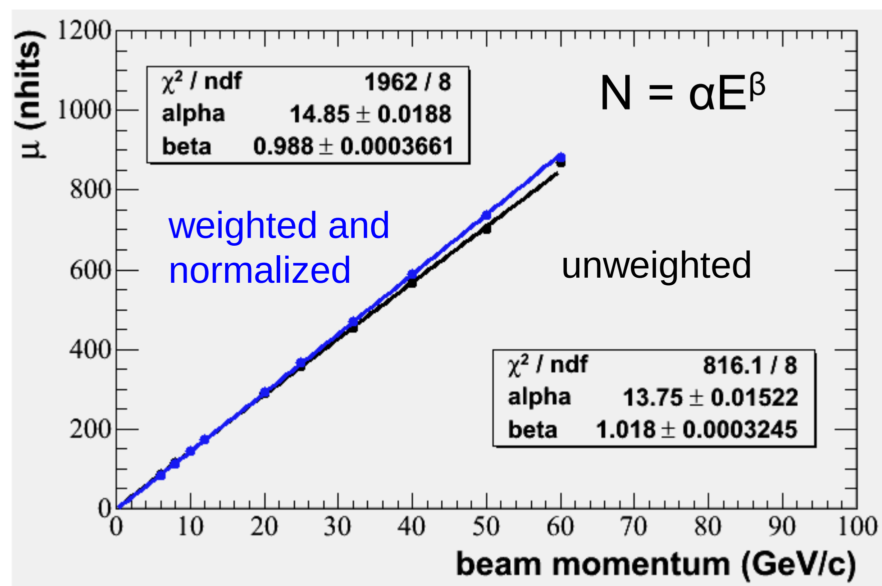
Normalization of Weights

Before normalization



$$w_i^n = \frac{\sum_{j=1} C_{i,j}^{-1} \mu_i}{r_E^{(n-1)} \cdots r_E^{(n-k)} \cdots r_E^{(0)}}$$

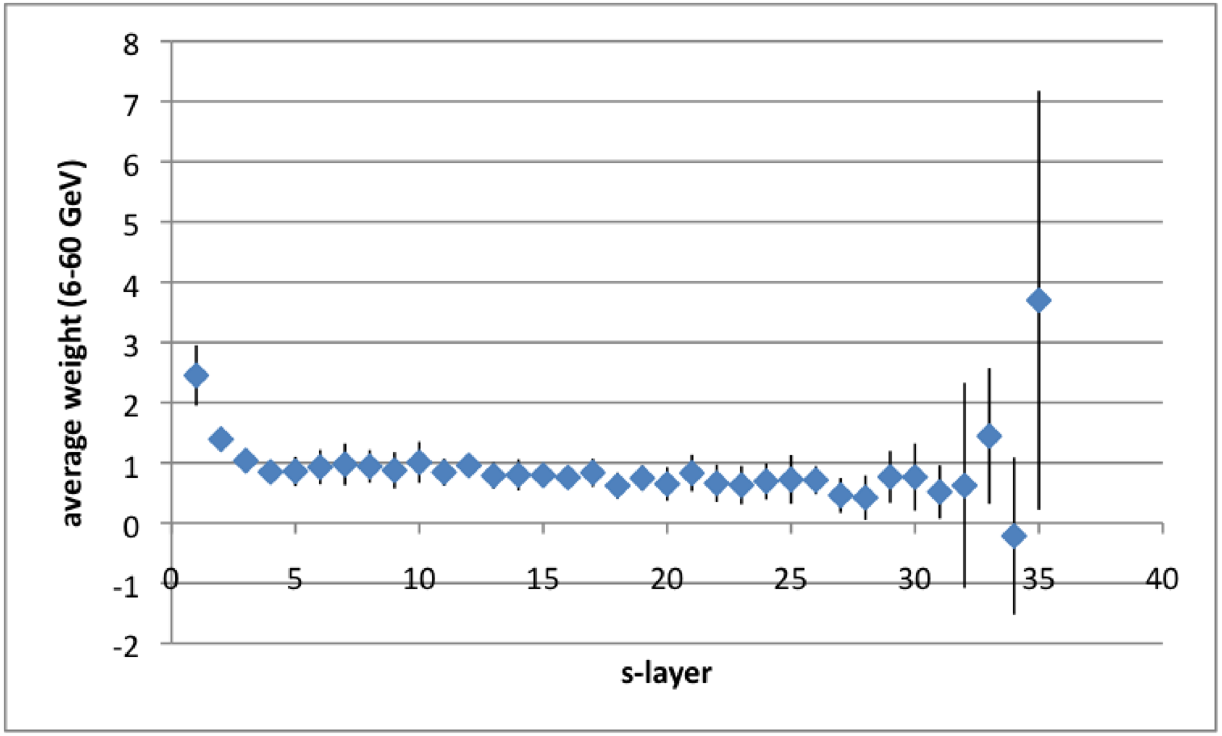
After two iterations



Normalization strategy: apply iterative corrections r^n_E to E^w_{rec}
$r^n_E = \mu^n_E/(E*14.74)$, where

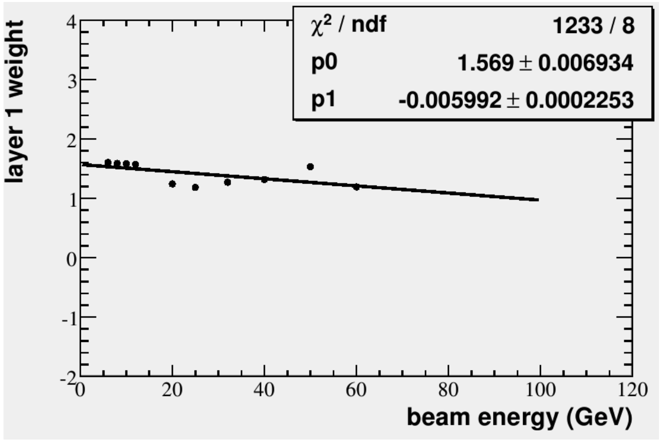
μ^n_E – blue points
14.74 hits/GeV – pion sampling term
n – iteration index

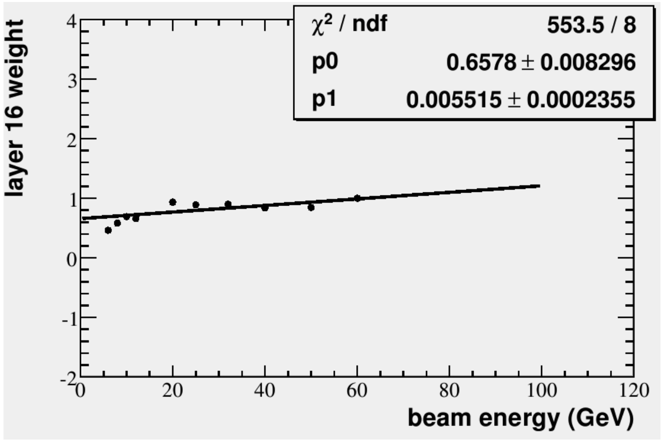
Average weights as function of layer number



Deviation from 1 for first layers
Slight decrease with increasing layer number

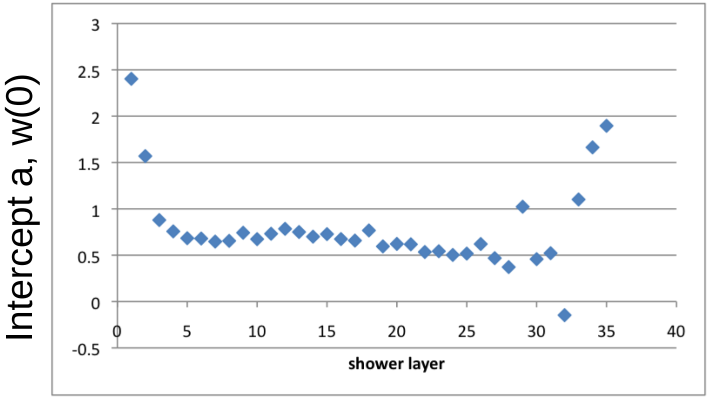
Parameterization of Weights in Each Layer

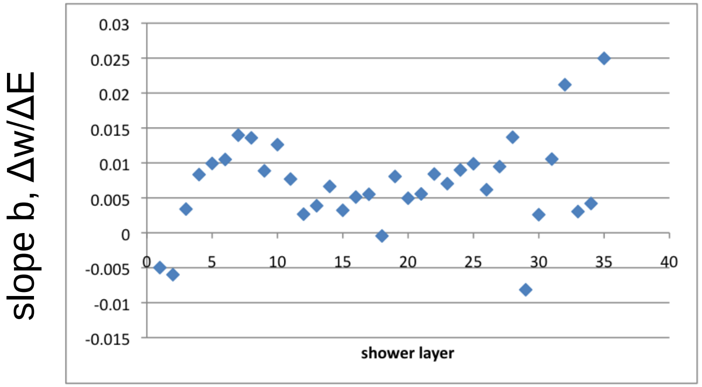




Fit weight in each layer to a straight line

$$w_i(E) = a_i + b_i*E$$





Estimate of Uncertainties in Weights

Propagate uncertainties in weights calculation
Mean μ and covariance matrix C^{-1} have correlated uncertainties
-> Monte-Carlo approach to calculate uncertainties of weights

1) diagonalize μ and C_{ij} to get uncorrelated uncertainties (diagonal C_{ij} means no correlations between layers)

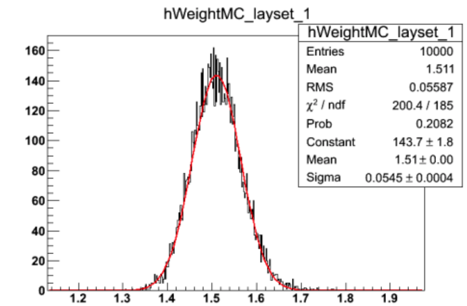
$$\hat{C}_{rs} = \sum_{i,j} U_{ri} U_{sj} C_{ij} \qquad \hat{\mu}_r = \sum_i U_{ri} \mu i$$

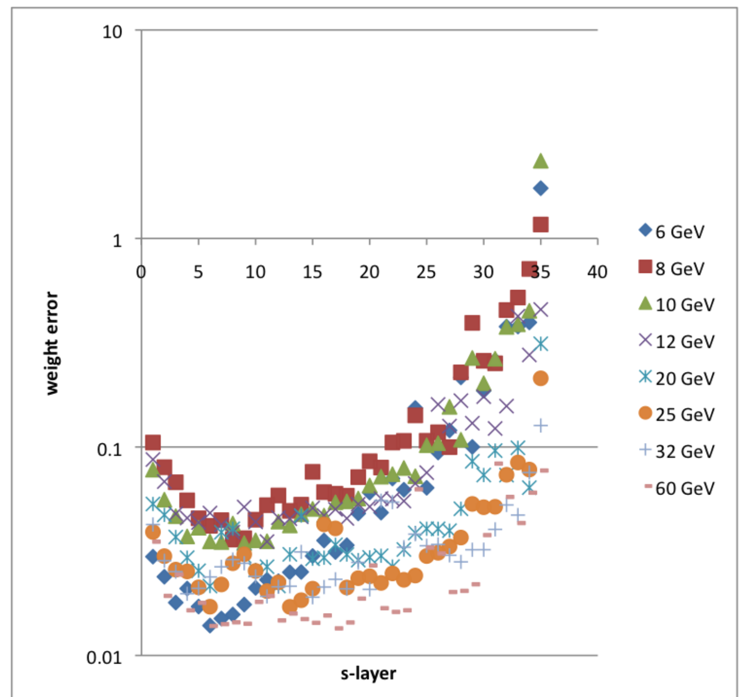
2) Monte-Carlo Procedure

a) smear C_{rs} with a χ^2 and mean μ_r with a Gaussian distribution
b) transform $\hat{C}_{rs}$ and μ_r back to real space, C^{*-1}, μ^*
c) calculate a smeared weight, w*

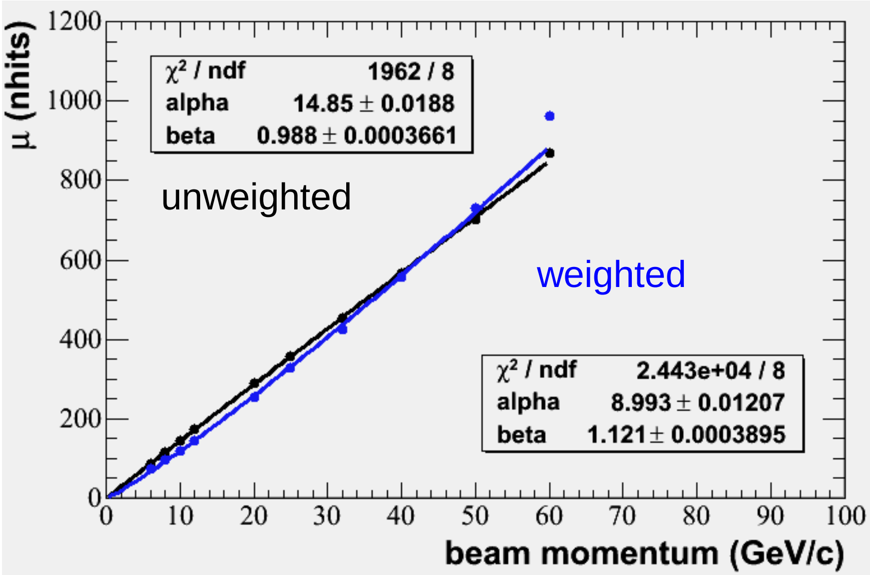
3) Get RMS from each generated w* distribution → uncertainty on w_i

$$w_i = \sum_{j=1} C^{-1}_{i,j} \mu_i$$



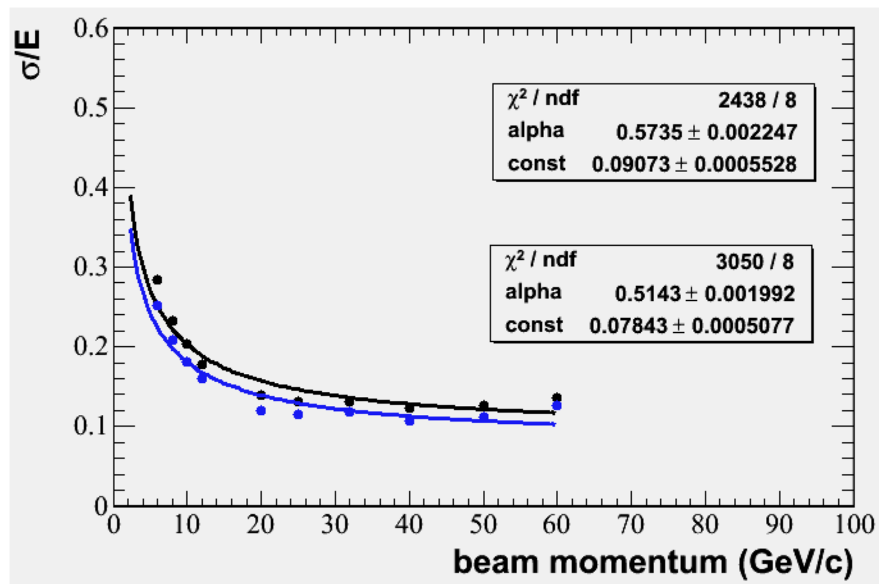


Weights Applied to Data Pions



Parameterizations aren't perfect

-> slight degradation of linearity

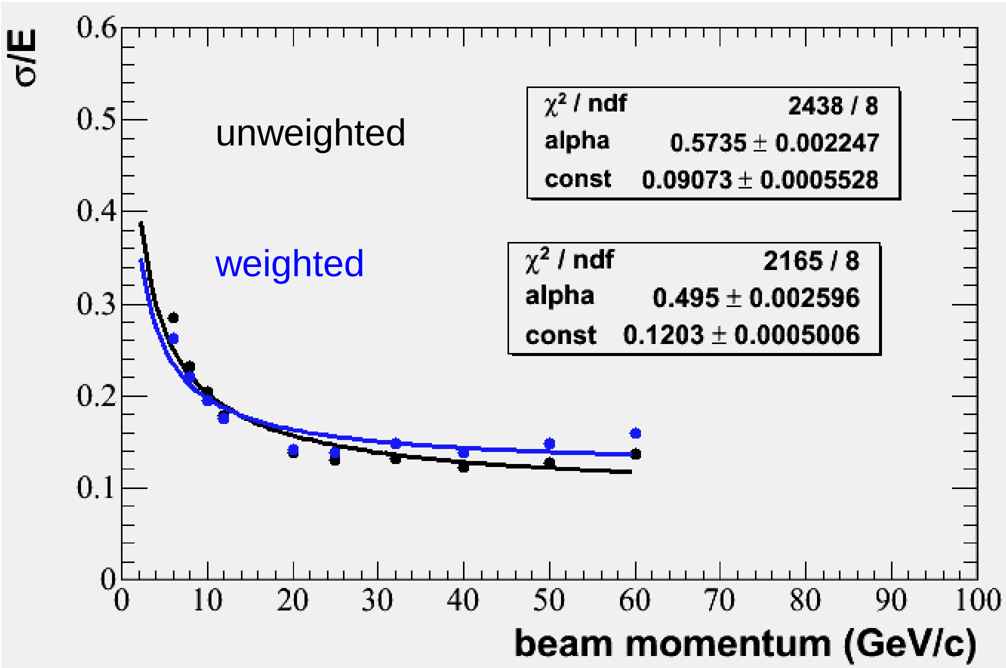


Resolution using the weights at the corresponding beam energy

-> 10% improvement in resolution

This is a cheat, as in a real experiment the energy of the incident particle is not known

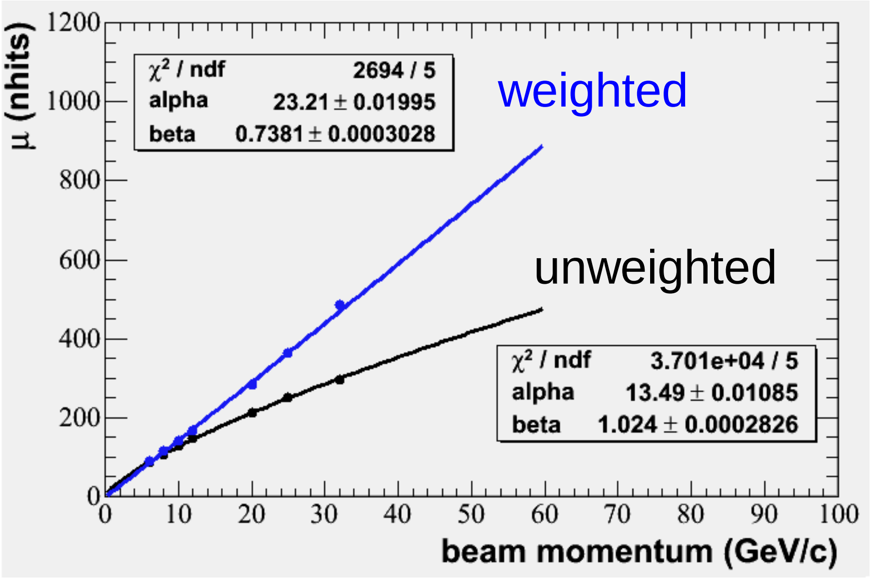
Weights Applied to Data Pions



Energy of event determined from N = ΣH_i

Slight improvement of resolution for E < 20 GeV
Worse resolution for E > 20 GeV

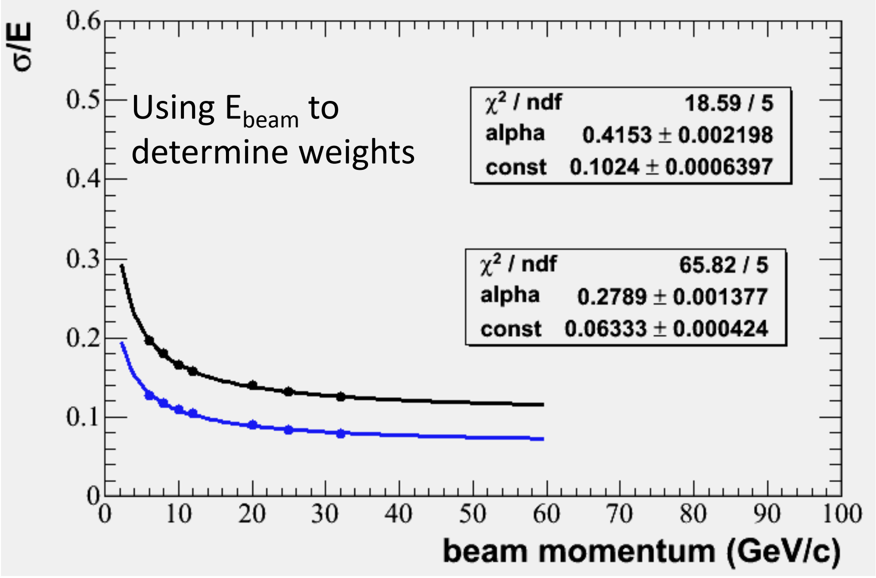
Weights Applied to Simulated Positrons

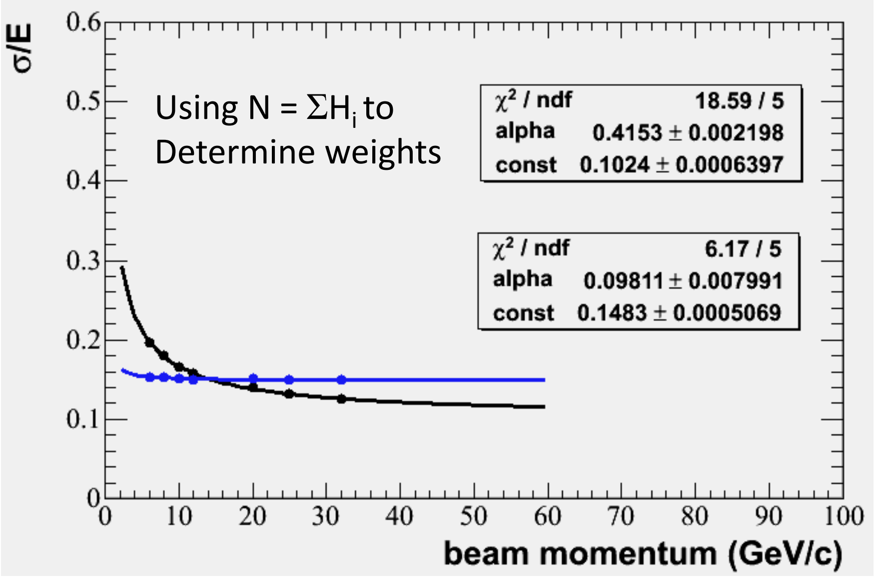


Same procedure applied to positrons

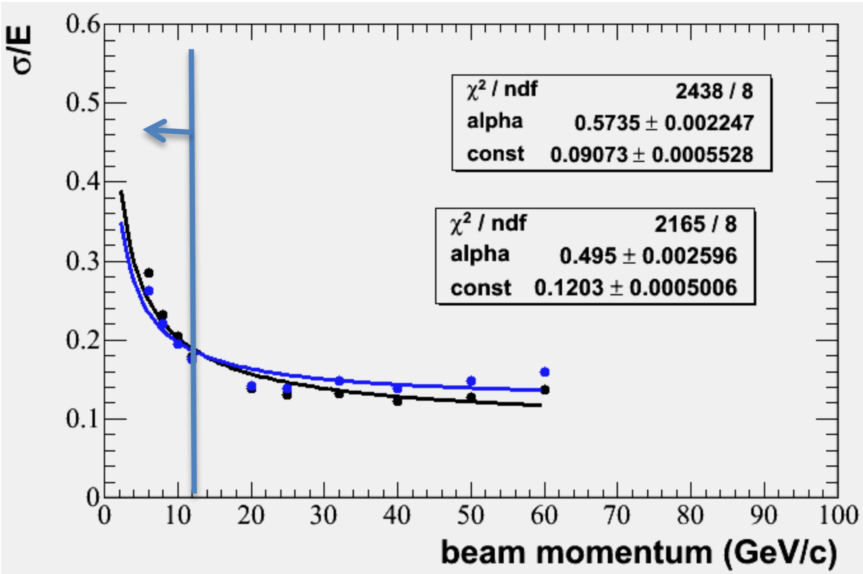
Resolution improves by 35% when using known E_{beam}

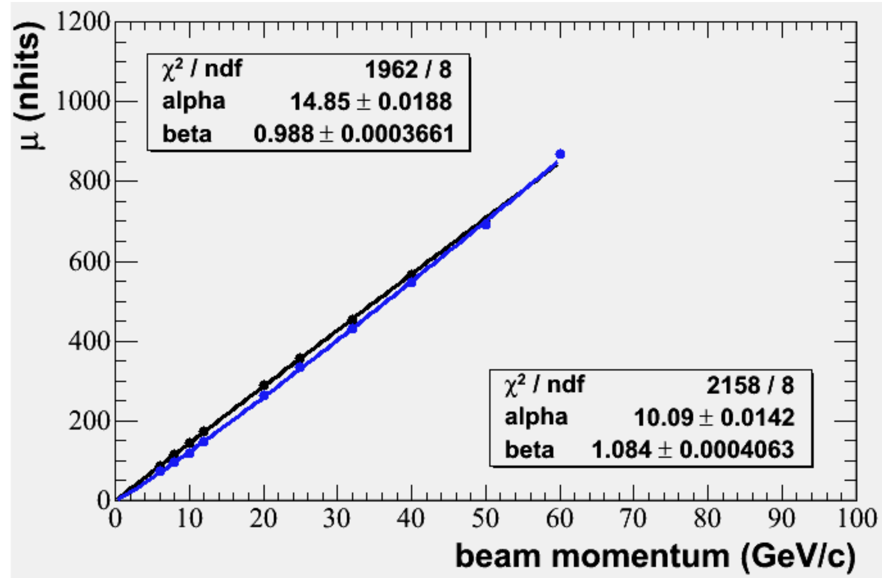
Using $N = \Sigma H_i$ improves resolution for E< 15 GeV, degrades resolution for E> 15 GeV





Weights in Improved E-Range

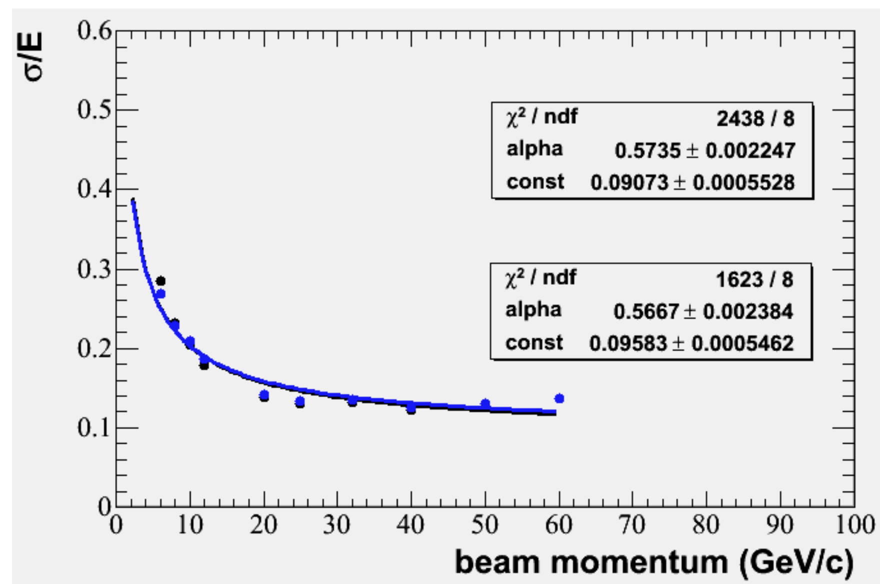




Apply longitudinal weights for E < 15 GeV = 220 hits

Apply unity weights for E > 15 GeV

-> Improvement gone!



Summary of Weights Application

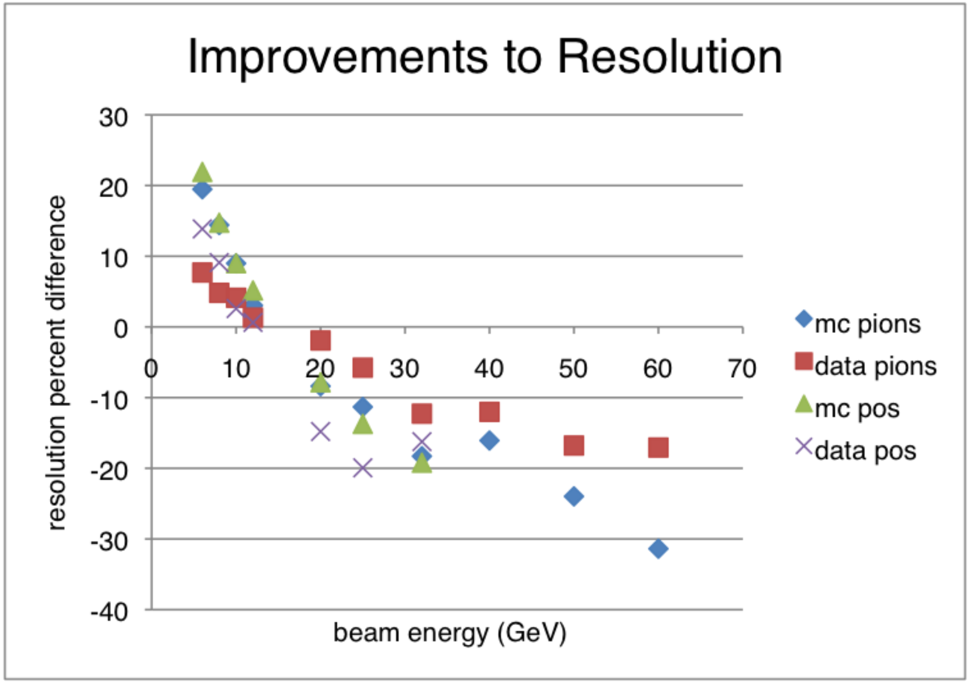
Apply weighting to…
Pions – MC and Data
Positrons – MC and Data

Plot the percent difference between weighted resolutions and unweighted resolutions

0 line is unweighted case
Positive values are improvement to energy resolution

In all cases resolution is improved below 12 GeV but degradation increases with energy

MC and Data only agree in behavior but actual values are quite different



Limitation on resolution seems to be dominated by fluctuation of $N = \Sigma H_i$

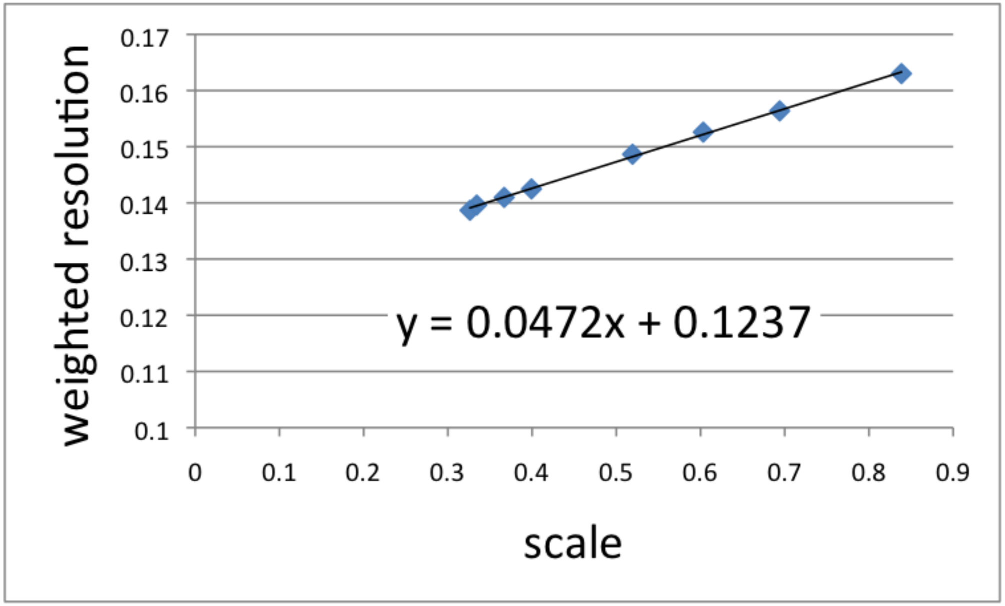
Limitation of Improvement

Artificially reduce the fluctuations from weight selection

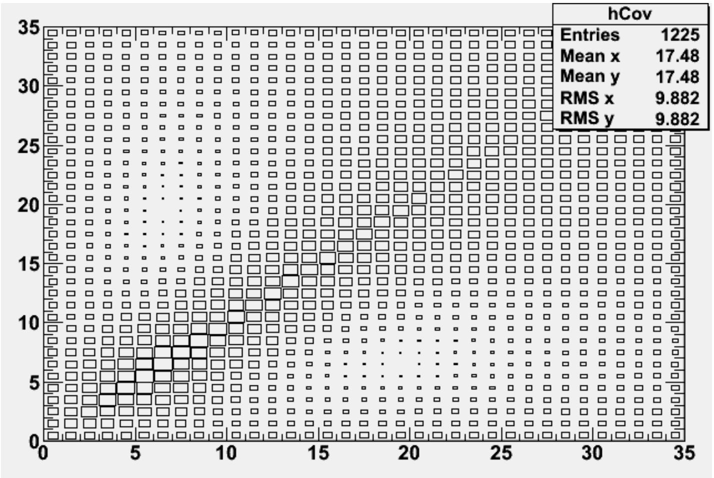
$\sigma_N' = \text{scale} * \sigma_N$

Improvement scales with resolution of $N = \Sigma H_i$

50 GeV Pions

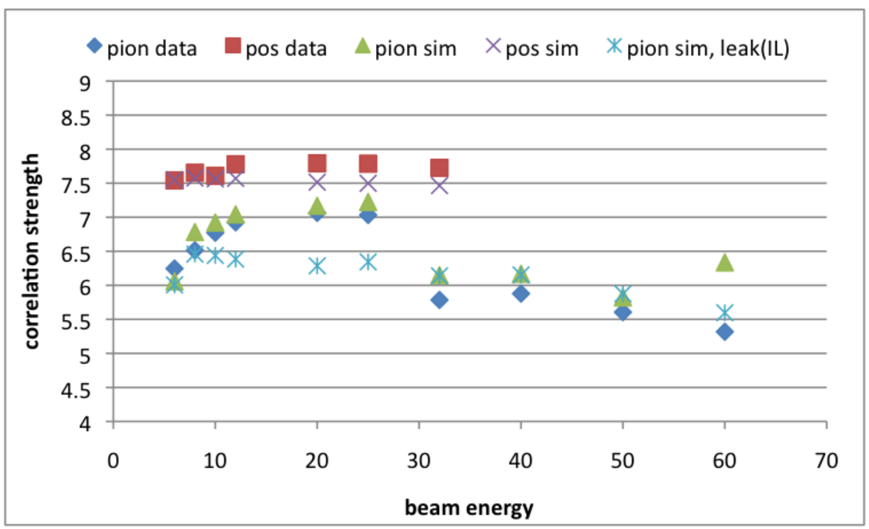


Correlation Strength



Does strength of correlations matter?
Correlation matrix for pions at 50 GeV
Diagonal matrix means no correlations
Larger S → larger correlations

$$S = \frac{\sum R_{ij} N_{ij}}{\sum N_{ij}}$$



Pions show a slight dependence on energy, positrons almost none

Changes in correlations strength do not explain why weighted resolutions improve for E < 12 GeV

Conclusions

- In order to improve the DHCAL energy resolution, hits in layers are weighted differently
- The method was applied to Pions/positrons Data/MC
- Resolutions are improved without knowledge of beam energy, but only for E < 15 GeV
- Application of the method depends on the initial resolution of the number of hits