Low vertical emittance tuning

Yannis PAPAPHILIPPOU
Accelerator and Beam Physics group
Beams Department
CERN

Eighth International Accelerator School for Linear Colliders
4-15 December 2013, Antalya

Outline

- Equilibrium emittances and optics conditions for different cells
  - FODO
  - Double Bend Achromat (DBA)
  - Theoretical Minimum Emittance (TME)
  - Multi-Bend Achromat (MBA)
  - Examples from low emittance rings
  - The ILC and CLIC DR optics
- Non-linear dynamics
  - Chromaticity and correcting sextupoles
  - Non-linear dynamics due to sextupoles and multipulos
  - Dynamic aperture
  - Frequency map analysis

Quantum vertical emittance limit

- Photons are emitted with a distribution with an angular width $1/\gamma$ about the direction of motion of the electron
- This leads to some vertical "recoil" that excites vertical betatron motion, resulting in a non-zero vertical emittance

$$\varepsilon_{y,\min} = \frac{13}{55}\frac{C_q}{j_y I_2}\int \frac{\beta_y}{|\rho|^3}\,ds$$

- For an isomagnetic lattice this can be written as

$$\varepsilon_y = 0.09\,\mathrm{pm}\cdot\frac{\langle\beta_y\rangle_{\mathrm{Mag}}}{\rho}$$

- Some examples
  - ASLS: 0.35 pm
  - PETRA-III: 0.04 pm
  - ILC DR: 0.1 pm
  - CLIC DR: 0.1 pm

Some factor higher than vertical emittance requirement of both CLIC and ILC

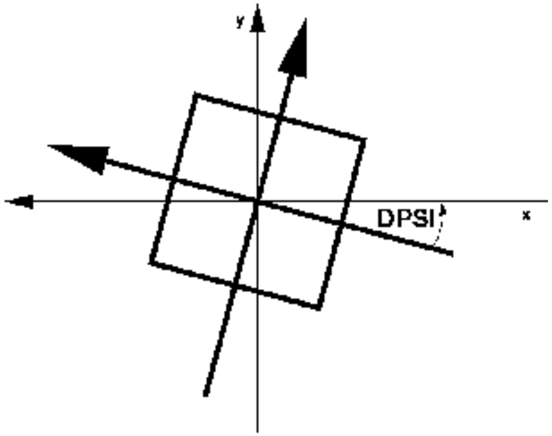
Vertical emittance dependences

- Vertical emittance in a flat storage ring is dominated by two effects
  - Residual vertical dispersion coupling longitudinal and vertical motion
  - Betatron coupling, which couples horizontal and vertical motion
- The dominant causes of residual vertical dispersion and betatron coupling are magnet alignment errors, in particular
  - Tilts of the dipoles around the beam axis
  - Vertical alignment errors on the quadrupoles
  - Tilts of the quadrupoles around the beam axis
  - Vertical alignment errors of the sextupoles

Vertical Steering Error

- Vertical steering error may be generated
  - Dipole roll producing an horizontal dipole component

$$\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$$

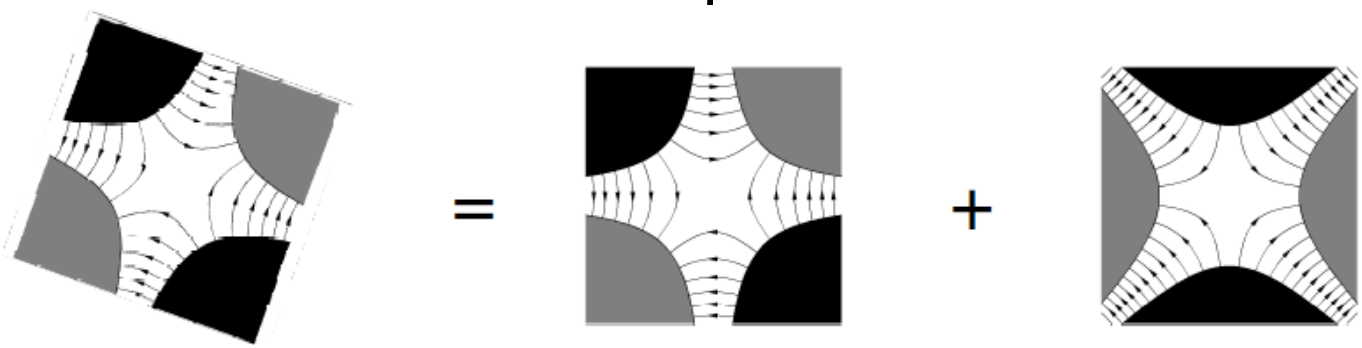


  - Vertical alignment errors on the quadrupoles so that there is a horizontal magnetic field at the location of the reference trajectory. Consider the displacement of a particle ***δy*** from the ideal orbit. The horizontal field in the quadrupole is

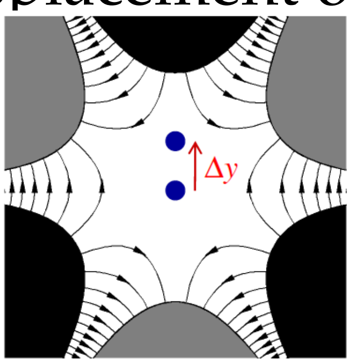
$$B_x = G\bar{y} = G(y + \delta y) = \underbrace{Gy}_{\text{quadrupole}} + \underbrace{G\delta y}_{\text{dipole}}$$

Coupling error

- Coupling errors lead to transfer of horizontal betatron motion and dispersion into the vertical plane in both cases, the result is an increase in vertical emittance.
- Coupling may result from rotation of a quadrupole, so that the field contains a skew component



- A vertical beam offset in a sextupole has the same effect as a skew quadrupole. The sextupole field for the displacement of a particle $\boldsymbol{\delta y}$ becomes



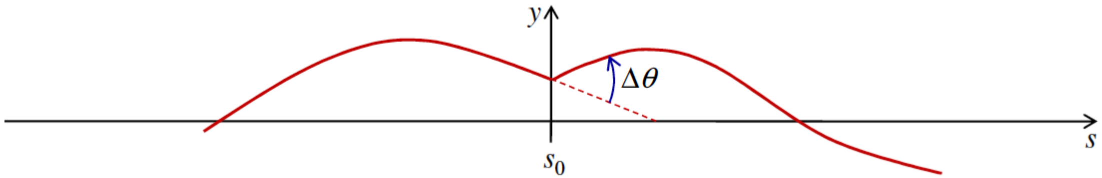
$$B_x = k_2 x\bar{y} = k_2 xy + \underbrace{k_2 x\delta y}_{\text{skew quadrupole}}$$

$$B_y = \frac{1}{2}k_2(x^2 - \bar{y}^2) = \overbrace{-k_2 y\delta y}^{\text{skew quadrupole}} + \frac{1}{2}k_2(x^2 - y^2) - \frac{1}{2}k_2\delta y^2$$

Effect of single dipole kick

- Consider a single dipole kick $\theta = \delta u'_0 = \delta u'(s_0) = \frac{\delta(Bl)}{B\rho}$ at s=s_0
- The coordinates before and after the kick are

$$\begin{pmatrix} u_0 \\ u'_0 - \theta \end{pmatrix} = \mathcal{M} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$



with the 1-turn transfer matrix

$$\mathcal{M} = \begin{pmatrix} \cos 2\pi Q + \alpha_0 \sin 2\pi Q & \beta_0 \sin 2\pi Q \\ -\gamma_0 \sin 2\pi Q & \cos 2\pi Q - \alpha_0 \sin 2\pi Q \end{pmatrix}$$

- The final coordinates ar$u_0 = \theta \frac{\beta_0}{2 \tan \pi Q}$ $\quad u'_0 = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan \pi Q}\right)$

- For any location around the ring it can be shown that

$$u(s) = \theta \underbrace{\frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)}}_{\text{Maximum distortion amplitude}} \cos(\pi Q - |\psi(s) - \psi_0|)$$

Maximum distortion amplitude

Transport of orbit distortion due to dipole kick

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$\begin{aligned} u_2 &= m_{11}u_1 + m_{12}u'_1 \\ u'_2 &= m_{21}u_1 + m_{22}u'_1 \end{aligned}$$

- Consider a single dipole kick at position 1 $\theta_1 = \dfrac{\delta(Bl)}{B\rho}$
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u'_1 + \theta_1) \rightarrow \delta u_2 = m_{12}\theta_1$$

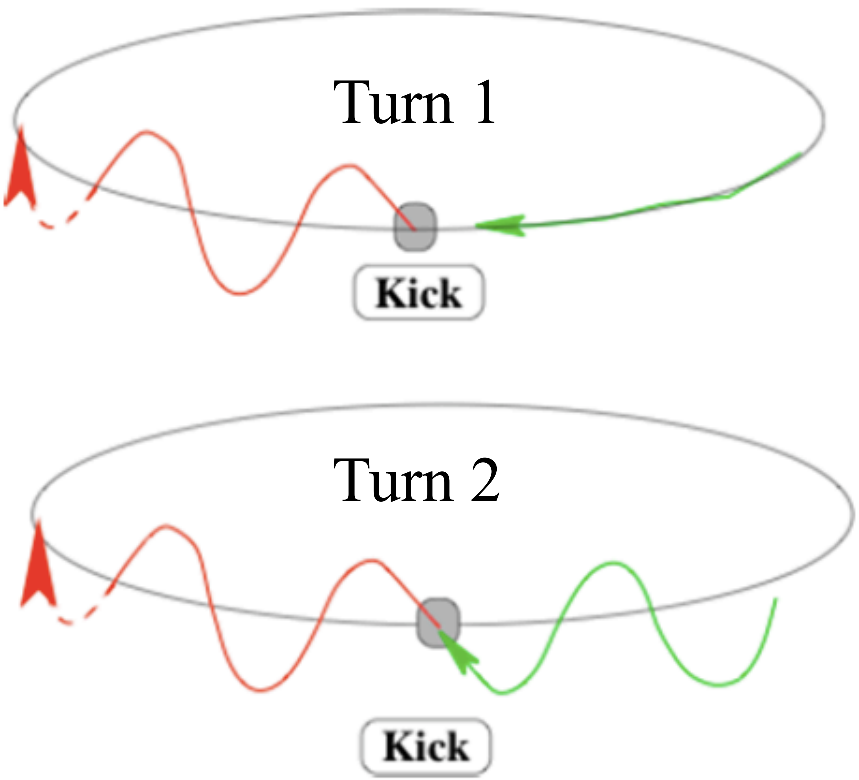
- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1 \beta_2} \sin(\psi_{12})\theta_1$$

$$\delta u'_2 = \sqrt{\frac{\beta_1}{\beta_2}} \left[\cos(\psi_{12})\theta_1 - \alpha_2 \sin(\psi_{12})\right]$$

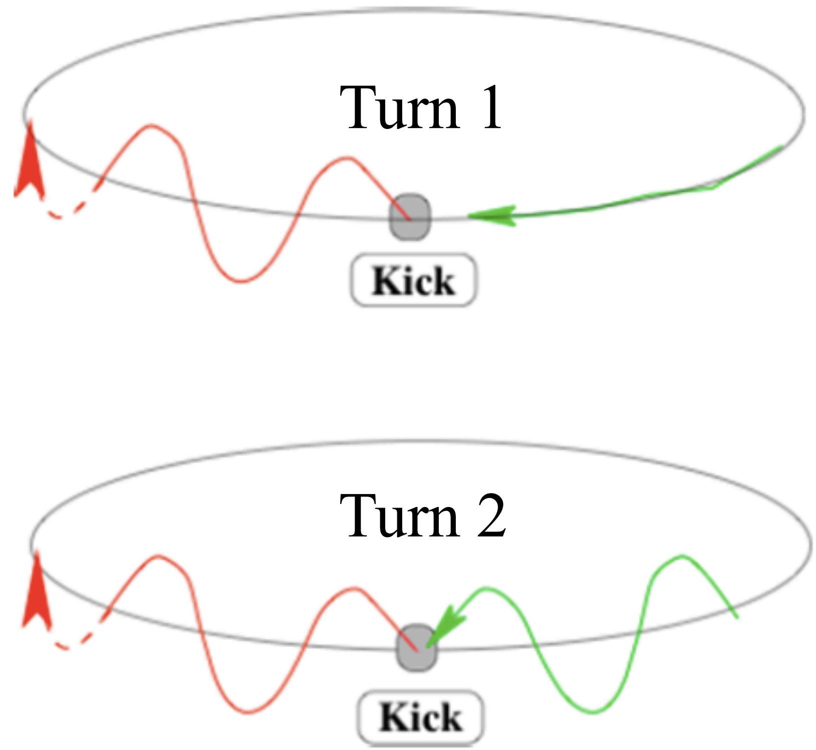
Integer and half integer resonance

- Dipole perturbations add-up in consecutive turns for $Q = n$
- Integer tune excites orbit oscillations (resonance)



$$\delta u_2 = \sqrt{\beta_1 \beta_2}\sin(\psi_{12})\theta_1$$

- Dipole kicks get cancelled in consecutive turns for $Q = n/2$
- Half-integer tune cancels orbit oscillations



$$\delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}}\left[\cos(\psi_{12})\theta_1 - \alpha_2 \sin(\psi_{12})\right]$$

Global orbit distortion

- Orbit distortion due to many errors

$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+C} \theta(\tau)\sqrt{\beta(\tau)} \cos(\pi Q - |\psi(s) - \psi(\tau)|) d\tau$$

- For a quadrupole of integrated focusing strength ($k_1 L$), vertically misaligned from the reference trajectory by ΔY, the steering is

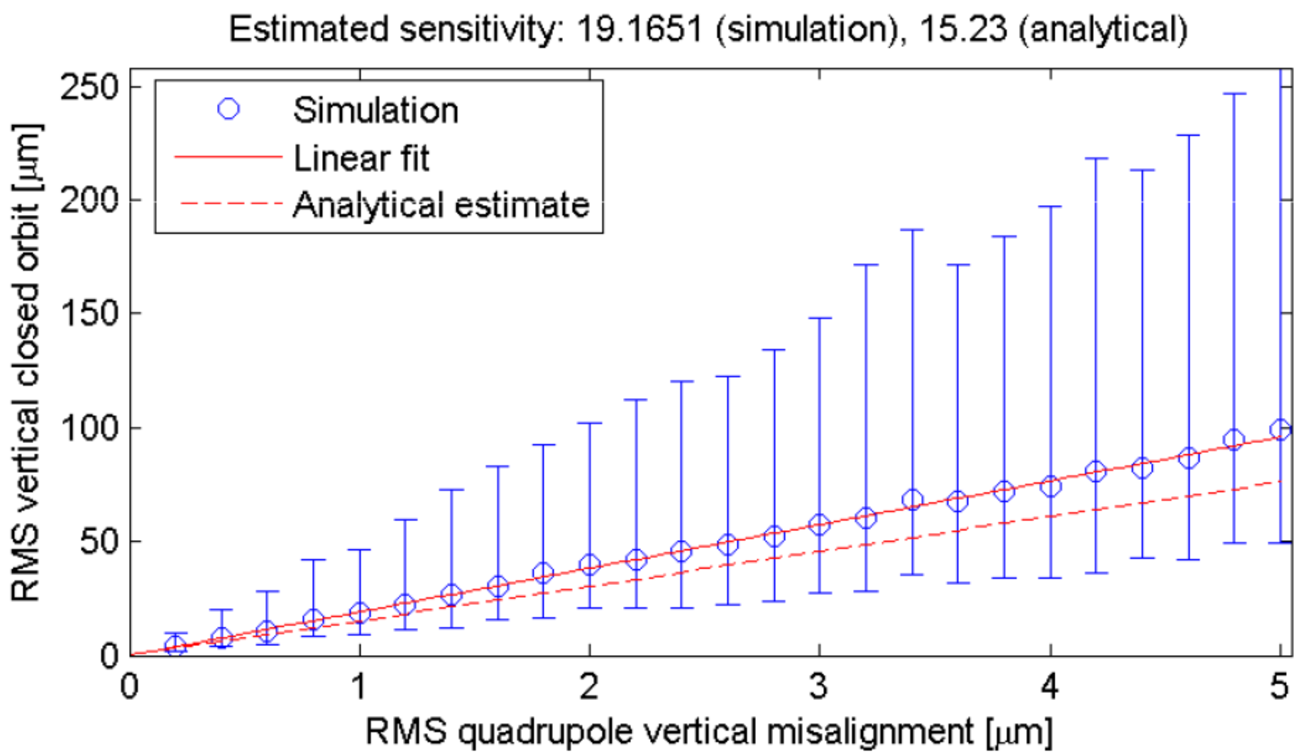
$$\frac{d\theta}{ds} = (k_1 L)\Delta Y$$

- Squaring the previous equation and averaging over many (uncorrelated) random alignment errors, we obtain

$$\left\langle \frac{y_{co}^2(s)}{\beta_y(s)} \right\rangle = \frac{\langle \Delta Y^2 \rangle}{8 \sin^2 \pi\nu_y} \sum_{\text{quads}} \beta_y (k_1 L)^2$$

Simulated orbit distortion

- "Orbit amplification factors" are commonly between 10 to 100
- This is a statistical quantity, over many different sets of misalignments and the orbit distortion may be much larger or smaller than expected from the rms quadrupole alignment error estimate



Reminder: General multi-pole perturbation

- Equations of motion including any multi-pole error term, in both planes

$$\frac{d^2\mathcal{U}_x}{d\phi_x^2} + \nu_{0x}^2\mathcal{U}_x = \overline{b_{n,r}}(\phi_x)\mathcal{U}_x^{n-1}\mathcal{U}_y^{r-1}$$

- Expanding perturbation coefficient in Fourier series and inserting the solution of the unperturbed system on the rhs gives the following series:

$$\mathcal{U}_x^{n-1} \approx \mathcal{U}_{0x}^{n-1} = \sum_{q_x=-n+1}^{n-1} \overline{W}_{q_x} e^{iq_x\nu_0\phi_x}$$

$$\overline{b_{nr}}(\phi_x) = \sum_{m=-\infty}^{\infty} \overline{b_{nrm}} e^{im\phi_x}$$

$$\mathcal{U}_y^{r-1} \approx \mathcal{U}_{0y}^{r-1} = \sum_{q_y=-r+1}^{r-1} \overline{W}_{q_y} e^{iq_y\nu_{0y}\phi_x}$$

- The equation of motion becomes

$$\frac{d^2\mathcal{U}_x}{d\phi_x^2} + \nu_{0x}^2\mathcal{U}_x = \sum_{m,q_x,q_y} \overline{b_{nrm}} W_{q_x}^x W_{q_y}^y e^{i(m+q_x\nu_{0x}+q_y\nu_{0y})\phi_x}$$

- In principle, same perturbation steps can be followed for getting an approximate solution in both planes

Linear Coupling

- For a localized skew quadrupole we have

$$\frac{d^2\mathcal{U}_x}{d\phi_x^2} + \nu_{0x}^2\mathcal{U}_x = \overline{b_{1,2}}(\phi_x)\mathcal{U}_y$$

- Expanding perturbation coefficient in Fourier series and inserting the solution of the unperturbed system gives the following equation:

$$\frac{d^2\mathcal{U}_x}{d\phi_x^2} + \nu_{0x}^2\mathcal{U}_x = \sum_{m=-\infty}^{\infty}\sum_{q_y=-1}^{q_y=1}\overline{b_{12m}}W_{q_y}^y e^{i(m+q_y\nu_{0y})\phi_x}$$

with $W_0^y = 0$

- The coupling resonance are found for $q_y = \pm 1$

$$m = \nu_{0x} + \nu_{0y} \qquad\qquad m = \nu_{0x} - \nu_{0y}$$

Linear sum resonance **Linear difference resonance**

- In the case of a thin skew quad: $\delta Q \propto |k_s|\sqrt{\beta_x\beta_y}$
- Coupling coefficients

$$|C_\pm| = \left|\frac{1}{2\pi}\oint ds k_s(s)\sqrt{\beta_x(s)\beta_y(s)}e^{i(\psi_x\pm\psi_y-(Q_x\pm Q_y-q_\pm)2\pi s/C)}\right|$$

- Tunes observed **on** difference resonance

$$Q_x - Q_y = q: \qquad Q_{1/2} = \tfrac{1}{2}(Q_x + Q_y) \pm \tfrac{1}{2}|\kappa_-|$$

- Betatron coupling from difference resonance

$$|\kappa_-| = \frac{1}{2\pi}\left|\oint ds\, a_2(s)\sqrt{\beta_x(s)\beta_y(s)}e^{i(\phi_x(s)-\phi_y(s)-(Q_x-Q_y+q)2\pi s/C)}\right|$$

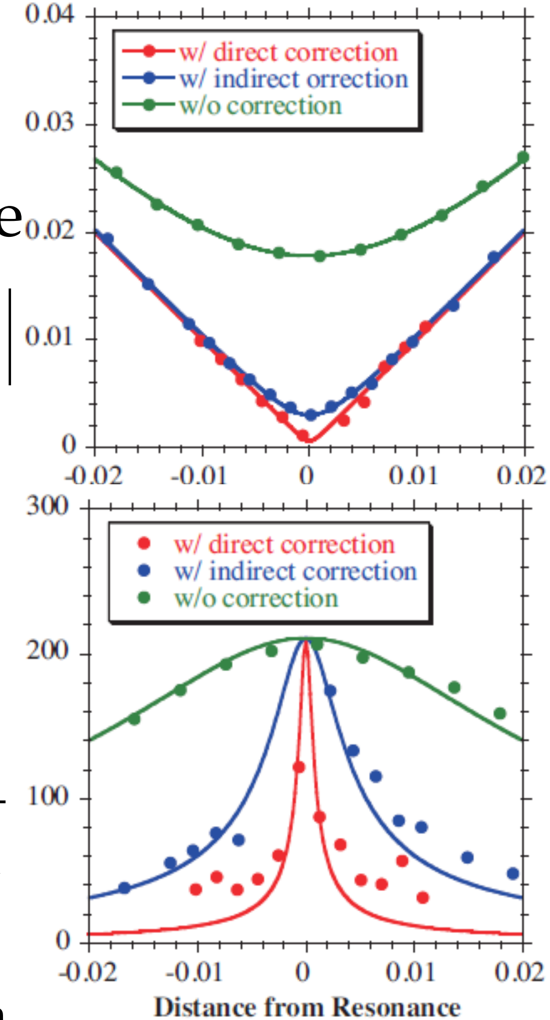
- Working point **off** resonance (but close)

$$\Delta Q_- = Q_x - Q_y - q = \sqrt{(Q_1 - Q_2)^2 - |\kappa_-|^2}$$

  - $Q_{x/y}$ uncoupled, $Q_{1/2}$ observed tunes
- Vertical emittance

$$\varepsilon_y = \varepsilon_x \frac{|\kappa_-|^2}{|\kappa_-|^2 + (\Delta Q_-)^2/2}$$

- Caution
  - assumes betatron coupling >> vertical dispersion
  - assumes difference >> sum coupling resonance
  - single resonance approximation



$|Q_1 - Q_2|$ and σ_y near resonance at SPRING-8.

Vertical dispersion

- The equation of motion for a particle with momentum P is

$$\frac{d^2y}{ds^2} = \frac{e}{P}B_x$$

- For small energy deviation δ, P is related to the reference momentum by $P \approx (1+\delta)P_0$
- We can write for the horizontal field (to first order in the derivatives)

$$B_x \approx B_{0x} + y\frac{\partial B_x}{\partial y} + x\frac{\partial B_x}{\partial x}$$

- If we consider a particle following an off-momentum closed orbit, so that:

$$y = \eta_y \delta, \qquad \text{and} \qquad x = \eta_x \delta,$$

- Combining the above equations, we find to first order in

$$\frac{d^2\eta_y}{ds^2} - k_1\eta_y \approx -k_{0s} + k_{1s}\eta_x$$

Vertical dispersion from alignment errors

- The previous equation is similar to the equation of the closed orbit

$$\frac{d^2 y_{co}}{ds^2} - k_1 y_{co} \approx -k_{0s} + k_{1s} x_{co}$$

- It is the reasonable to generalize the relationship between the closed orbit and the quadrupole misalignments, to find

$$\left\langle \frac{\eta_y^2}{\beta_y} \right\rangle = \frac{\langle \Delta Y_Q^2 \rangle}{8 \sin^2 \pi \nu_y} \sum_{\text{quads}} \beta_y (k_1 L)^2 + \frac{\langle \Delta \Theta_Q^2 \rangle}{8 \sin^2 \pi \nu_y} \sum_{\text{quads}} \eta_x^2 \beta_y (k_1 L)^2 + \frac{\langle \Delta Y_S^2 \rangle}{8 \sin^2 \pi \nu_y} \sum_{\text{sexts}} \eta_x^2 \beta_y (k_2 L)^2$$

- Skew dipole terms assumed to come from vertical alignment errors on the quads **Qi**, and the
- Skew quads assumed to come from
  - Tilts on the quadrupoles
  - Vertical alignment errors on the sextupoles,
- All alignment errors are considered uncorrelated.

Vertical emittance from vertical dispersion

- The natural emittance in the vertical plane can be written as the horizontal one

$$\varepsilon_y = C_q \gamma^2 \frac{I_{5y}}{j_y I_2}$$

- the synchrotron radiation integrals are given by

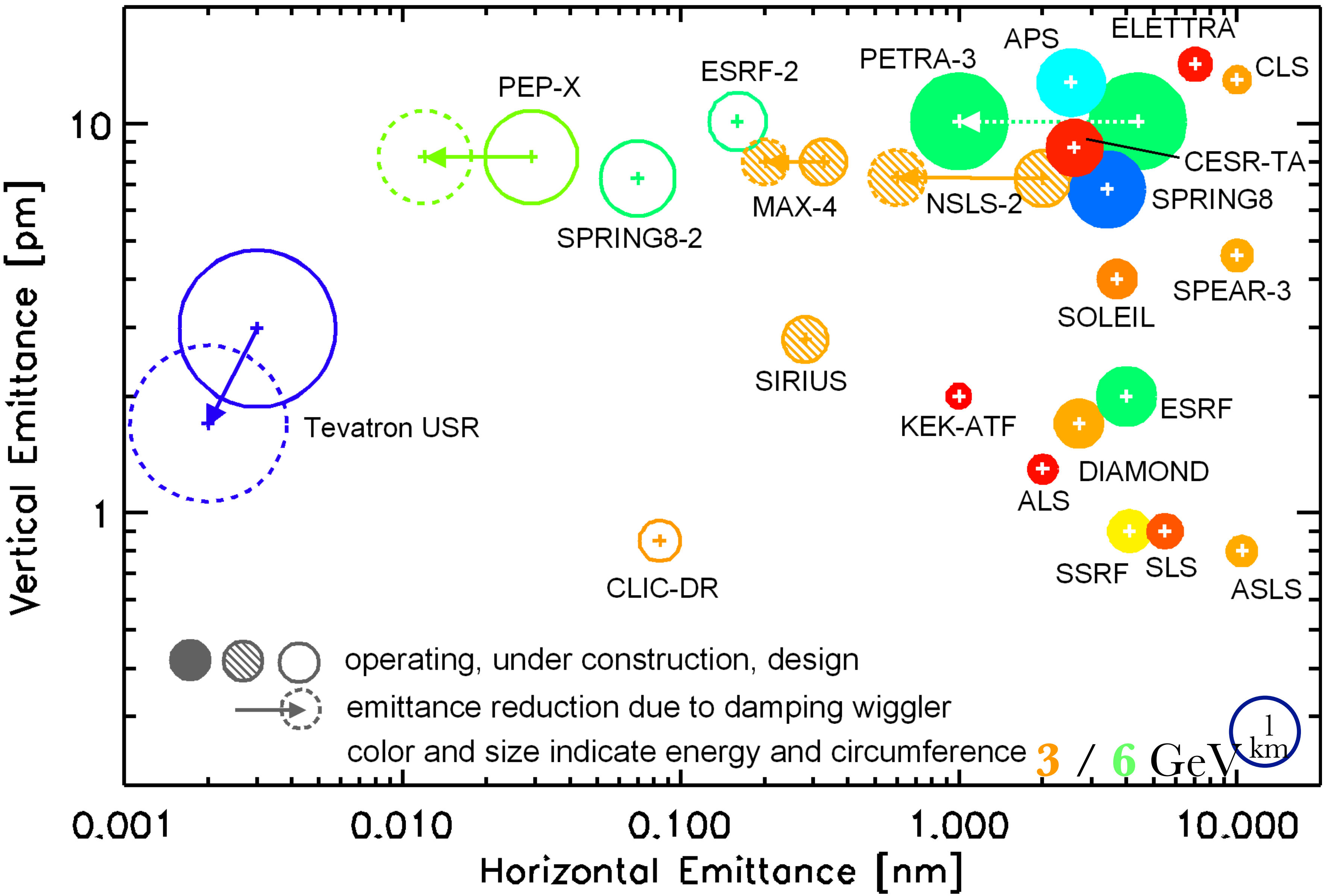
$$I_{5y} = \oint \frac{\mathcal{H}_y}{|\rho|^3} ds \approx \langle \mathcal{H}_y \rangle \oint \frac{1}{|\rho|^3} ds = \langle \mathcal{H}_y \rangle I_3 \quad \text{and} \quad I_2 = \oint \frac{1}{\rho^2} ds$$

with $$\mathcal{H}_y = \gamma_y \eta_y^2 + 2\alpha_y \eta_y \eta_{py} + \beta_y \eta_{py}^2$$

- Then the vertical emittance is $\varepsilon_y \approx C_q \gamma^2 \langle \mathcal{H}_y \rangle \frac{I_3}{j_y I_2}$ or in terms of the energy spread $\varepsilon_y \approx \frac{j_z}{j_y} \langle \mathcal{H}_y \rangle \sigma_\delta^2$, with $\sigma_\delta^2 = C_q \gamma^2 \frac{I_3}{j_z I_2}$
- Note that $\left\langle \frac{\eta_y^2}{\beta_y} \right\rangle = \frac{1}{2} \langle \mathcal{H}_y \rangle$ and finally

$$\varepsilon_y \approx 2 \frac{j_z}{j_y} \left\langle \frac{\eta_y^2}{\beta_y} \right\rangle \sigma_\delta^2$$

Emittances achieved and planned

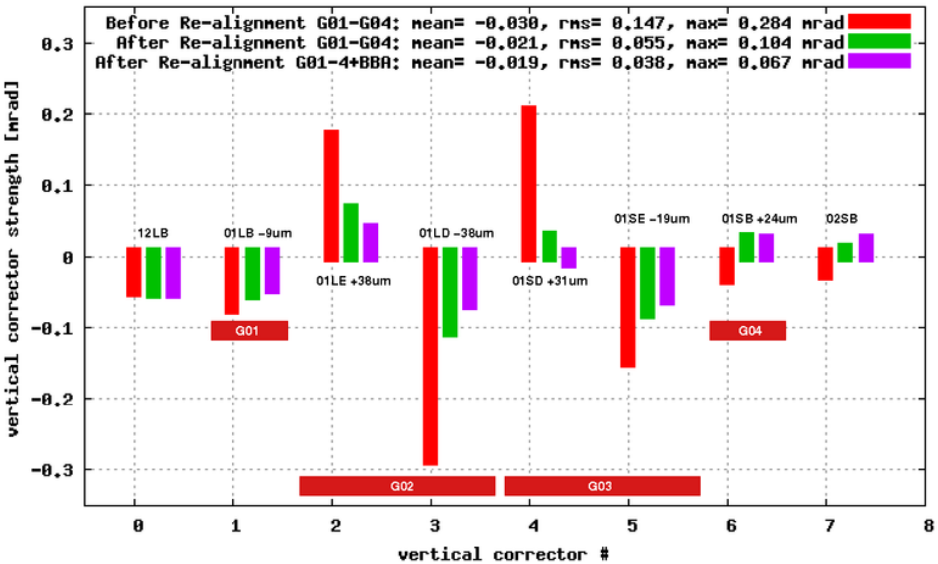


Methods for coupling control

- Measurement or estimation of BPM roll errors to avoid “fake” vertical dispersion measurement.
- Realignment of girders / magnets to remove sources of coupling and vertical dispersion.
- Model based corrections:
  - Establish lattice model: multi-parameter fit to orbit response matrix (using LOCO or related methods) to obtain a calibrated model.
  - Use calibrated model to perform correction or to minimize derived lattice parameters (e.g. vertical emittance) in simulation and apply to machine.
  - Application to coupling control: correction of vertical dispersion, coupled response matrix, resonance drive terms using skew quads and orbit bumps, or direct minimization of vertical emittance in model.
- Model independent corrections:
  - empirical optimization of observable quantities related to coupling (e.g. beam size, beam life time).
- Coupling control in operation: on-line iteration of correction

Magnet / girder realignment

- Magnet misalignment = source of coupling
  - steps between girders: vertical dispersion from vertical corrector dipoles
- BBGA (= beam based girder alignment)
  - Misalignments from orbit response
- BAGA (= beam assisted girder alignment)
  - girder misalignment data from survey
  - girder move with stored beam and running orbit feedback
  - vertical corrector currents confirm move.



BAGA (SLS):
Corrector strengths (sector 1) before and after girder alignment, and after beam based BPM calibration (BBA)

⇓

V-Corrector rms strengths reduced by factor 4 (147→38 mrad)

Resonance drive terms

- Single resonance approximation for large machines
  - high periodicity, few systematic resonances
  - working point nearer to difference than to sum coupling resonance e.g. ESRF 36.45 / 13.39
- Lattice model from ORM or TBT data
  - assume many error sources for fitting (quad rolls etc.)
  - calculate difference and sum coupling resonance drive terms (RDT) and vertical dispersion.
- Response matrix for existing skew quad correctors
- Empirical weights a_1, a_2 for RDTs vs. vertical dispersion

⇨ Vertical emittance 2.6 ±1.1 pm
  - Definition: mean and rms of 12 beam size monitors

$$\begin{pmatrix} a_1 \vec{f}_{1001} \\ a_1 \vec{f}_{1010} \\ a_2 \vec{D}_y \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_{c}$$

$$f_{\substack{1001\\1010}} = \frac{\sum_{w}^{W} J_{w,1}\sqrt{\beta_x^w \beta_y^w}\, e^{i(\Delta\phi_{w,x} \mp \Delta\phi_{w,y})}}{4(1 - e^{2\pi i (Q_u \mp Q_v)})}$$

LOCO (Linear Optics from Closed Orbit)

- Applied to general optics correction and to coupling control
- Low statistical error: response matrix = many, highly correlated data
- Low measurement error: high precision of BPMs in stored beam mode

Fit parameters (almost any possible)

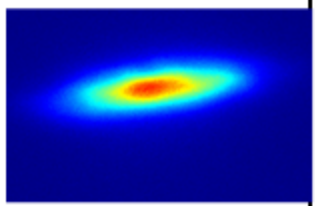
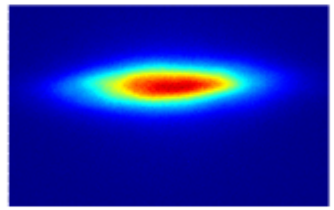
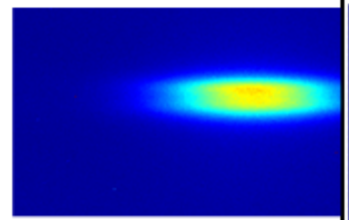
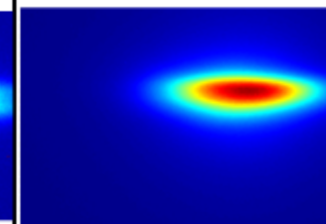
- Quadrupole gradients and roll errors
- BPM and corrector calibrations and roll errors
- Sextupole misalignments
- Not possible: dipole errors ⇦ quad misalignments

Vertical emittance minimization

- Minimizing coupled response matrix using existing skew quad correctors does not necessarily give the lowest vertical emittance
- Establish model with many skew quad error sources
- Use existing skew quads to minimize vertical emittance in model

Results of coupling suppression with LOCO

- Example: SSRF

	Initial	20 skews	60skews	After realignment
Beam profile				
Coupling （LOCO）	0.44%	0.26%	0.18%	0.022%
ertical Emittance (pmrad)	17	10	7	0.9

- more LOCO calibrated model vertical emittances:
  - ASLS 0.3 pm (meas. 0.8 ± 0.1 pm)
  - ALS 1.3 pm (meas. ~2 pm)

LET algorithm (low emittance tuning)

- Principle: double linear system
- Measurement vectors
  - vertical orbit
  - vertical dispersion
  - off-diagonal (coupling)...
  - horizontal orbit
  - horizontal dispersion
  - diagonal (regular)...

  ...parts of the orbit response matrix
- Knob vectors
  - vertical correctors
  - skew quadrupoles
    - and BPM roll errors
  - horizontal correctors
- Weight factors (α , ω)
- Supresss vertical dispersion and coupling
  - DIAMOND (1.7 pm)
  - SLS (1.3 pm)

$$\begin{pmatrix} (1-\alpha-2\omega)\,\vec{y} \\ \alpha\,\vec{\eta}_y \\ \omega\,\vec{ORM}_{y,\theta_H} \\ \omega\,\vec{ORM}_{x,\theta_V} \end{pmatrix} = \mathcal{M}_v \begin{pmatrix} \vec{\theta}_V \\ \vec{K} \\ \vec{T} \end{pmatrix}$$

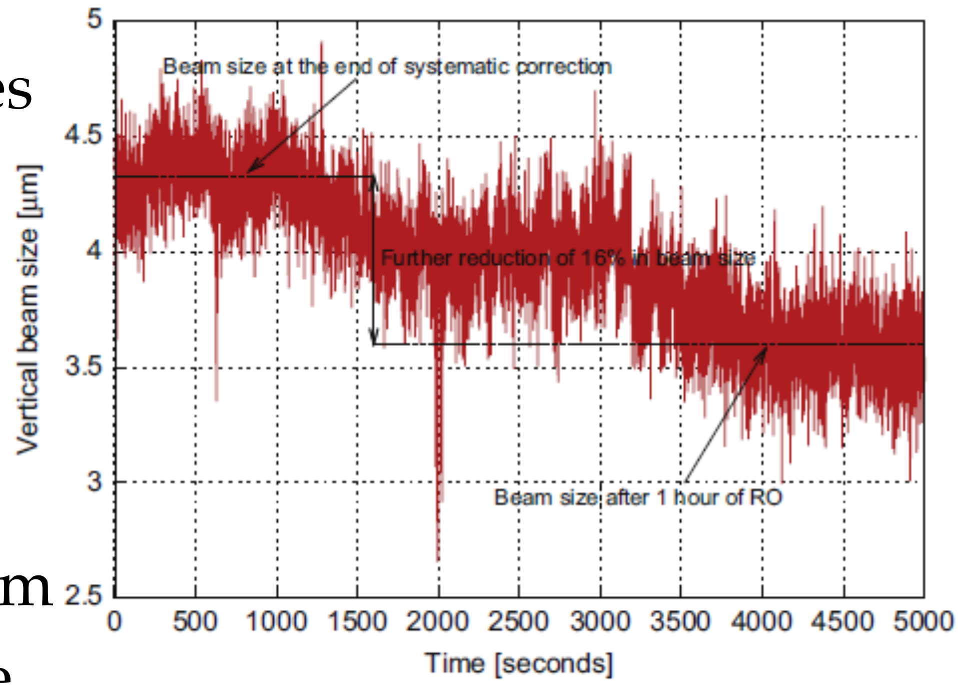
$$\begin{pmatrix} (1-\alpha-2\omega)\,\vec{x} \\ \alpha\,\vec{\eta}_x \\ \omega\,\vec{ORM}_{x,\theta_H} \\ \omega\,\vec{ORM}_{y,\theta_V} \end{pmatrix} = \mathcal{M}_x \begin{pmatrix} \vec{\theta}_H \\ \vec{T} \end{pmatrix}$$

Model independent methods

- Overcome model deficiencies (and BPM limitations)
  - potential to further improve the ***best*** model based solutions
- Requires stable and precise observable of performance
  - beam size or lifetime as observables related to vertical emittance
  - beam-beam bremsstrahlung rate as observable of luminosity
- requires actuators (knobs)
  - skew quadrupoles and orbit bumps for vertical emittance minimization
  - sextupole correctors for lifetime optimization
  - beam steerers for beam-beam overlap
- optimization procedures
  - capable to handle noisy penalty functions (filtering, averaging)
  - algorithms: random walk, simplex, genetic (MOGA) etc.
  - needs good starting point: best model based solution
  - works in simulation and in real machine

Model independent optimization example

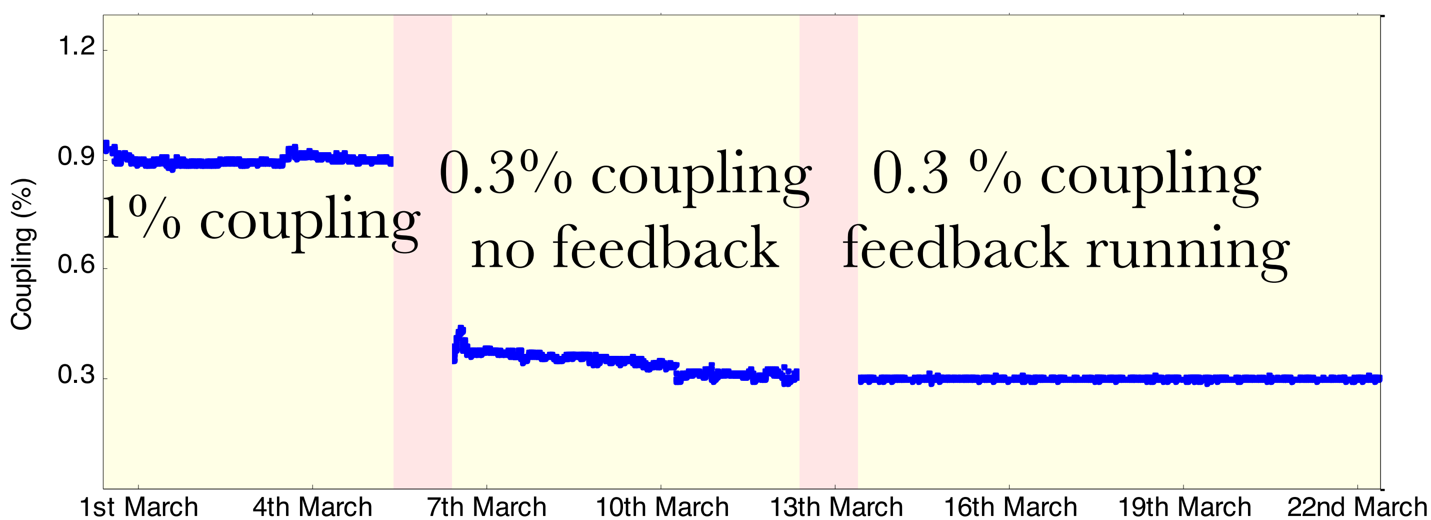
- Coupling minimization at SLS observable: vertical beam size from monitor
- Knobs: 24 skew quadrupoles
- Random optimization: trial & error (small steps)
- Start: model based correction: $e_y = 1.3$ pm
- 1 hour of random optimization $e_y \rightarrow 0.9 \pm 0.4$ pm



- Measured coupled response matrix off-diagonal terms were reduced after optimization
- Model based correction limited by model deficiencies rather than measurement errors.

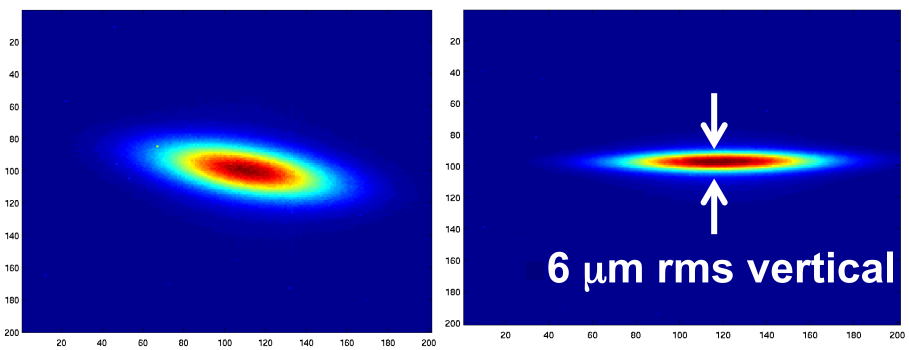
Coupling control in operation

- Keep vertical emittance constant during ID gap changes
- Example from DIAMOND
- Offset δSQ to ALL skew quads generates dispersion wave and increases vert. emittance without coupling.
- Skew quads from LOCO for low vert .emit. of ~ 3pm
- Increase vertical emit to 8 pm by increasing the offset δSQ
- Use the relation between vertical emittance and δSQ in a slow feedback loop (5 Hz)

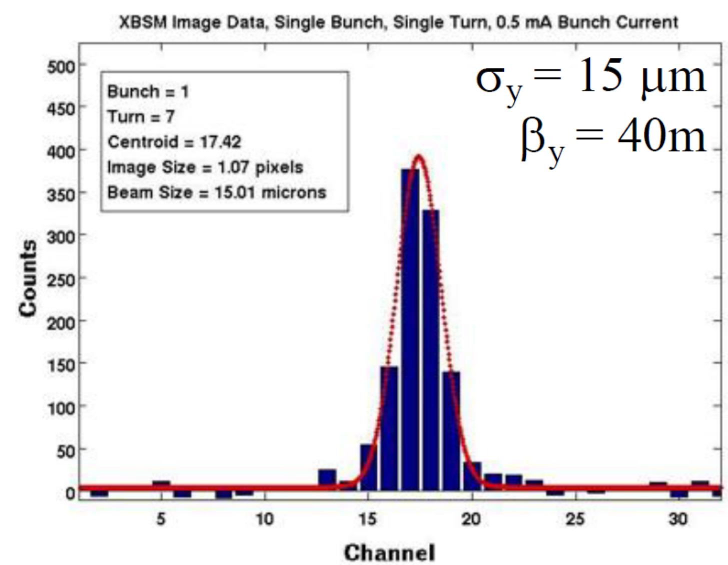


Vertical beam size monitor

- Gives local apparent emittance = $[\sigma_y(s)]^2/\beta_y(s)$
- Requires beta function measurement
  - [dispersion & energy spread measurement too]
- Different methods (e.g. π-polarization)
- Model based evaluation of measurement
  - e.g. diffraction effects in imaging



Pinhole camera images before/after coupling correction at DIAMOND



1-D X-ray diode array camera at CESR-TA

Summary

- Derived approximate formulae for estimating the sensitivity of the vertical emittance to a range of magnet alignment errors
- Described briefly some methods for accurate emittance computation in storage rings with specified coupling and alignment errors
- Outlined some of the practical techniques used for low-emittance tuning in actual low emittance rings in operation