Klystron Theory

Consider a klystron consisting of two cavities, a "buncher" and a "catcher," both gridded. (Fig.1). Let a beam of electrons, which has been accelerated by a potential $\underline{V_o}$ to a velocity $\underline{u_o}$, traverse the first pair of grids, where it is acted upon by an rf voltage $V_1 \sin\omega t$, reduced by a "coupling coefficient" $\underline{M}$. The latter modifies the voltage across the grids to produce the effective voltage modulating the electron beam. Expressions for the coupling coefficient M (always less than 1) will be derived later.

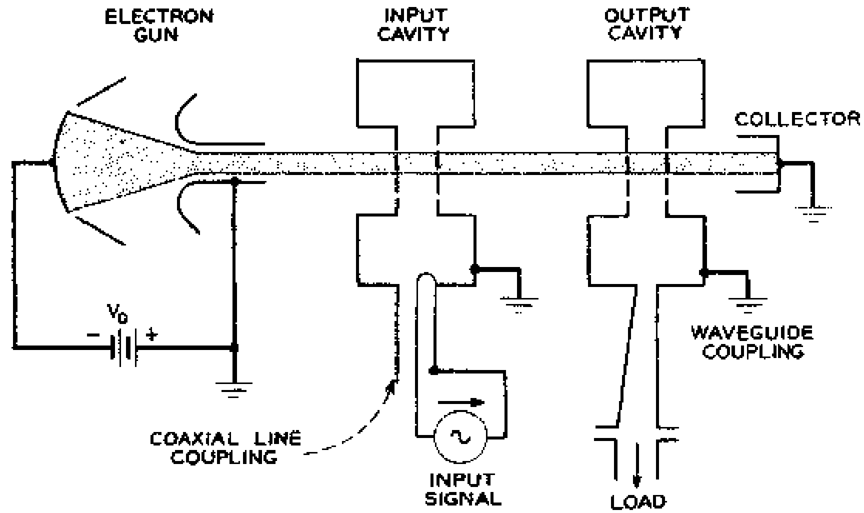


Fig. 1

The electrons in the beam enter the gridded gap with energy,

$$\frac{1}{2} m u_0^2 = eV_0 \tag{1}$$

where the electron charge $\underline{e}$ does not carry its own negative sign. The electron energy is modified by the rf field at the gap and the following relationship can be written for the exit velocity $\underline{u}$:

$$\frac{1}{2} m u^2 - \frac{1}{2} m u_0^2 = eMV_1 \sin\omega t \tag{2}$$

from (1) and (2), it follows,

$$u = u_0 \sqrt{1 + \frac{MV_1}{V_0} \sin\omega t} \tag{3}$$

If we assume that $V_1 << V_o$ (which is an good assumption for the first cavity of a two-cavity klystron), then (4)

$$u \cong u_0 \left(1 + \frac{MV_1}{2V_0} \sin \omega t\right)$$

We consider, for now, that the first interaction gap is very narrow, and that we can neglect the finite transit time of the entering electrons. (Later we will inquire into the happenings within both interaction gaps). The electrons then enter and leave the first gap at time t_1, then drift for a distance l, and arrive at the center of the second gap at time t_2. Then, (invoking again the small-signal assumption $V_1/V_0<<1$),

$$t_2 = t_1 + \frac{l}{u} = t_1 + \frac{l}{u_0\left(1 + \frac{MV_1}{2V_0}\sin \omega t_1\right)} \approx t_1 + \frac{l}{u_0} - \frac{lMV_1}{2u_0V_0}\sin \omega t_1 \qquad (5)$$

or, in terms of phase,

$$\omega t_2 = \omega t_1 + \theta_0 - X \sin \omega t_1 \qquad (6)$$

$$X = \frac{MV_1\theta_0}{2V_0}$$

X is the "bunching parameter", and $\theta_0=\omega l/u_0$.
Obviously, when X >1, ωt_2 is multivalued and there are electrons overtaking

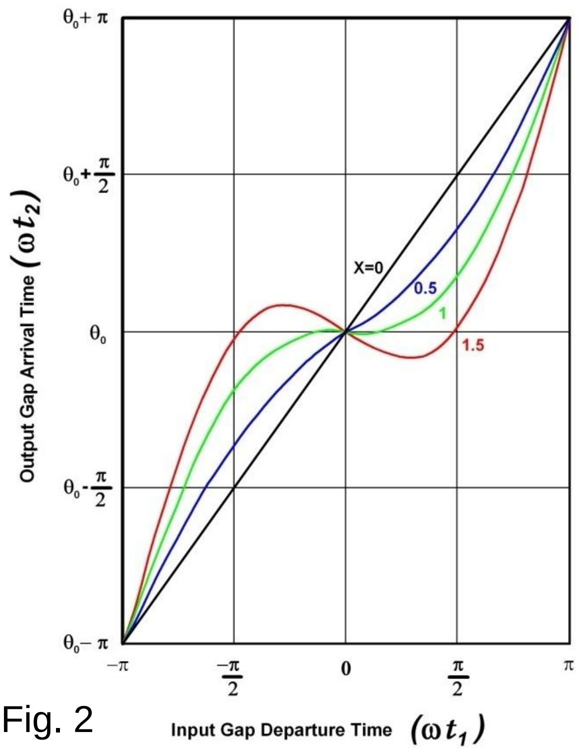


Fig. 2

The quantity of charge leaving the buncher in the time interval t_1 to $t_1 + dt_1$ is $I_o dt_1$, where I_o is the beam DC current entering the buncher. This charge, after drifting, enters the catcher in the interval t_2 to t_2+dt_2. If I_t (total current, dc and rf) is the current transported by the beam to the entrance to the catcher, then through conservation of charge,

$$I_o dt_1 = I_t dt_2 \tag{7}$$

We have, differentiating (6)

$$\frac{dt_2}{dt_1} = 1 - X\cos\omega t_1 \tag{8}$$

From (7) and (8), can now write

$$I_t = I_o/(dt_2/dt_1) \tag{9}$$

And, replacing dt_2/dt_1 by its value in Eq. (9)

$$I_t = \frac{I_o}{(1 - X\cos\omega t_1)} \tag{10}$$

For X = 1, the current at the catcher becomes infinite, since by inspection of Fig. 2, the finite charge transported from the buncher at $t_1 = 0$ arrives at the catcher in a zero time interval ($dt_2/dt_1 = 0$ at $t_1 = 0$)

To calculate I_t, one must then sum the absolute values of all current contributions to I_t from time segments t_{11}, t_{12}, etc, at the buncher as follows,

$$I_t = I_0\left[\frac{1}{\left|1 - X\cos\omega t_{11}\right|} + \frac{1}{\left|1 - X\cos\omega t_{12}\right|} + \ldots.\right] \tag{11}$$

The current waveforms at the buncher are shown in Fig. 3 below

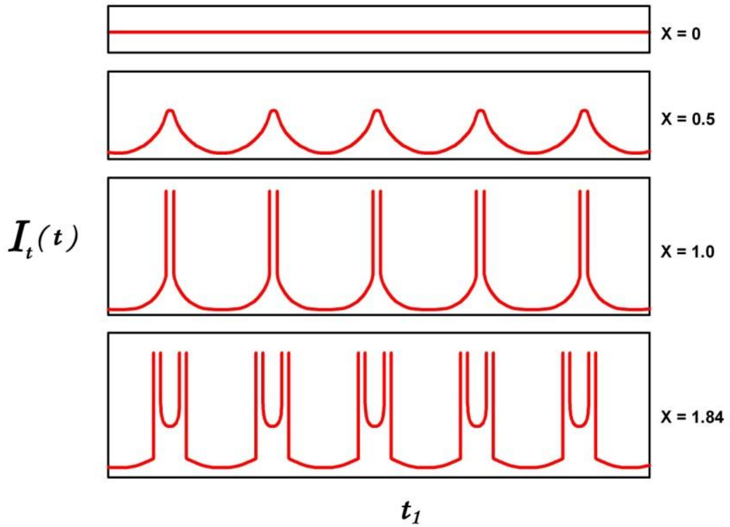


Fig 3

Now, since I_t is clearly a periodic function of ωt, it can be expanded in a Fourier series, as follows,

$$I_t = I_o + \sum_1^{\infty}[a_n \cos n(\omega t_2 - \theta_0) + b_n \sin n(\omega t_2 - \theta_0)] \tag{12}$$

the coefficients are given by,

$$a_n = 1/\pi \int_{\theta_0-\pi}^{\theta_0+\pi} I_t \cos n(\omega t_2 - \theta_0) d(\omega t_2) \tag{13}$$

and

$$b_n = 1/\pi \int_{\theta_0-\pi}^{\theta_0+\pi} I_t \sin n(\omega t_2 - \theta_0) d(\omega t_2) \tag{14}$$

Using (7), we can now write

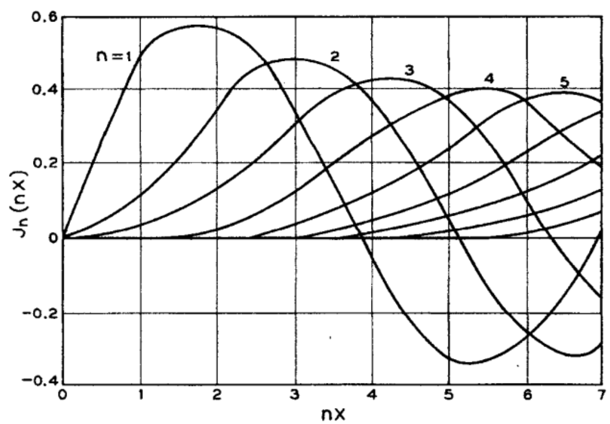
$$a_n = \frac{I_0}{\pi} \int_{-\pi}^{\pi} \cos n(\omega t_1 - X \sin \omega t_1) d(\omega t_1) \tag{15}$$

and

$$b_n = \frac{I_0}{\pi} \int_{-\pi}^{\pi} \sin n(\omega t_1 - X \sin \omega t_1) d(\omega t_1)$$

b_n is identically equal to zero, since the integrand above is an odd function of ωt_1.
It turns out that the expression (15) for the an coefficients is also a representation of the Bessel functions of the first kind and nth order (Fig. 4).

$$a_n = 2J_n(nX) \qquad (16)$$



Bessel functions of various orders. The maximum value of J_1 occurs at $X = 1.84$ and is equal to 0.582.

Fig 4.

Therefore, the catcher rf current I_t can be written as the following series

$$I_t = I_0 + 2I_0 \sum_{1}^{\infty} J_n(nX) \cos n(\omega t_1 - \theta_0) \qquad (17)$$

The n = 1 harmonic (the fundamental) is simply,

$$\begin{aligned} I_1 &= 2I_0 J_1(X) \cos(\omega t - \theta_0) \\ &= \mathrm{Re}\left[2I_0 J_1(X) e^{j(\omega t - \theta_0)}\right] \end{aligned} \qquad (18)$$

When X<1, the series converges (17) for all values of t_2. For X=1, and X>1, there are discontinuities at various t_2 values as shown in Fig 3 (which would disappear if space charge were taken into account). The harmonic amplitudes correspond to the peaks of the Bessel functions (Fig. 4). We can now calculate the output power from the fundamental (n = 1), using (16) and the maximum value of $J_1(X)$, which is 0.582 and occurs at X = 1.84. The output power is the product of the rf current I_1 and the maximum voltage that can be developed across the output gap without reflecting electrons, which is the beam voltage V_0. Both are peak values, so,

$$P_{out} = \frac{1.16 Io}{\sqrt{2}} x \frac{Vo}{\sqrt{2}} = 0.58 I_0 V_0 = 0.58 P_{in} \qquad (19)$$

Consequently, for the two-cavity klystron, without space charge and with sinusoidal voltage modulation, the maximum efficiency is 58 percent. The above derivation is completely valid, even when there is electron overtaking. The small-signal approximation used to formulate the expressions used in launching the velocity modulated beam into the drift space is not used beyond the buncher in arriving at the above result.

As we shall develop in following sections, however, the effects of space charge and a number of other issues force a much lower efficiency in the two-cavity klystron case. The mathematics becomes too complex for the purposes of these lectures, but it can be shown that the use of a third cavity, or an additional 2nd harmonic cavity, or multiple cavities properly arranged, can produce I_1/I_o ratios as high as 1.8. In one case, a multi-cavity experimental klystron efficiency of 74 percent has been a result of such optimum bunching.