Parameter Optimisation

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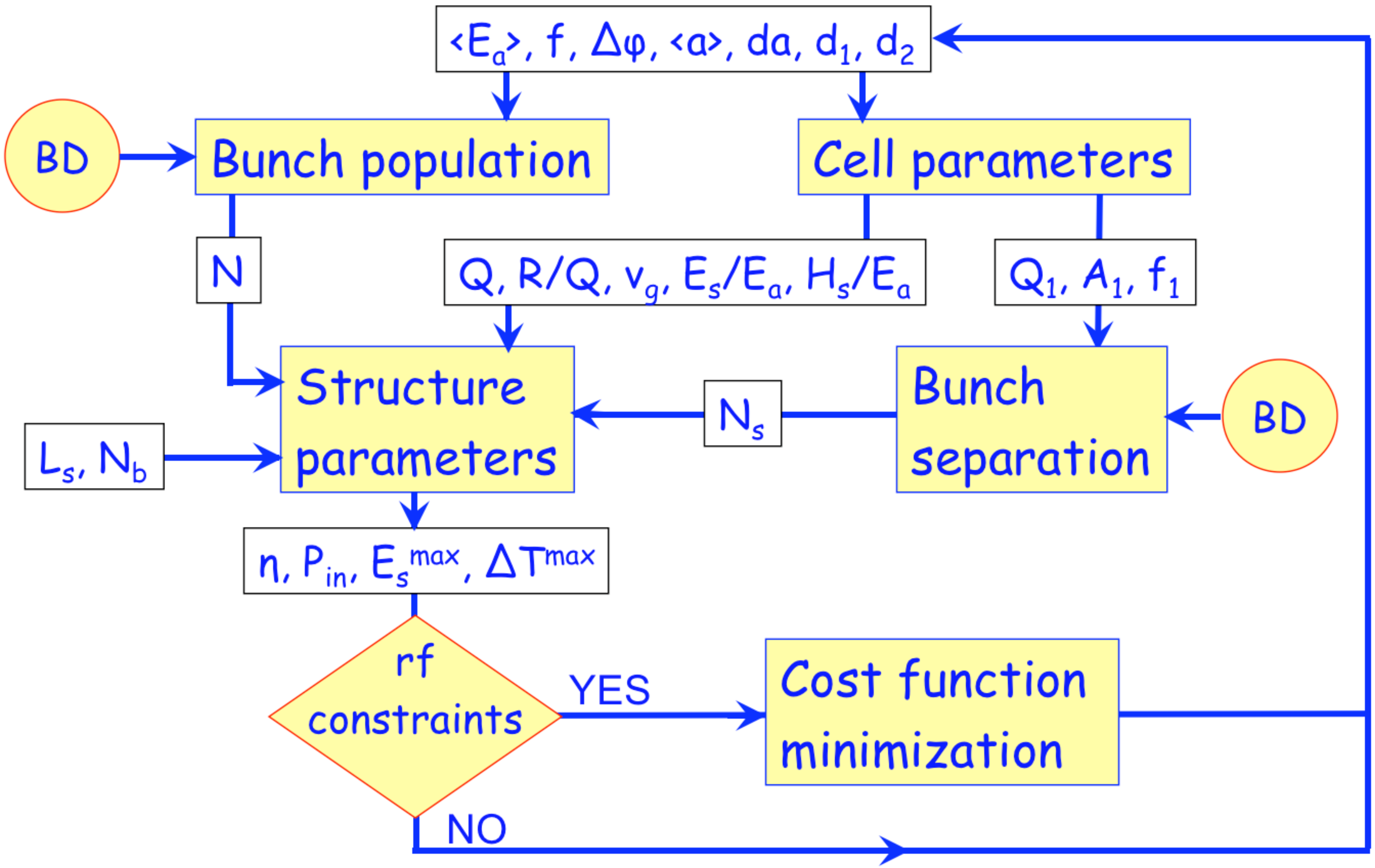
Linear Collider School, December 2013

Overview

- Parameter optimisation requires to remember the previous lectures
- We will go through the relevant steps again

Work Flow as seen by RF Expert (Alexej Grudiev)



Luminosity

Simplified treatment and approximations used throughout

$$\mathcal{L} = H_D \frac{N^2 f_{rep} n_b}{4\pi\sigma_x\sigma_y}$$

$$\mathcal{L} \propto H_D \frac{N}{\sqrt{\beta_x\epsilon_x}\sqrt{\beta_y\epsilon_y}} \eta P$$

$$\epsilon_x = \epsilon_{x,DR} + \epsilon_{x,BC} + \epsilon_{x,BDS} + \ldots$$

$$\epsilon_y = \epsilon_{y,DR} + \epsilon_{y,BC} + \epsilon_{y,linac} + \epsilon_{y,BDS} + \epsilon_{y,growth} + \epsilon_{y,offset} \ldots$$

$$\sigma_{x,y} \propto \sqrt{\beta_{x,y}\epsilon_{x,y}/\gamma}$$

$$N f_{rep} n_b \propto \eta P$$

typically $\epsilon_x \gg \epsilon_y$, $\beta_x \gg \beta_y$

Fundamental limitations from

- beam-beam: $N/\sqrt{\beta_x\epsilon_x}$, $N/\sqrt{\beta_x\epsilon_x\beta_y\epsilon_y}$
- emittance generation and preservation: $\sqrt{\beta_x\epsilon_x}$, $\sqrt{\beta_y\epsilon_y}$
- main linac RF: η

Potential Limitations

- Efficiency η:
  depends on beam current that can be transported
  Decrease bunch distance $\Rightarrow$ long-range transverse wakefields in main linac
  Increase bunch charge $\Rightarrow$ short-range transverse and longitudinal wakefields in main linac, other effects

- Horizontal beam size σ_x
  beam-beam effects, final focus system, damping ring, bunch compressors

- vertical beam size σ_y
  damping ring, main linac, beam delivery system, bunch compressor, need to collide beams, beam-beam effects

- Will try to show how to derive $L_{bx}(f, a, \sigma_a, G)$

Beam Size Limit at IP

- The vertical beam size had been $\sigma_y = 1\,\mathrm{nm}$ (BDS)

  $\Rightarrow$ challenging enough, so keep it $\Rightarrow \epsilon_y = 10\,\mathrm{nm}$

- Fundamental limit on horizontal beam size arises from beamstrahlung

  Two regimes exist depending on beamstrahlung parameter

$$\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E_0} \propto \frac{N\gamma}{(\sigma_x+\sigma_y)\sigma_z}$$

  $\Upsilon \ll 1$: classical regime, $\Upsilon \gg 1$: quantum regime

  At high energy and high luminosity $\Upsilon \gg 1$

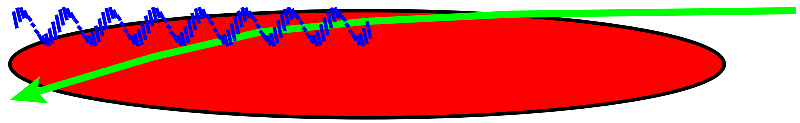
$$\mathcal{L} \propto \Upsilon\sigma_z/\gamma P\eta$$

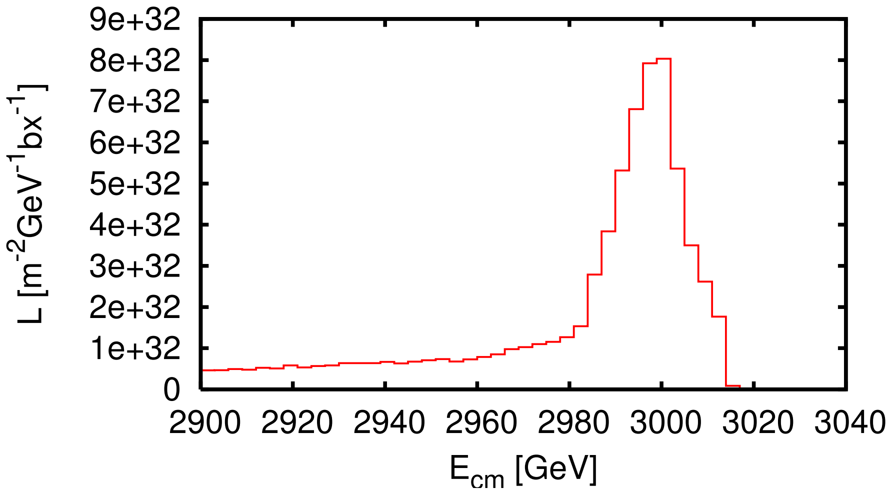
  $\Rightarrow$ partial suppression of beamstrahlung

  $\Rightarrow$ coherent pair production

  In CLIC $\langle\Upsilon\rangle \approx 6$, $N_{coh} \approx 0.1N$

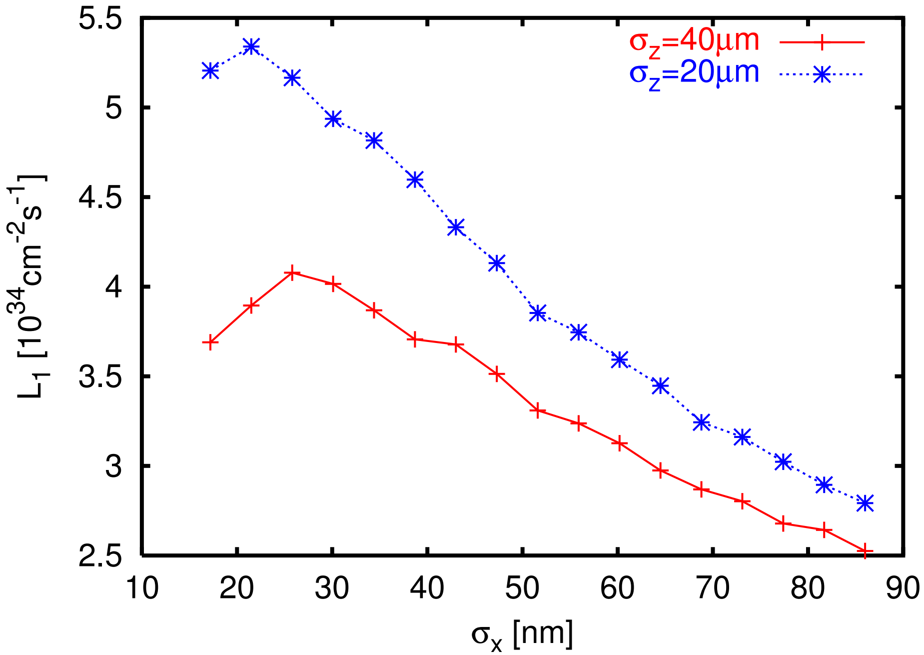
  $\Rightarrow$ somewhat in quantum regime

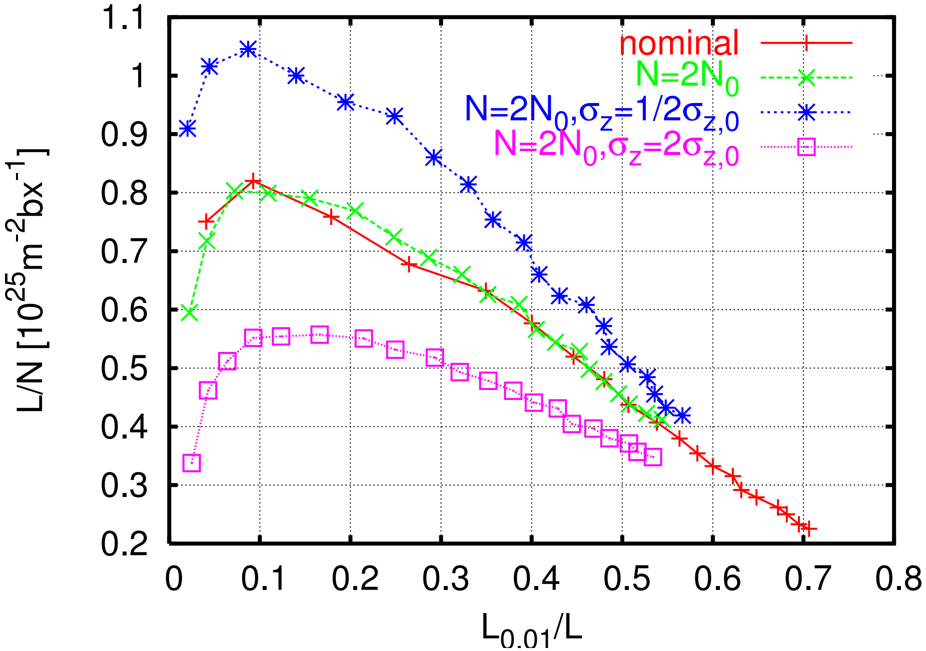




$\Rightarrow$ Use luminosity in peak as figure of merit

Luminosity Optimisation at IP





Total luminosity for $\Upsilon \gg 1$

$$\mathcal{L} \propto \frac{N}{\sigma_x}\frac{\eta}{\sigma_y} \propto \frac{n_\gamma^{3/2}}{\sqrt{\sigma_z}}\frac{\eta}{\sigma_y}$$

large $n_\gamma \Rightarrow$ higher $\mathcal{L} \Rightarrow$ degraded spectrum

chose n_γ, e.g. maximum $L_{0.01}$ or $L_{0.01}/L = 0.4$ or ...

$$\mathcal{L}_{0.01} \propto \frac{\eta}{\sqrt{\sigma_z}\sigma_y}$$

Other Beam Size Limitations

- Final focus system squeezes beams to small sizes with main problems:
  - beam has energy spread (RMS of $\approx 0.35\%$) $\Rightarrow$ avoid chromaticity
  - synchrotron radiation in bends $\Rightarrow$ use weak bends $\Rightarrow$ long system
  - radiation in final doublet (Oide Effect)
- Large $\beta_{x,y} \Rightarrow$ large nominal beam size
- Small $\beta_{x,y} \Rightarrow$ large distortions
- Beam-beam simulation of nominal case: effective $\sigma_x \approx 40\,\mathrm{nm}$, $\sigma_y \approx 1\,\mathrm{nm}$

$\Rightarrow$ lower limit of $\sigma_x \Rightarrow$ for small N optimum n_γ cannot be reached

  - new FFS reaches $\sigma_x \approx 40\,\mathrm{nm}$, $\sigma_y \approx 1\,\mathrm{nm}$

- Assume that the transverse emittances remain the same
  - not strictly true
  - emittance depends on charge in damping ring (e.g $\epsilon_x(N = 2 \times 10^9) = 450\,\mathrm{nm}$, $\epsilon_x(N = 4 \times 10^9) = 550\,\mathrm{nm}$)

Beam Dynamics Work Flow

- The parameter optimisation has been performed keeping the main linac beam dynamics tolerances at the same level as for the original 30 GHz design
- The minimum spot size at the IP is dominated by BDS and damping ring
  - adjusted N/σ_x for large bunch charges to respect beam-beam limit
- For each of the different frequencies and values of a/λ a scan in bunch charge N has been performed
  - the bunch length has been determined by requiring the final RMS energy spread to be $\sigma_E/E = 0.35\%$ and running 12° off-crest
  - the transverse wake-kick at $2\sigma_z$ has been determined
  - the bunch charge which gave the same kick as the old parameters has been chosen
- The wakefields have been calculated using some formulae from K. Bane
  - used them partly outside range of validity

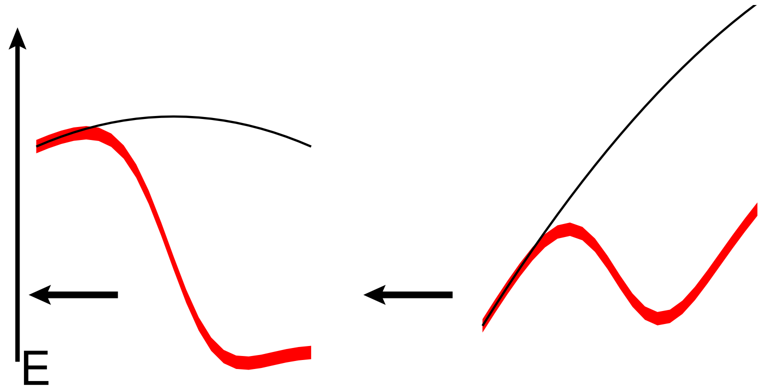
    $\Rightarrow$ but still a good approximation, confirmed by RF experts

$\Rightarrow$ N and $L_{bx}(f, a, \sigma_a, G)$ given to RF experts

Beam Loading and Bunch Length

- Aim for shortest possible bunch (wakefields)
- Energy spread into the beam delivery system should be limited to about 1% full width or 0.35% RMS
- Multi-bunch beam loading compensated by RF
- Single bunch longitudinal wakefield needs to be compensated

  $\Rightarrow$ accelerate off-crest



- Limit around average $\Delta\Phi \leq 12^{\circ}$

  $\Rightarrow \sigma_z = 44\,\mu\mathrm{m}$ for $N = 3.72 \times 10$

Specific Wakefields

- Longitudinal wakefields contain more than the fundamental mode
- We will use wakefields based on fits derived by Karl Bane

  l length of the cell

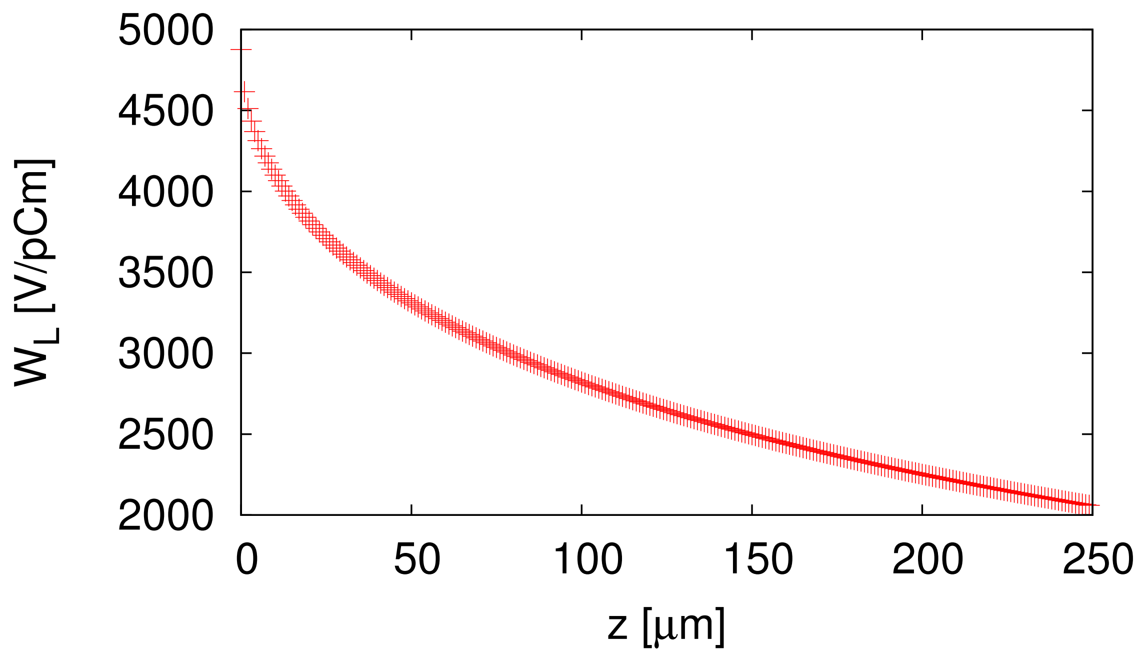
  a radius of the iris aperture

  g length between irises

$$s_0 = 0.41 a^{1.8} g^{1.6} \left(\frac{1}{l}\right)^{2.4}$$

$$W_L = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

- Use CLIC structure parameters



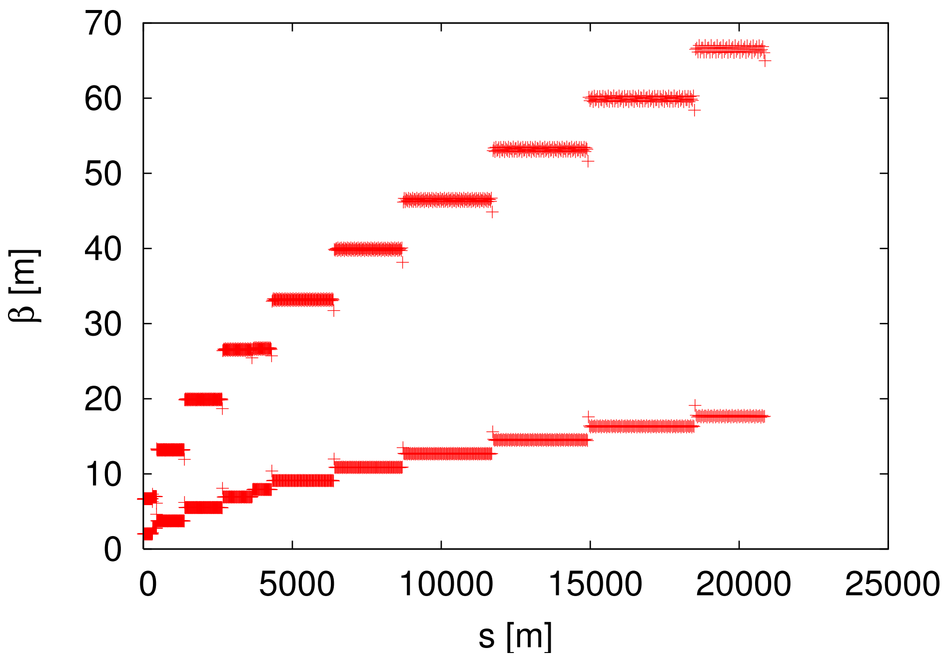
- Summation of an infinite number of cosine-like modes
  - calculation in time domain or approximations for high frequency modes

Recipe for Choosing the Bunch Parameters

- Decide on the average RF phase
  - OK, we fix 12°
- Decide on an acceptable energy spread at the end of the linac
  - OK, we chose 0.35%
- Determine $\sigma_z(N)$
  - chose a bunch charge
  - vary the bunch length until the final energy spread is acceptable
  - chose next charge
- Determine which bunch charge (and corresponding bunch length) can be transported stably

CLIC Lattice Design

- Used $\beta \propto \sqrt{E}$, $\Delta\Phi = \mathrm{const}$
  - balances wakes and dispersion
  - roughly constant fill factor
  - phase advance is chosen to balance between wakefield and ground motion effects
- Preliminary lattice
  - made for $N = 3.7 \times 10^9$
  - quadrupole dimensions need to be confirmed
  - some optimisations remain to be done
- Total length 20867.6m
  - fill factor 78.6%



- 12 different sectors used
- Matching between sectors using 7 quadrupoles to allow for some energy bandwidth

CLIC Fill Factor

- Want to achieve a constant fill factor

  - to use all drive beams efficiently

- Scaling $f = f_0\sqrt{E/E_0}$ yields

$$L_q \propto \frac{E}{\sqrt{\frac{E}{E_0}}} \propto \sqrt{E}$$

using a quadrupole spacing of $L = L_0\sqrt{E/E_0}$ leads to

$$\frac{L_q}{L} \propto \frac{\sqrt{E}}{\sqrt{E}} \propto \mathrm{const}$$

⇒ The choice allows to maintain a roughly constant fill factor

⇒ It maximises the focal strength along the machine

Magnet Considerations

- The maximum strength of a focusing magnet is limited
  - for a normal conducting design rule of thumb is $1\,\mathrm{T}$ at the poletip

$\Rightarrow$ Required integrated magnet strength is

$$\frac{\mathrm{T}}{\mathrm{m}}\frac{E}{0.3\,\mathrm{GeV}}\frac{\mathrm{m}}{f}$$

- For CLIC poletip radius is given by practical considerations of magnet design $a \approx 5\,\mathrm{mm}$ yielding a gradient of $200\,\mathrm{T/m}$
- We chose about 10% of the machine to be quadrupoles

  $\Rightarrow$ fill factor is $\approx 80\%$
  - 10% are lost for flanges (mainly on structures)
- Use $L_0 = 1.5\,\mathrm{m}$ and $f_0 = 1.3\,\mathrm{m}$ yields

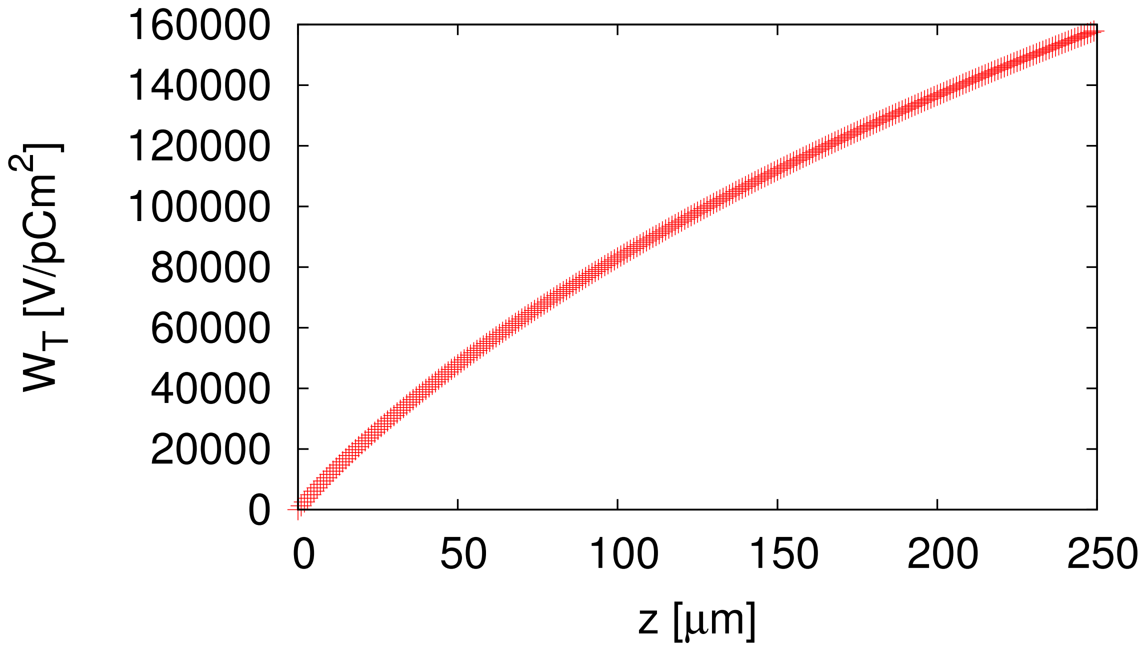
$$\eta_q = \frac{E_0}{0.3\,\mathrm{GeV}}\frac{\mathrm{T/m}}{200\,\mathrm{T/m^2}}\frac{\mathrm{m}}{f_0}\frac{1}{L_0}$$

$$\Rightarrow \eta_q \approx 7.7\%$$

- We use discrete lengths hence we loose a bit more

Example of a Transverse Wakefield (CLIC)

Fit obtained by K. Bane
For short distances the wakefield rises linear
Summation of an infinite number of sine-like modes with different frequencies



$$s_0 = 0.169 a^{1.79} g^{0.38} \left(\frac{1}{l}\right)^{1.17}$$

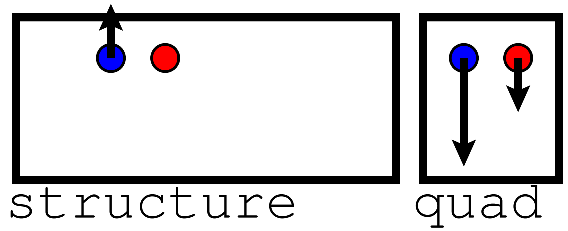
$$w_\perp(z) = 4\frac{Z_0 c s_0}{\pi a^4}\left[1 - \left(1 + \sqrt{\frac{z}{s_0}}\right)\exp\left(-\sqrt{\frac{z}{s_0}}\right)\right]$$

$$w_\perp(z) \approx 4\frac{Z_0 c s_0}{\pi a^4}\left[1 - \left(1 + \sqrt{\frac{z}{s_0}}\right)\left(1 - \sqrt{\frac{z}{s_0}}\right)\right] = 4\frac{Z_0 c s_0}{\pi a^4}\left[1 - \left(1 - \frac{z}{s_0}\right)\right] = 4\frac{Z_0 c z}{\pi a^4}$$

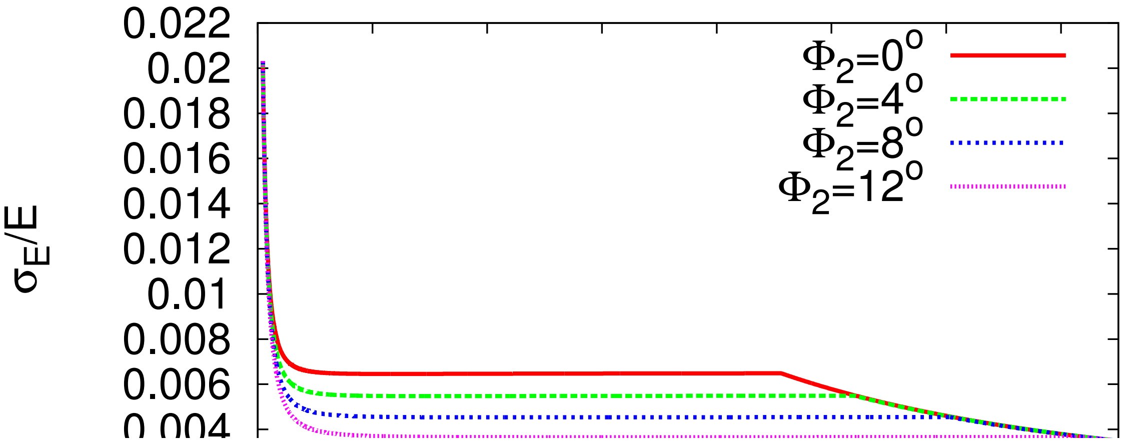
Energy Spread and Beam Stability

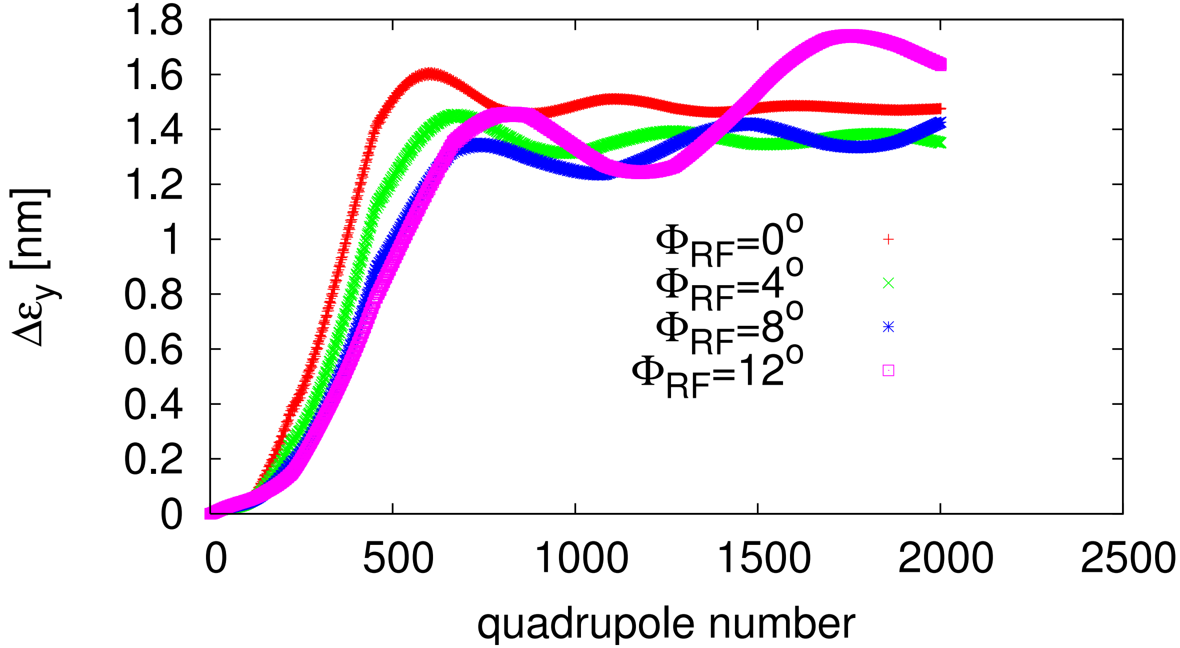
- Trade-off in fixed lattice
  - large energy spread is more stable
  - small energy spread is better for alignment

⇒ Beam with $N = 3.7 \times 10^9$ can be stable



⇒ Tolerances are not a unique number





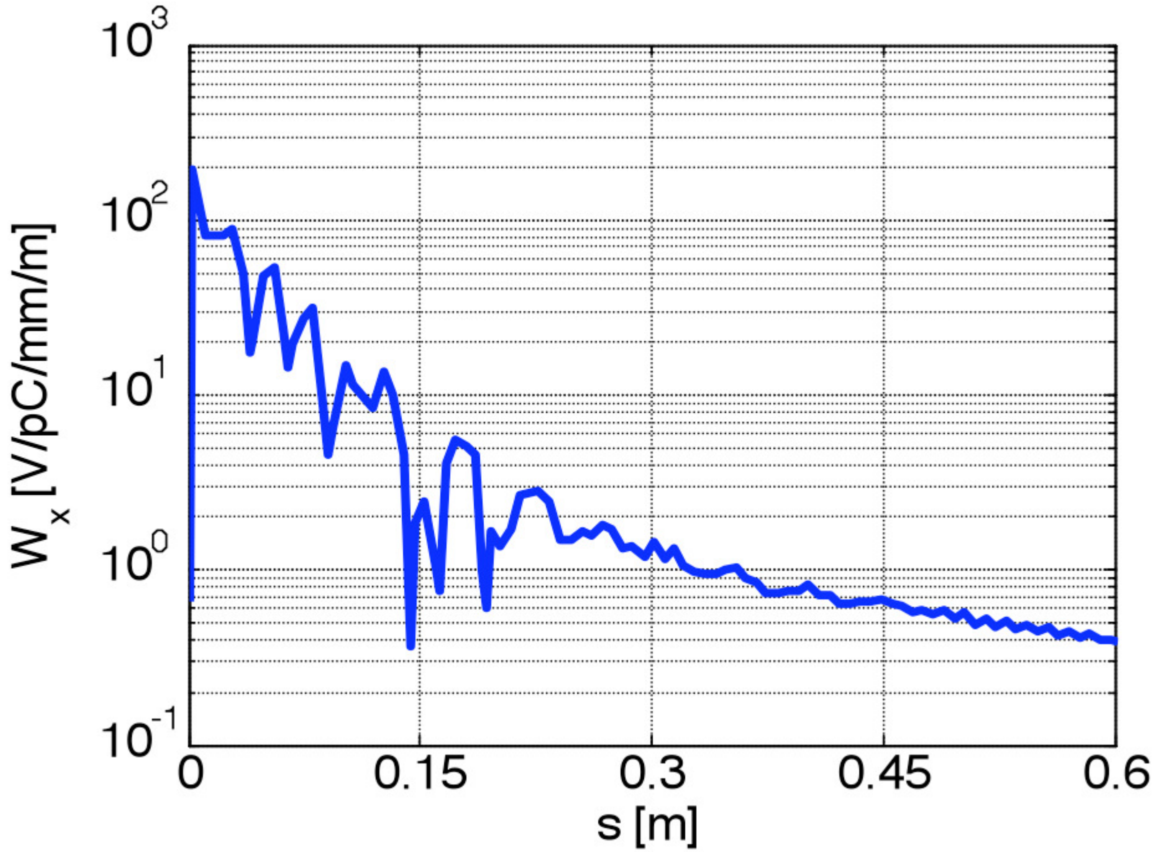
Remember: Multi-Bunch Wakefields

- Long-range transverse wakefields have the form

$$W_\perp(z) = \sum_i^\infty 2k_i \sin\left(2\pi\frac{z}{\lambda_i}\right)\exp\left(-\frac{\pi z}{\lambda_i Q_i}\right)$$

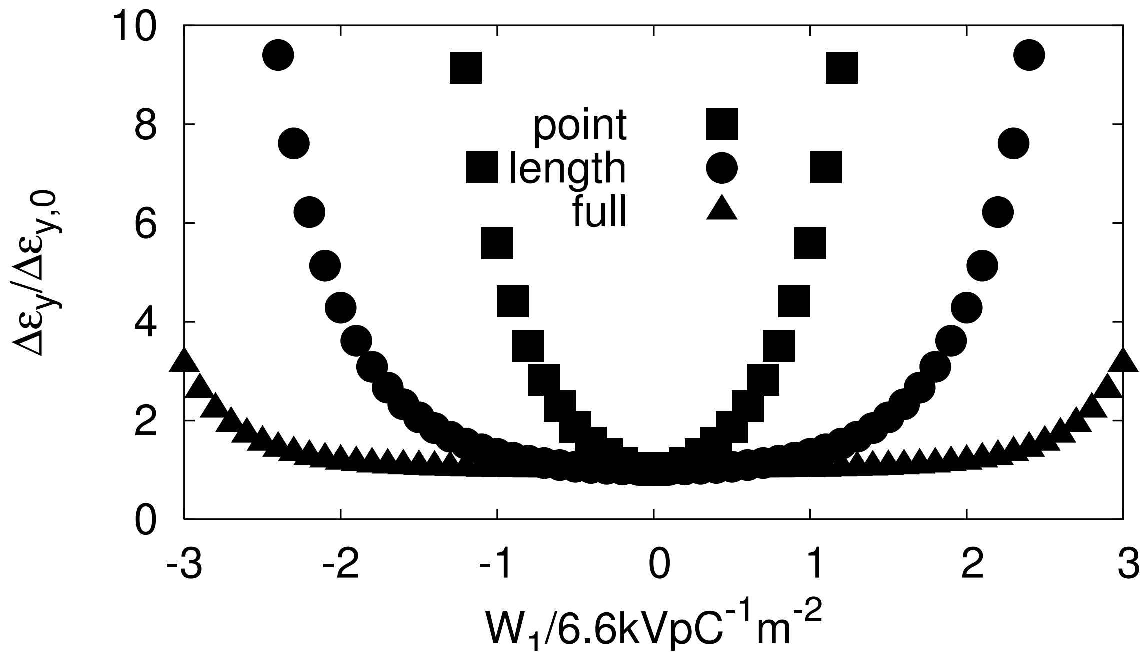
- In practice need to consider only a limited number of modes

- There impact can be reduced by detuning and damping



Multi-Bunch Jitter

- If bunches are not point-like the results change
  - an energy spread leads to a more stable case
- Simulations show
  - point-like bunches
  - bunches with energy spread due to bunch length
  - including also initial energy spread



⇒ Point-like bunches is a pessimistic assumption for the dynamic effects

Final Emittance Growth (CLIC)

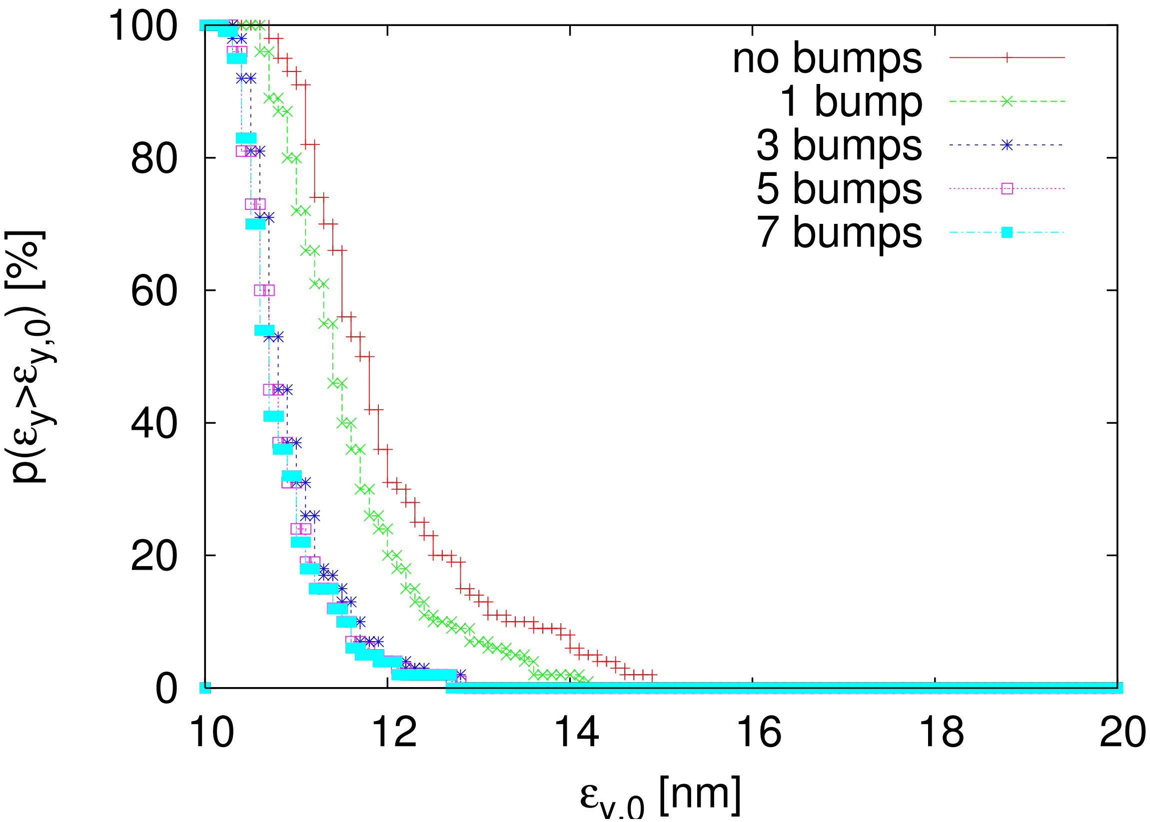
imperfection	with respect to	symbol	value	emitt. growth
BPM offset	wire reference	σ_{BPM}	14 μm	0.367 nm
BPM resolution		σ_{res}	0.1 μm	0.04 nm
accelerating structure offset	girder axis	σ_4	10 μm	0.03 nm
accelerating structure tilt	girder axis	σ_t	200 μradian	0.38 nm
articulation point offset	wire reference	σ_5	12 μm	0.1 nm
girder end point	articulation point	σ_6	5 μm	0.02 nm
wake monitor	structure centre	σ_7	5 μm	0.54 nm
quadrupole roll	longitudinal axis	σ_r	100 μradian	$\approx$ 0.12 nm

- Selected a good DFS implementation

  - trade-offs are possible

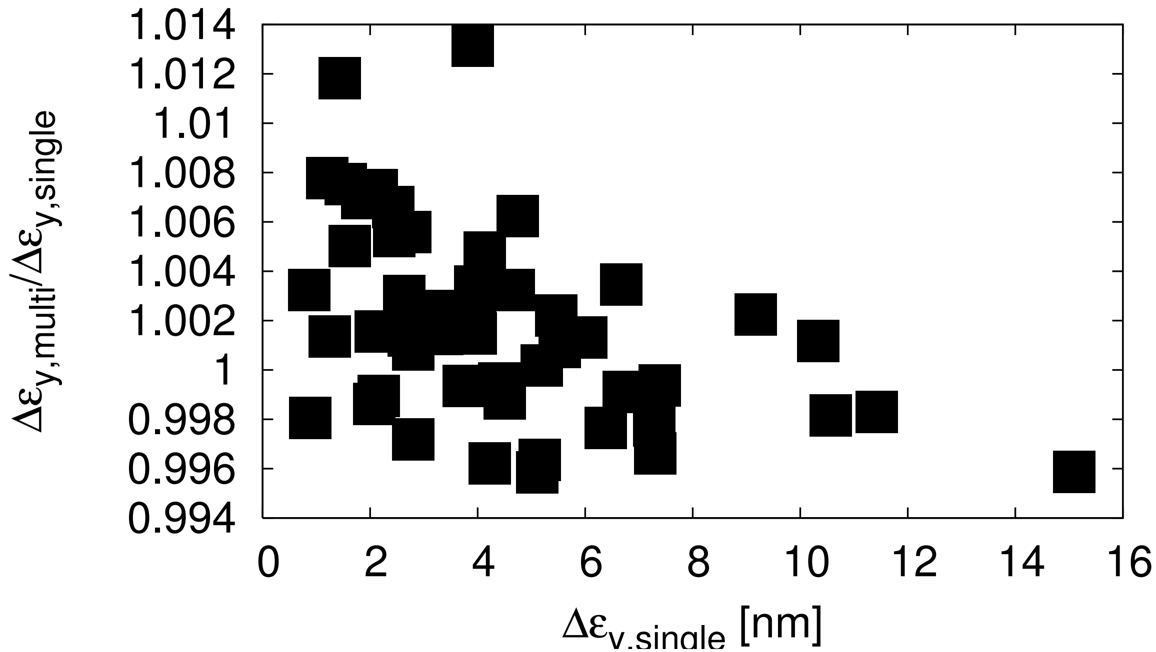
- Multi-bunch wakefield misalignments of $10\,\mu$m lead to $\Delta\epsilon_y \approx 0.13\,$nm

- Performance of local prealignment is acceptable



Multi-Bunch Static Imperfections

- In CLIC
  - we misalign all structures
  - perform one-to-one steering with a multi-bunch beam
  - perform one-to-one steering with a single bunch
  - compare the emittance growth



CLIC Example of Fast Imperfection Tolerances

- Many sources exist

Source	Luminosity budget	Tolerance
Damping ring extraction jitter	1%	
Magnetic field variations	?%	
Bunch compressor jitter	1%	
Quadrupole jitter in main linac	1%	$\Delta\epsilon_y = 0.4\,\mathrm{nm}$ $\sigma_{jitter} \approx 1.8\,\mathrm{nm}$
Structure pos. jitter in main linac	0.1%	$\Delta\epsilon_y = 0.04\,\mathrm{nm}$ $\sigma_{jitter} \approx 800\,\mathrm{nm}$
Structure angle jitter in main linac	0.1%	$\Delta\epsilon_y = 0.04\,\mathrm{nm}$ $\sigma_{jitter} \approx 400\,\mathrm{nradian}$
RF jitter in main linac	1%	
Crab cavity phase jitter	1%	$\sigma_\phi \approx 0.01^\circ$
Final doublet quadrupole jitter	1%	$\sigma_{jitter} \approx 0.1\,\mathrm{nm}$
Other quadrupole jitter in BDS	1%	
...	?%	

RF Constraints

- To limit the breakdown rate and the severeness of the breakdowns
- The maximum surface field has to be limited

$$\hat{E} < 260\,\mathrm{MV/m}$$

- The temperature rise at the surface needs to be limited

$$\Delta T < 56\,\mathrm{K}$$

- The power flow needs to be limited

  - related to the badness of a breakdown

  empirical parameter is

$$P/(2\pi a)\tau^{\frac{1}{3}} < 18\,\frac{\mathrm{MW}}{\mathrm{mm}}\mathrm{ns}^{\frac{1}{3}}$$

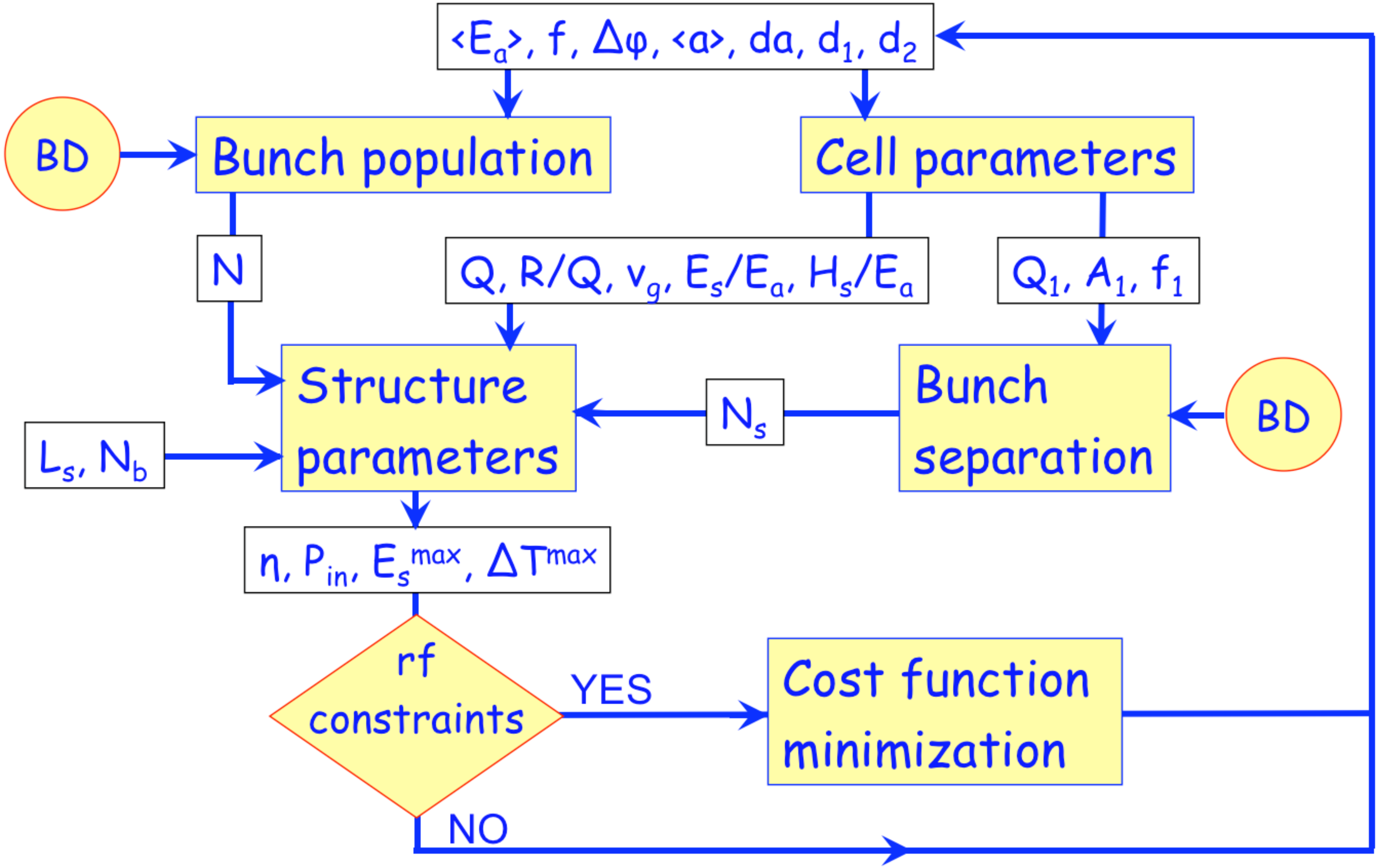
RF Work Flow

- Calculate RF properties of cells with different a/λ
  - structures can be constructed by interpolating between these values
- Remove all structures with a too high surface field
- Determine the pulse length supported by the structure
- Estimate long-range wake and chose bunch distance
  - bunch charge is given by beam dynamics
- Calculate RF to beam efficiency for the structure

Cost Model

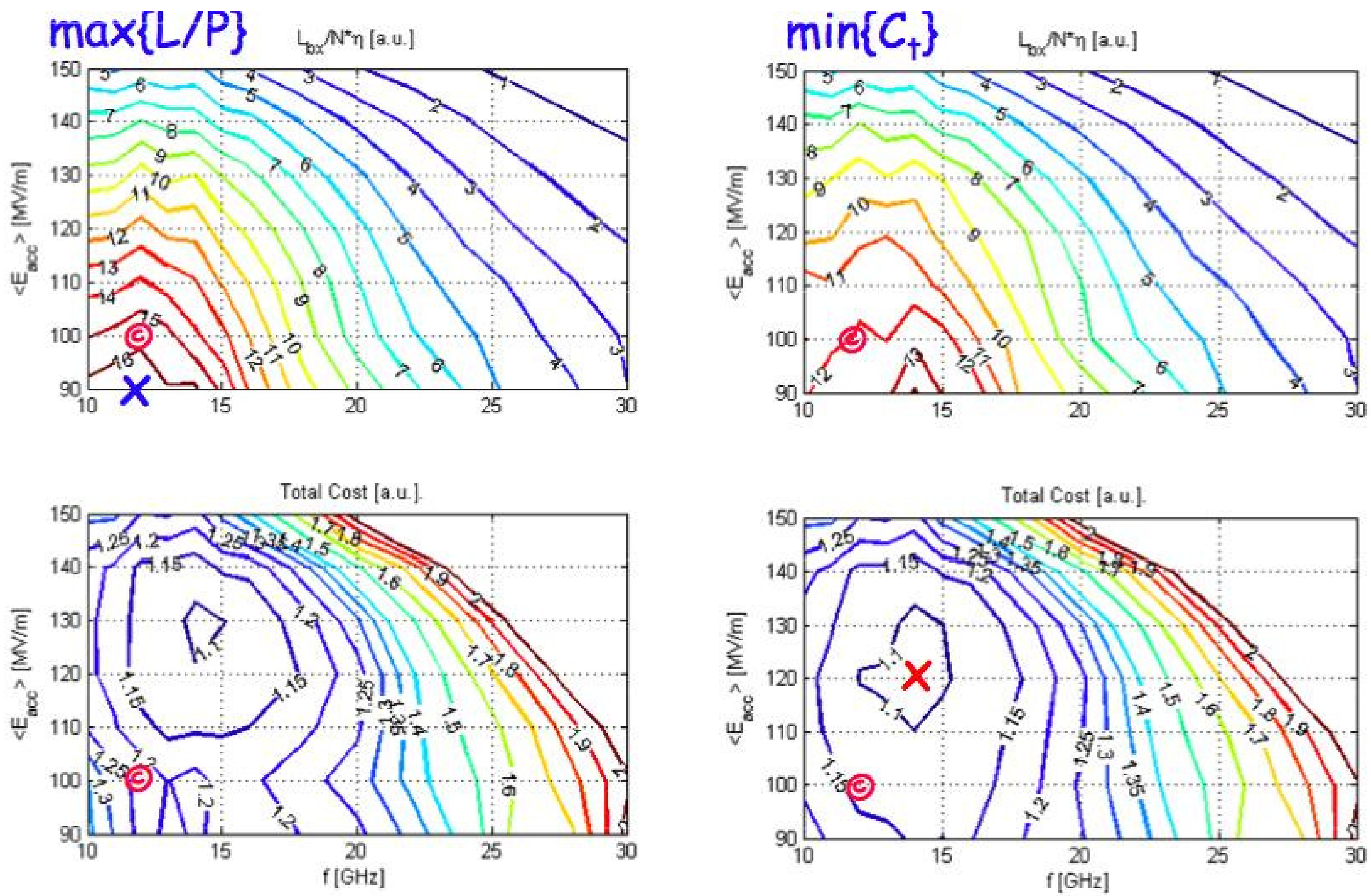
- The machine should be optimised for lowest cost
  - power consumption will also limit the choice
- A simplified cost model can den developed
  - e.g. cost per unit length of linac
  - energy to be stored in drive beam accelerator modulators per pulse
  - ...
- With this model one can identify the cheapest machine

Work Flow



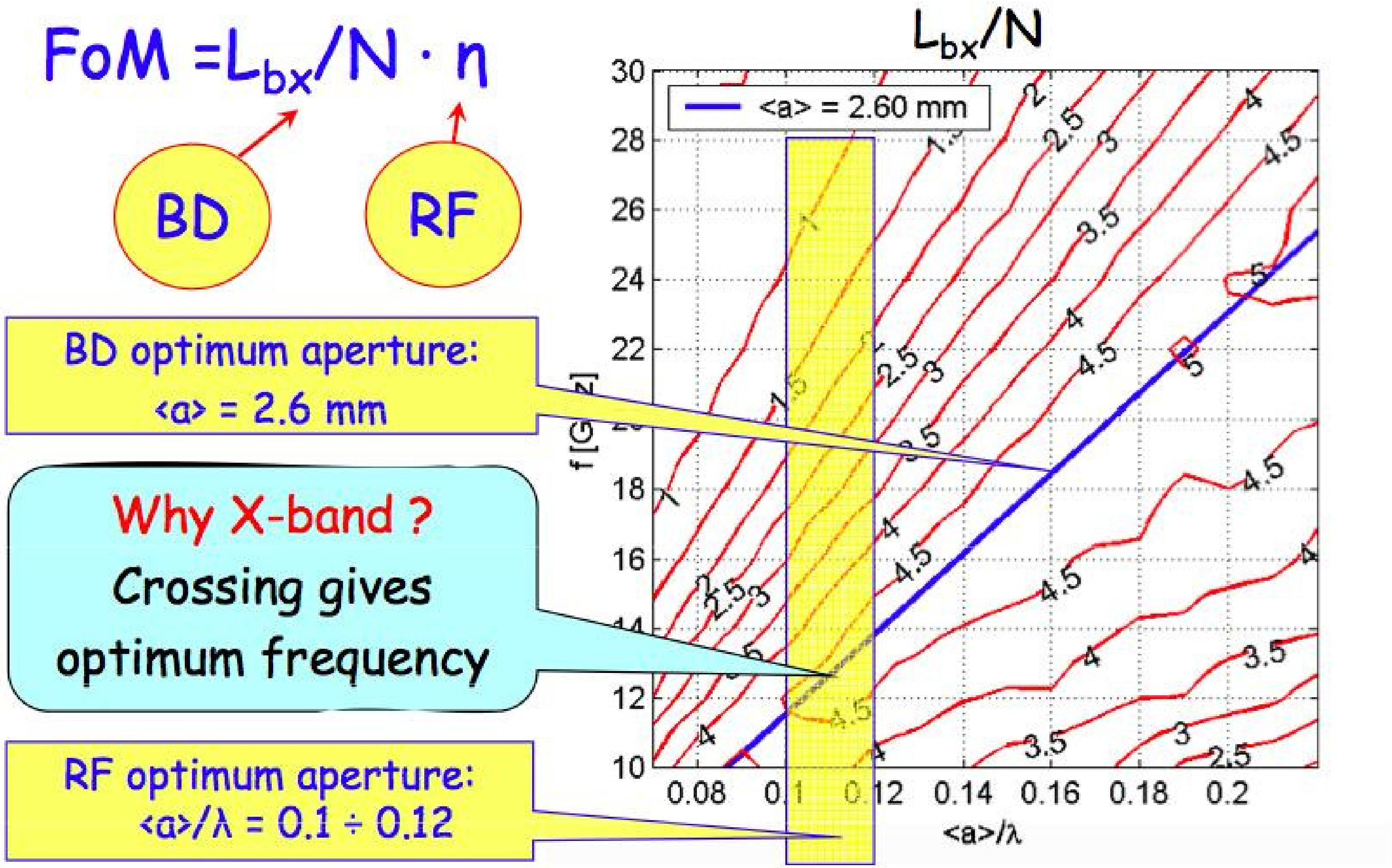
<Ea>, f, Δφ, <a>, da, d1, d2
BD
Bunch population
Cell parameters
N
Q, R/Q, vg, Es/Ea, Hs/Ea
Q1, A1, f1
Structure parameters
Ns
Bunch separation
BD
Ls, Nb
η, Pin, Es max, ΔTmax
rf constraints
YES
Cost function minimization
NO

Results



max{L/P}
min{C_t}
L_bx/N*η [a.u.]
L_bx/N*η [a.u.]
Total Cost [a.u.]
Total Cost [a.u.]
<E_acc> [MV/m]
f [GHz]

Results 2

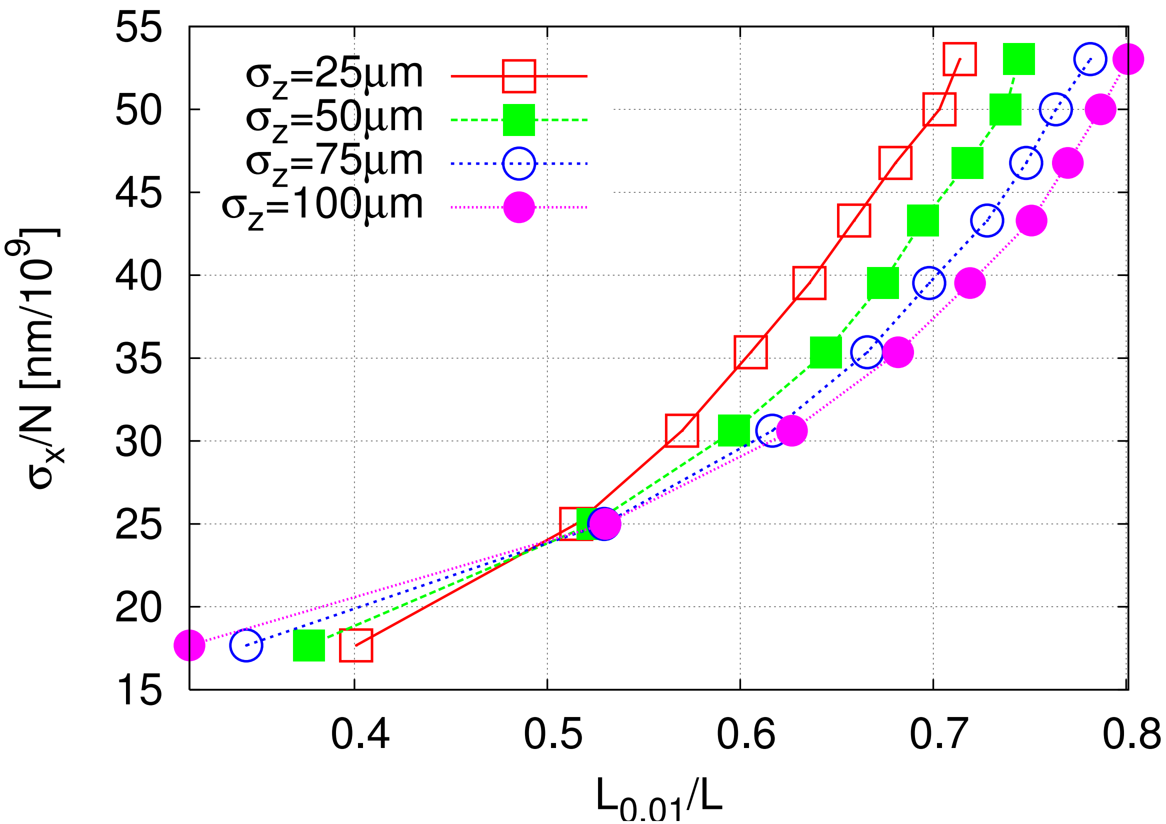

FoM =Lbx/N · η
BD
RF
BD optimum aperture:
<a> = 2.6 mm
Why X-band ?
Crossing gives
optimum frequency
RF optimum aperture:
<a>/λ = 0.1 ÷ 0.12
Lbx/N
<a> = 2.60 mm
f [GHz]
<a>/λ
0.08
0.1
0.12
0.14
0.16
0.18
0.2
10
12
16
18
22
24
26
28
30

Lattice at Lower Energy

Required Beam Size (CLIC 500GeV)

- Roughly constant luminosity spectrum quality for constant N/σ_x
- Required is beam size is between 25 and 40 nm for beam with $N = 10^9$ particles
  - scales with the square of the charge
- For $\beta_x = 10\,\mathrm{mm}$ and $N = 4\times10^9$ requires $\epsilon_x \approx 1\,\mu\mathrm{m}$

$$\epsilon_{x,opt} \approx \left(\frac{N}{4\times10^9}\right)^2 \frac{10\,\mathrm{mm}}{\beta_x}\,\mu\mathrm{m}$$



Relative Luminosity

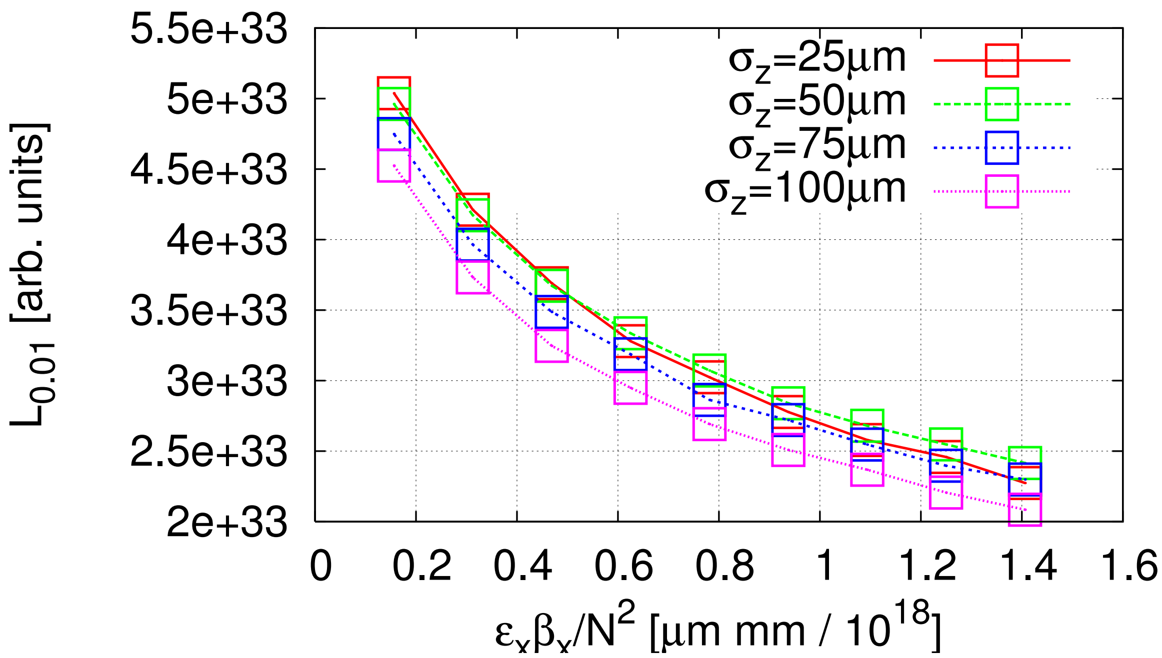
- Relevant parameter is

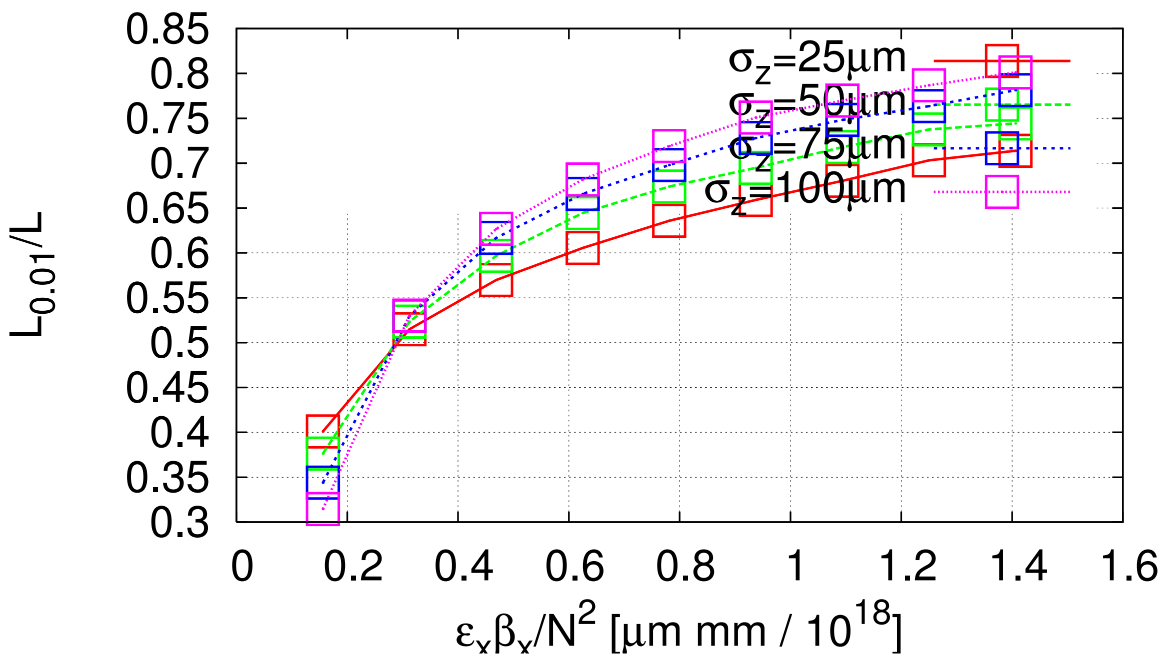
$$D = \frac{\beta_x}{\mathrm{mm}} \frac{\epsilon_x}{\mu\mathrm{m}} \left(\frac{10^9}{N}\right)^2$$

$$\frac{L_{bx}}{N} \propto \frac{1}{\sqrt{D}}$$

- Require this value to be in the range 0.3–0.7
  - $\approx 30\%$ more luminosity for lower value
- NLC had $N = 7.5\times10^9$ $\beta_x = 10\,\mathrm{mm}$ and $\epsilon_x = 4\,\mu\mathrm{m}$
  - $D = 0.7$

$\Rightarrow$ close to optimum





Beam Jitter at Lower Energy

- Two main limitations
  - local beam stability
  - integrated residual effect along the machine
- To keep the local beam stability constant yields the limitation
  - $Nw_\perp(2\sigma_z) = \mathrm{const}$
  - keeps the beam energy spread constant
- A second limitation arises from the integral effect

$$\frac{d}{ds}\frac{\Delta y'/\sigma'_y}{y/\sigma_y} \propto \frac{Nw_\perp \sigma_y}{E\sigma'_y}$$

  - Integral using lattice scaling $\beta = \beta_0\sqrt{E(s)/E_0}$ yields

$$\frac{\Delta y'/\sigma'_y}{y/\sigma_y} \propto \frac{Nw_\perp \beta_0}{G}\sqrt{\frac{E_f}{E_0}}$$

- $Nw_\perp(2\sigma_z) = \mathrm{const}$ is stronger limitation as long as
  - $G \geq \sqrt{E_f/E_{f,0}}G_0$
  - For 500 GeV $G \geq 41\,\mathrm{MV/m}$

Emittance Growth at Lower Energy

- Express structure induced emittance growth as function of energy and gradient

$$\frac{d}{ds}\frac{\Delta\epsilon(s)}{\epsilon} \propto \left(\frac{Nw_\perp(2\sigma_z)\Delta y L_{cav}}{E(s)}\frac{1}{\sigma'_y(s)}\right)^2 \frac{1}{L_{cav}}$$

using the lattice scaling $\beta = \beta_0\sqrt{E(s)/E_0}$ one finds

$$\Delta\epsilon_{cav} \propto \frac{N^2 w_\perp^2(2\sigma_z)\Delta y^2 \beta_0 L_{tot,cav}}{G}\sqrt{\frac{E_f}{E_0}}$$

$\Rightarrow$ Could increase $Nw_\perp(2\sigma_z)$ by factor 2.4 at 500 GeV

- for constant gradient

- For constant $Nw_\perp$ and L_{cav} we find $G \geq 41\,\mathrm{MV/m}$
- For constant $Nw_\perp$ and doubled L_{cav} we find $G \geq 82\,\mathrm{MV/m}$
  - but for $G = 50\,\mathrm{MV/m}$ still only 1.6 times as high as at $3\,\mathrm{TeV}$
- Dispersive emittance growth scales as

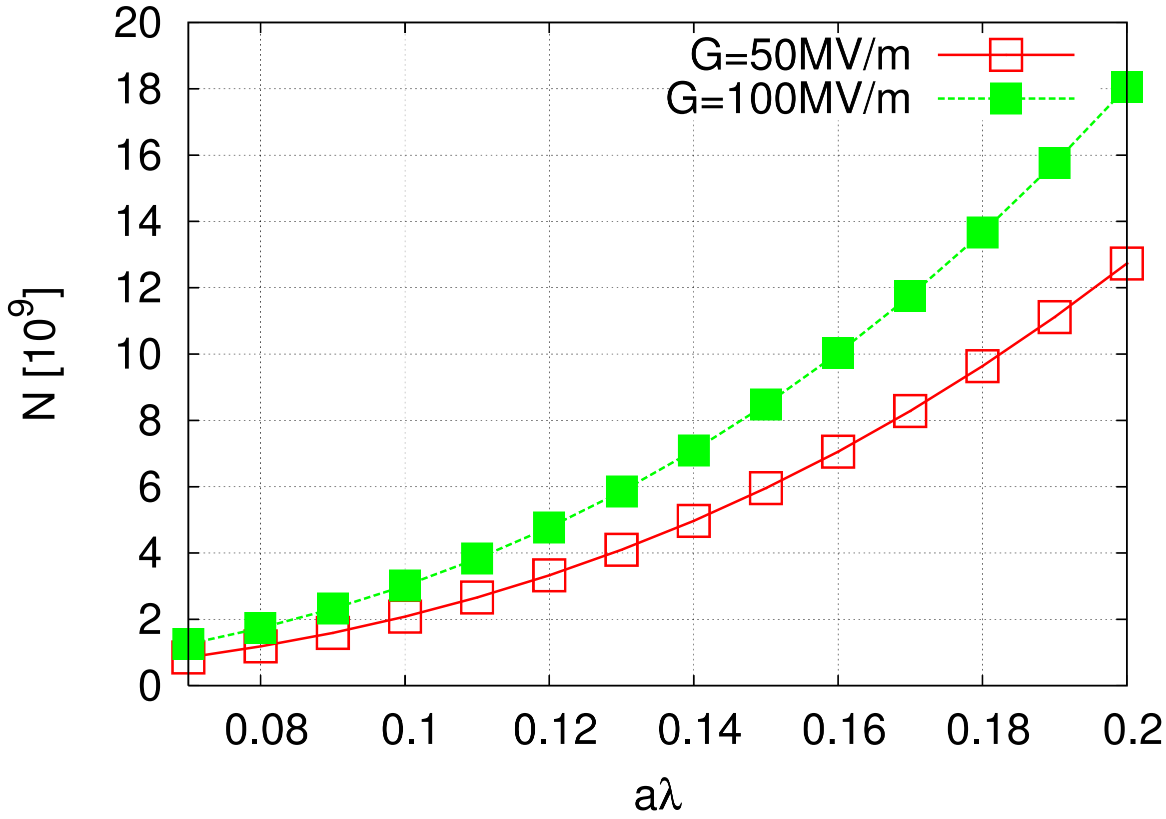
$$\Delta\epsilon_{tot,disp} \propto \frac{\Delta E^2 \Delta y^2}{G}\sqrt{\frac{E_f}{E_0}}$$

$\Rightarrow$ independent of structure length

- Total emittance growth should not increase much, first simulations confirm this

Aperture and Bunch Charge

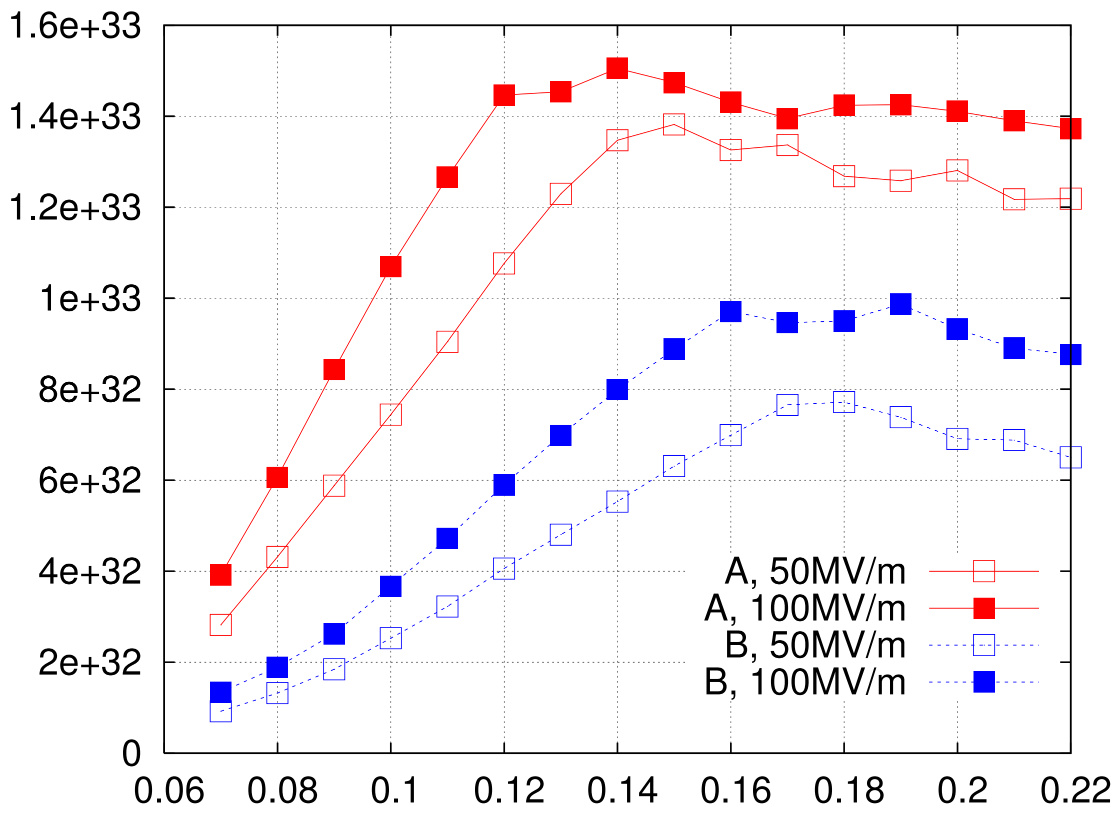
- Procedure is similar to the one for 3 TeV
  - $\sigma_y(N)$ from single bunch longitudinal wake
  - N, σ_z from transverse single bunch wake
- Keep local beam stability constant
  - leads to less bunch charge than for 3 TeV
  - but longer bunches



Luminosity

Assume the following

- case A
  - emittance from 3 TeV
  - beta-functions of $\beta_x = 10\,\mathrm{mm}$ and $\beta_y = 0.1\,\mathrm{mm}$ at the interaction point
- case B
  - horizontal emittance from $\epsilon_x = 3\,\mu\mathrm{m}$ at the damping ring to $\epsilon_x = 4\,\mu\mathrm{m}$ at the interaction point
  - vertical emittance from $\epsilon_y = 10\,\mathrm{nm}$ at the damping ring to $\epsilon_y = 40\,\mathrm{nm}$ at the interaction point
  - beta-functions of $\beta_x = 8\,\mathrm{mm}$ and $\beta_y = 0.1\,\mathrm{mm}$ at the interaction point



Summary

- You had a glimpse on the most important main linac topics
- To really understand experiments are nice
  - a cheap way is to use a simulation code
  - and play with it

Thanks

Many thanks to you for listening (I hope) and to those who helped prearing lecture