Problems A1-2: Static and Dynamic Imperfections

1) A linac that transports a beam with very small vertical and large horizontal emittance suffers from quadrupole vibrations. All quadrupoles have the same jitter amplitude. Unfortunately due to budget limitations only some quadrupoles can be stabilised. How should the quadrupoles be selected for stabilisation? You can assume thin quadrupoles.

2) A badly designed linac shows slow systematic drifts due to temperature changes after it has been switched on. After $100\ \mathrm{s}$ these drifts result in an emittance growth of $\Delta\epsilon_{100} = 100\ \mathrm{nm}$.
a) What is the expected average emittance growth after $\Delta\epsilon_{200}$ $200\ \mathrm{s}$?
b) What is the emittance growth if the beam orbit is measured every second and 10% of the value is corrected?

3) A transport line consists of a FODO lattice with quadrupole spacing L_0 and focal strength f_0. The line is rebuilt changing the quadrupole spacing to $L = 2L_0$ and the focal strength to $f = 2f_0$.
How does the quadrupole jitter tolerance σ_q change?

Solutions

1) We need to stabilise the beam in the vertical plane since the emittance is much smaller in this plane. The angular deflection by a quadrupole with offset d is

$$\Delta y' = \frac{d}{f}$$

We want to minimise the deflection in the normalised phase space

$$\Delta y'_N = \frac{\Delta y'}{\sqrt{1/(\beta\gamma)}}$$

$$\Rightarrow \Delta y'_N = \frac{\sqrt{\beta\gamma}}{f} d$$

Hence we have to chose the quadrupoles with the largest values

$$\frac{\sqrt{\beta\gamma}}{f}$$

Solutions

2)

a) We use the fact that the orbit grows linearly with time and hence the emittance with the square of the time.

Hence,

$$\Delta\epsilon_{200} = \left(\frac{200\ \mathrm{s}}{100\ \mathrm{s}}\right)^2 \Delta\epsilon_{100} = 400\ \mathrm{nm}$$

b) Correcting 10% every second corresponds to $g = 0.1$. So we calculate the emittance growth per second

$$\Delta\epsilon_0 = \left(\frac{1\ \mathrm{s}}{100\ \mathrm{s}}\right)^2 \Delta\epsilon_{100} = 0.01\ \mathrm{nm}$$

with a gain of $g = 0.1$ we then find

$$\Delta\epsilon = \Delta L_0 \frac{1}{g^2} = 1\ \mathrm{nm}$$

Solutions

3) Since the ratio of focal length to quadrupole spacing remains constant ($f/L = f_0/L_0$) the phase advance per cell remains constant. Therefore the beta-function is now twice the original value $\beta = 2\beta_0$. The multi-pulse emittance growth due the jitter for one quadrupole is

$$\Delta\epsilon \propto \frac{\beta}{f^2}\sigma_q^2$$

The emittance growth per length of the line is given by

$$\frac{\Delta\epsilon}{L} \propto \frac{\beta}{f^2 L}\sigma_q^2$$

$$\rightarrow \frac{\beta}{f^2 L}\sigma_q^2 = \frac{\beta_0}{f_0^2 L_0}\sigma_{q,0}^2$$

from the scaling we calculate

$$\sigma_q = 2\sigma_{q,0}$$