Course B: Superconductive RF

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LC school 2013

5 - 15 Dec. 2013, Antalya, Turkey

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RF Cavity Fundamental

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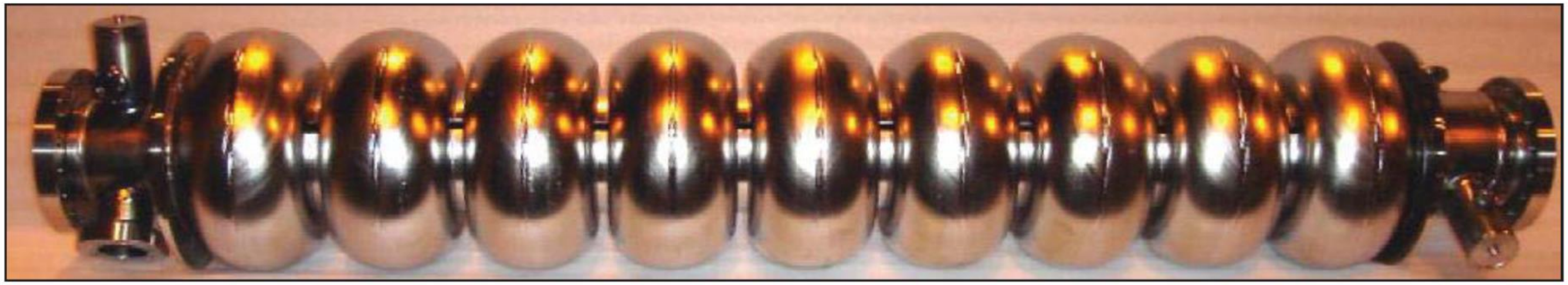
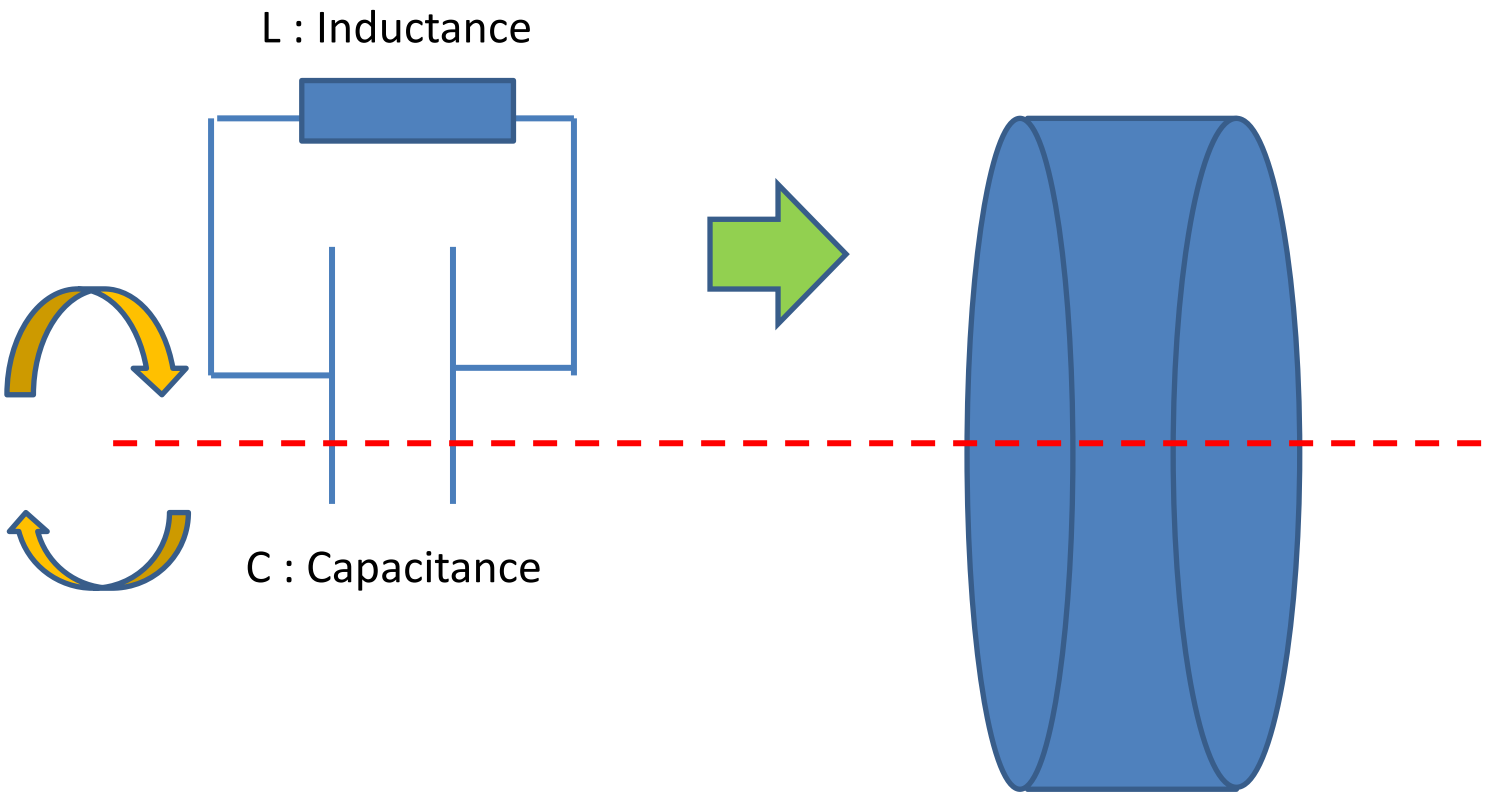
1.3 GHz elliptical 9-cell cavity



Pill Box Cavity



Pill Box Cavity

Hollow right cylindrical enclosure

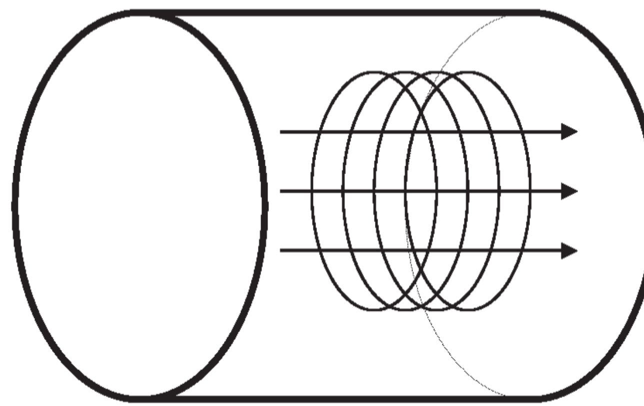
Operated in the TM_{010} mode $H_z = 0$

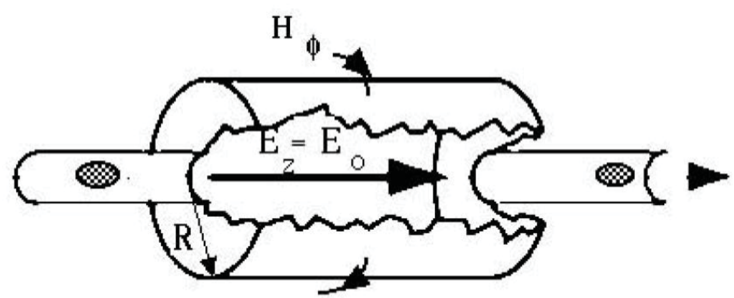
$$\frac{\partial^2 E_z}{\partial^2 r} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial^2 t} \qquad \omega_0 = \frac{2.405c}{R}$$

$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

TM_{010} mode





Modes in pill-box cavity

- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length
- TM_{0mn}
  - Monopoles modes that can couple to the beam and exchange energy
- TM_{1mn}
  - Dipole modes that can deflect the beam
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam

TM-modes in pill-box cavity

$$\frac{E_r}{E_0} = -\frac{n\pi}{x_{lm}}\frac{R}{L}J_l'\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{E_\varphi}{E_0} = \frac{ln\pi}{x_{lm}^2}\frac{R^2}{rL}J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

$$\frac{E_z}{E_0} = J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\omega_{lmn} = c\sqrt{\left(\frac{x_{lm}}{R}\right)^2 + \left(\frac{\pi n}{L}\right)^2}$$

$$\frac{H_r}{E_0} = -i\omega\varepsilon\frac{l}{x_{lm}^2}\frac{R^2}{r}J_l\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

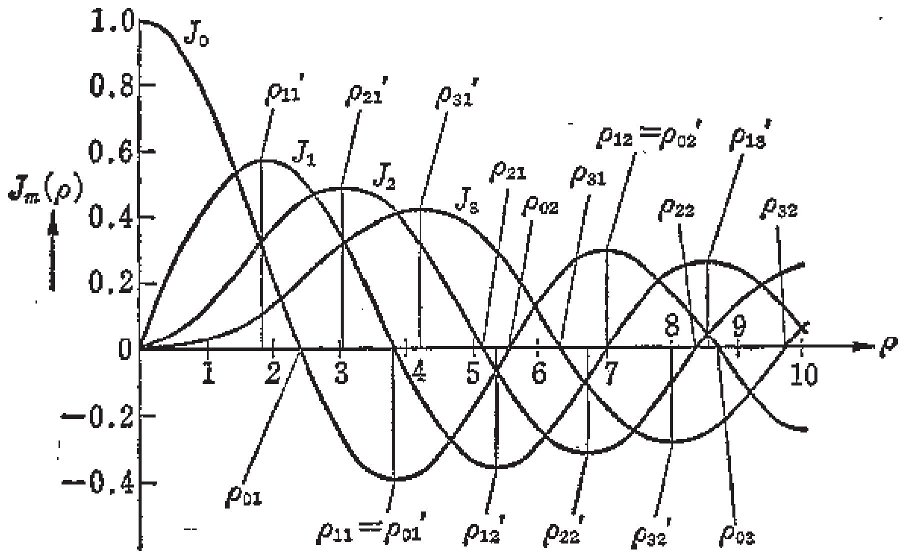
x_{lm} is the mth root of $J_l(x)$

Bessel function

$$\frac{H_\varphi}{E_0} = -i\omega\varepsilon\frac{R}{x_{lm}}J_l'\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{H_z}{E_0} = 0$$

Bessel function



Bessel function

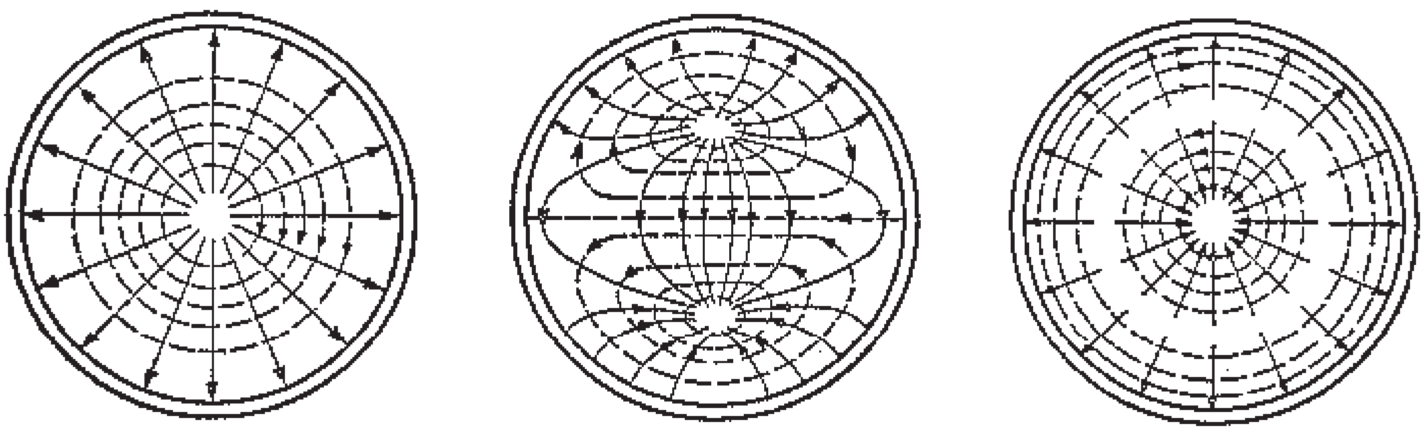
Root of Bessel function

n \ m	0	1	2	3
1	3.8317	1.8412	3.0542	4.2012
2	7.0156	5.3314	6.7061	8.0152
3	10.1735	8.5363	9.9695	11.3459
4	13.3237	11.7060	13.1704	14.5858

TM-modes

Electric field →

Magnetic field - - - →

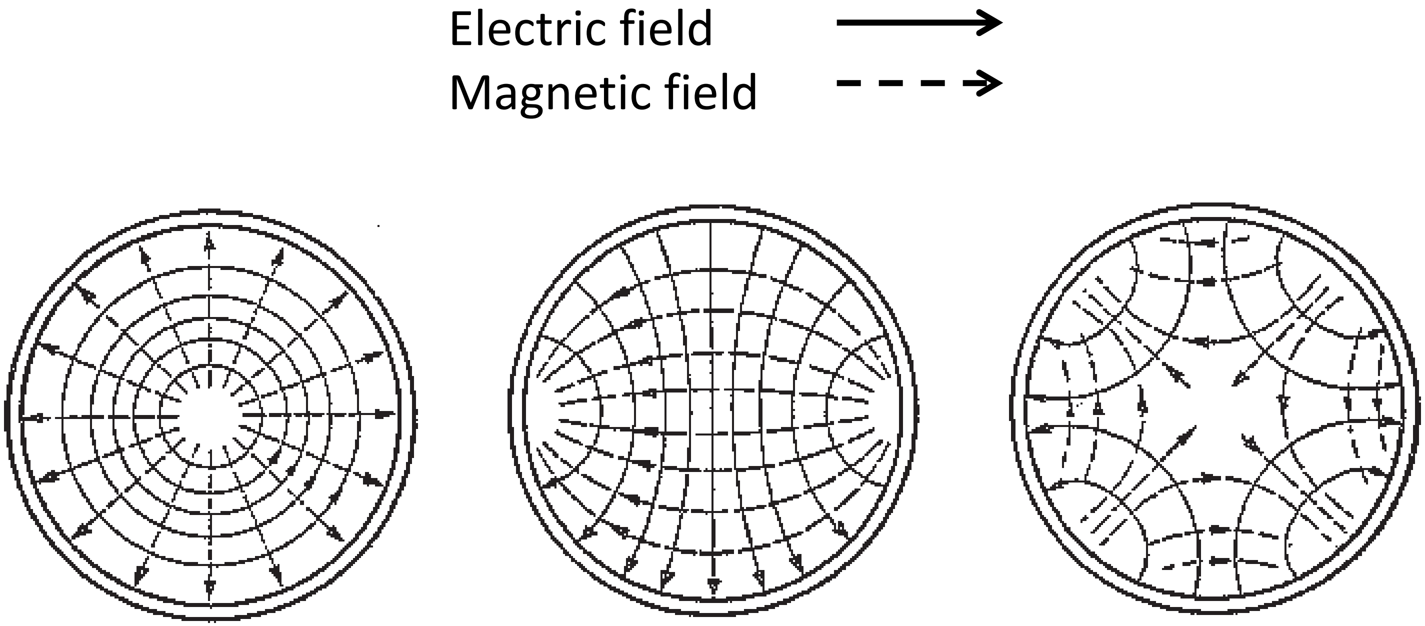


(a) TM_{01} (b) TM_{11} (c) TM_{02}

TE-modes

Electric field

Magnetic field



(a) TE_{01} (b) TE_{11} (c) TE_{21}

TM_{010} mode in a pill-box cavity

$$E_r = E_\varphi = 0 \qquad E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right)$$

$$H_r = H_z = 0 \qquad H_\varphi = -i\omega\varepsilon E_0 \frac{R}{x_{01}} J_1\left(x_{01}\frac{r}{R}\right)$$

$$\omega = x_{01}\frac{c}{R} \qquad x_{01} = 2.405$$

$$R = \frac{x_{01}}{2\pi}\lambda = 0.383\lambda$$

TM_{010} mode in a pill-box cavity

Energy content

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

Power dissipation

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R+L)R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

Geometrical factor

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R+L)}$$

TM_{010} mode in a pill-box cavity

Energy Gain

$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

Gradient

$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

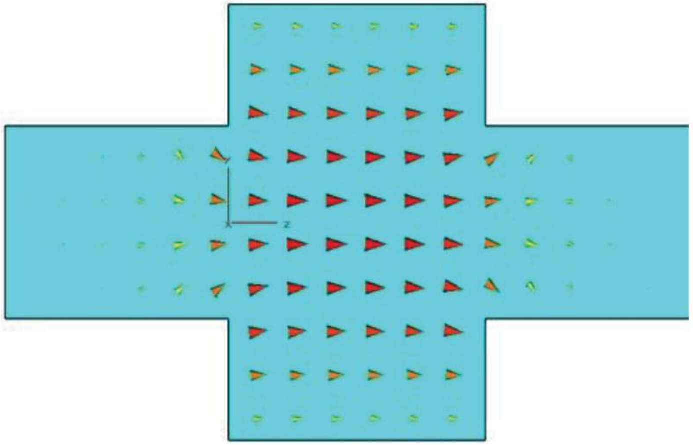
Shunt impedance

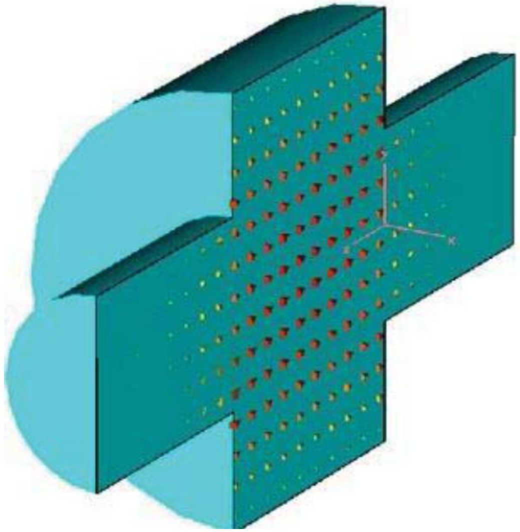
$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2\left(\frac{\pi L}{\lambda}\right)$$

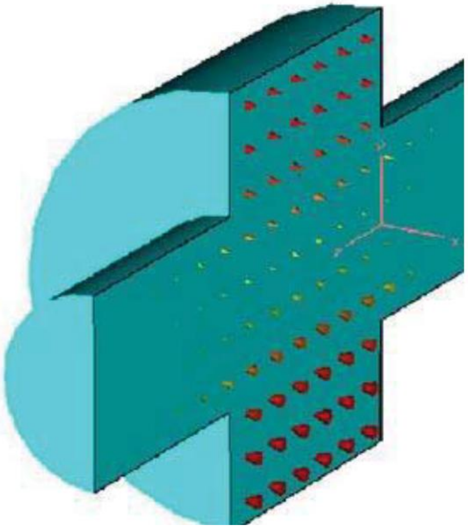
Pill-box cavity to real cavity

Beam tubes reduce the electric field on axis

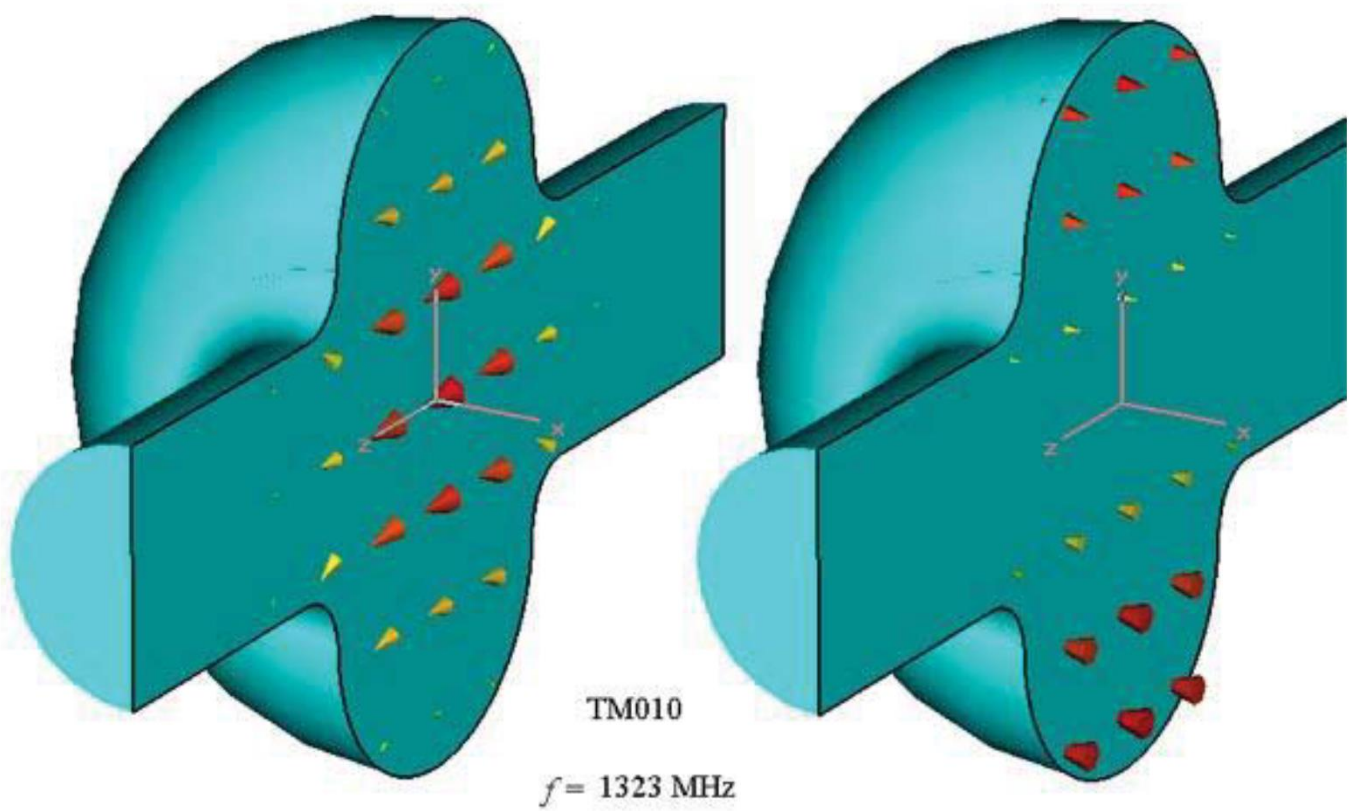
- **Gradient decreases**
- **Peak fields increase**
- **R/Q decreases**



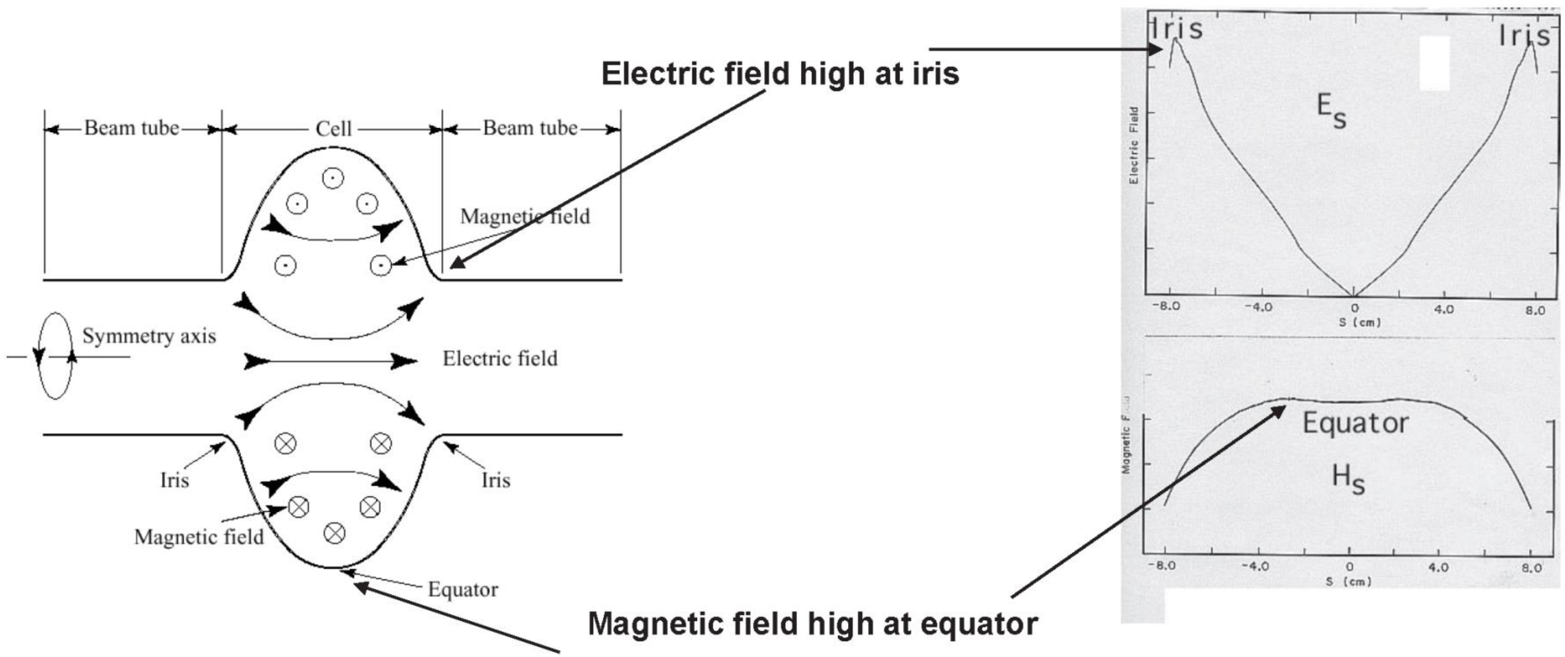




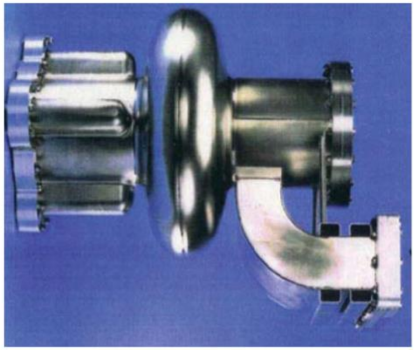
Pill-box cavity to real cavity

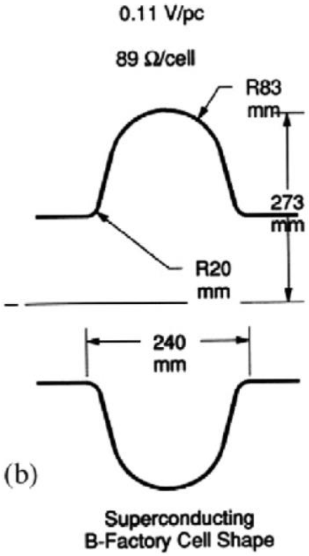


Single-cell cavity



Single-cell cavity





Quantity	Cornell SC 500 MHz	Pillbox
G	**270 ohmΩ**	257 Ω
R_a/Q_0	**88 ohm/cell**	196 Ω/cell
E_{pk}/E_{acc}	**2.5**	1.6
H_{pk}/E_{acc}	**52 Oe/MV/m**	30.5 Oe/(MV/m)

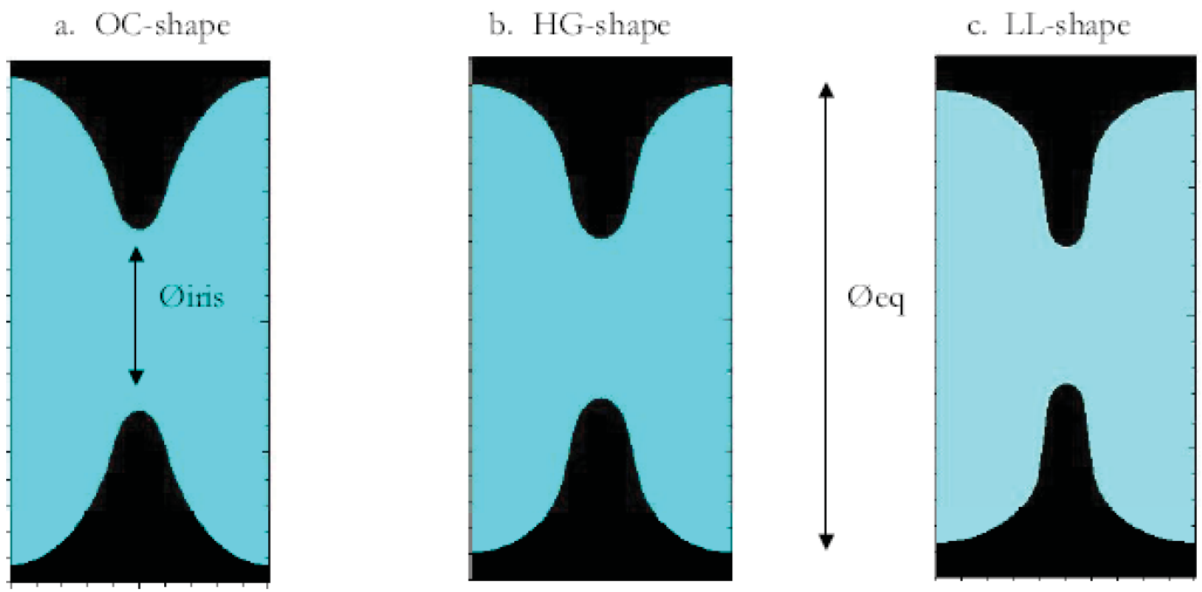
Cell shape design

- What is the purpose of the cavity?
- What EM parameters should be optimized to meet the design specs?

The "perfect" shape does not exist, it all depends on your application

Example: CEBAF upgrade

- "High Gradient" shape: lowest E_p/E_{acc}
- "Low Loss" shape: lowest cryogenic losses G(R/Q)



CEBAF upgrade cell-shape comparison

Table 1. Parameters of inner dumbbells

Parameters	Unit	OC-shape	HG-Shape	LL-Shape
Øeq	[mm]	187.03	180.50	174.00
Øiris	[mm]	70.00	61.40	53.00
k_{cc}	[%]	3.29	1.72	1.49
E_{peak}/E_{acc}	-	2.56	1.89	2.17
B_{peak}/E_{acc}	$[mT\cdot(MV/m)^{-1}]$	4.56	4.26	3.74
Lorentz factor*) k_L	$[Hz\cdot(MV/m)^{-2}]$	-1.35	-1.1	-1.2
R/Q	[Ω]	96.5	111.9	128.8
r/q = (R/Q)/length	[Ω/m]	965	1119	1288
G	[Ω]	273.8	265.5	280.3
R/Q*G	[Ω*Ω]	26421	29709	36102

CEBAF Upgrade: cryo-budget limit of 30W/cavity. Higher energy gain can be obtained using LL-shape.

Trend in TM-mode cavity dedign

• The **field emission is not a hard limit** in the performance of sc cavities if the surface preparation is done in the right way.

• Unlikely this, **magnetic flux on the wall** limits performance of a sc cavity (Q_0 decreases or/and quench). Hard limit **~180 mT** for Nb.

↓

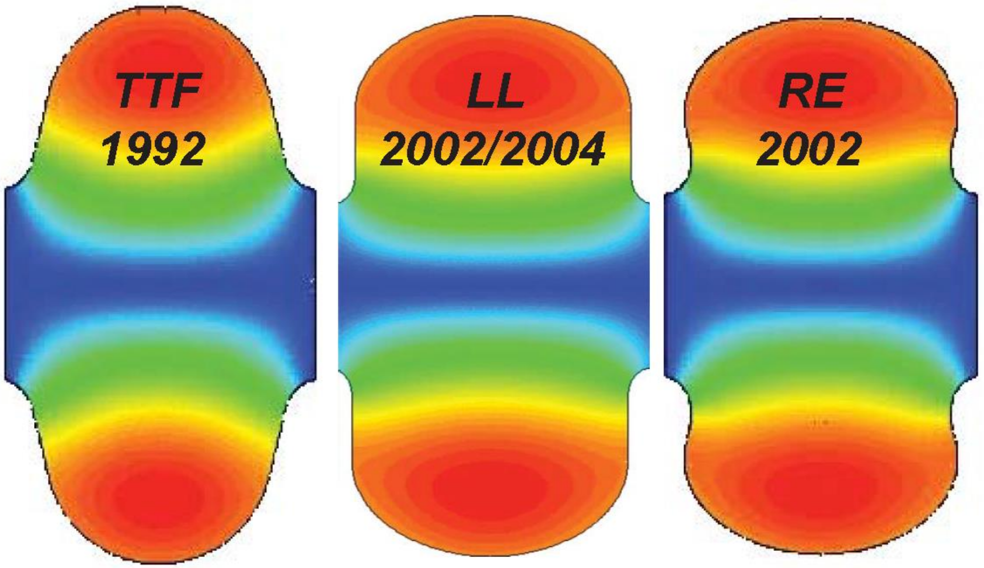
B_{peak} / E_{acc} should be low

↓

1. Cavities may operate at higher gradients. ⟷ *2. Cavities may operate at lower cryogenic load.*

$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G \cdot (R/Q)}$$

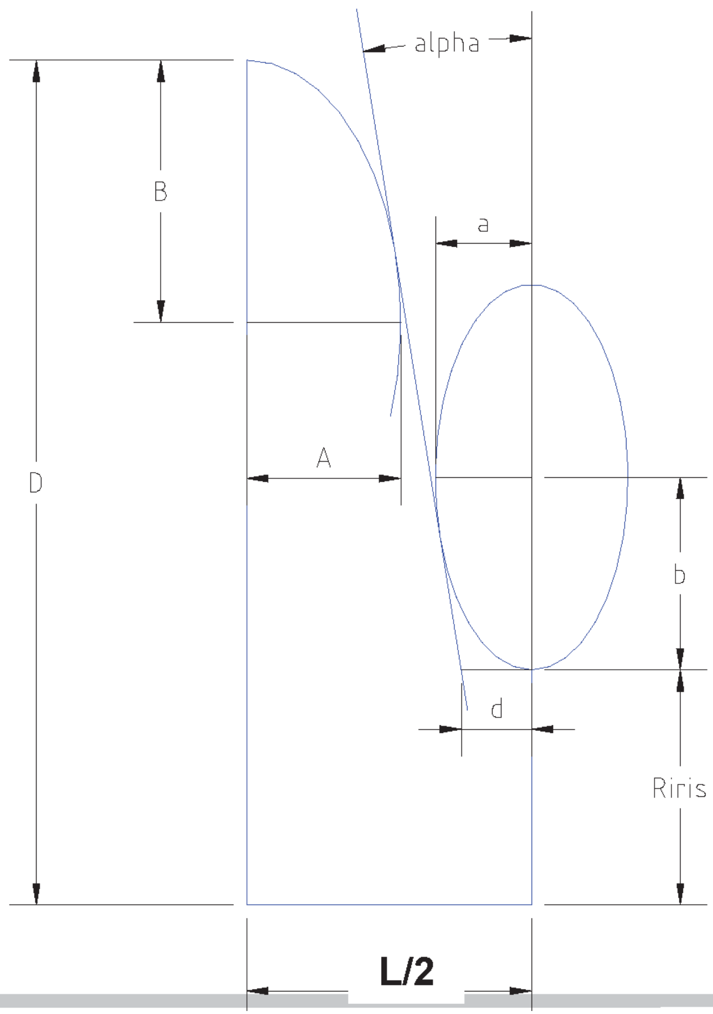
New advanced shape for ILC



r_{iris}	[mm]	35	30	33
k_{cc}	[%]	1.9	1.52	1.8
E_{peak}/E_{acc}	-	1.98	2.36	2.21
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.61	3.76
R/Q	[Ω]	113.8	133.7	126.8
G	[Ω]	271	284	277
R/Q*G	[Ω*Ω]	30840	37970	35123

Cell-shape parametrization

- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - ✓ Ellipse ratio at the equator (R=B/A)
    ruled by mechanics
  - ✓ Ellipse ratio at the iris (r=b/a)
    E_{peak}
  - ✓ Side wall inclination (α) and position (d)
    E_{peak} vs. B_{peak} tradeoff and coupling
    k_{cc}
  - ✓ Cavity iris radius R_{iris}
    coupling k_{cc}
  - ✓ Half-cell Length $L/2=\lambda\beta/4$
    β
  - ✓ Cavity radius D
    used for frequency tuning
- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters

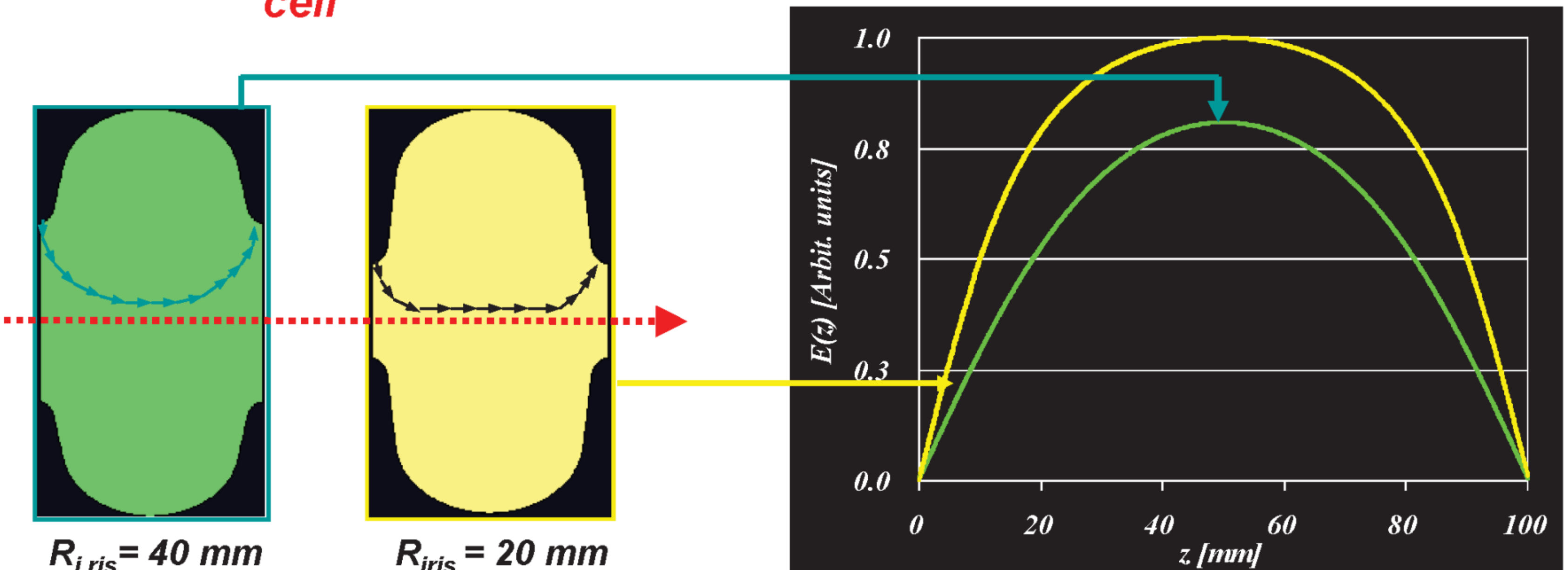


R-iris

Why for a smaller aperture (R_{iris})?

- (R/Q) is bigger
- E_{peak}/E_{acc} , B_{peak}/E_{acc} is lower

E_{acc} is higher at the same stored energy in the cell



E_z (z) for small and big iris radius

R-iris

We know that a smaller aperture makes:

- *(R/Q) higher*
- B_{peak}/E_{acc} , E_{peak}/E_{acc} *lower*

} (+)

but unfortunately a smaller aperture makes:

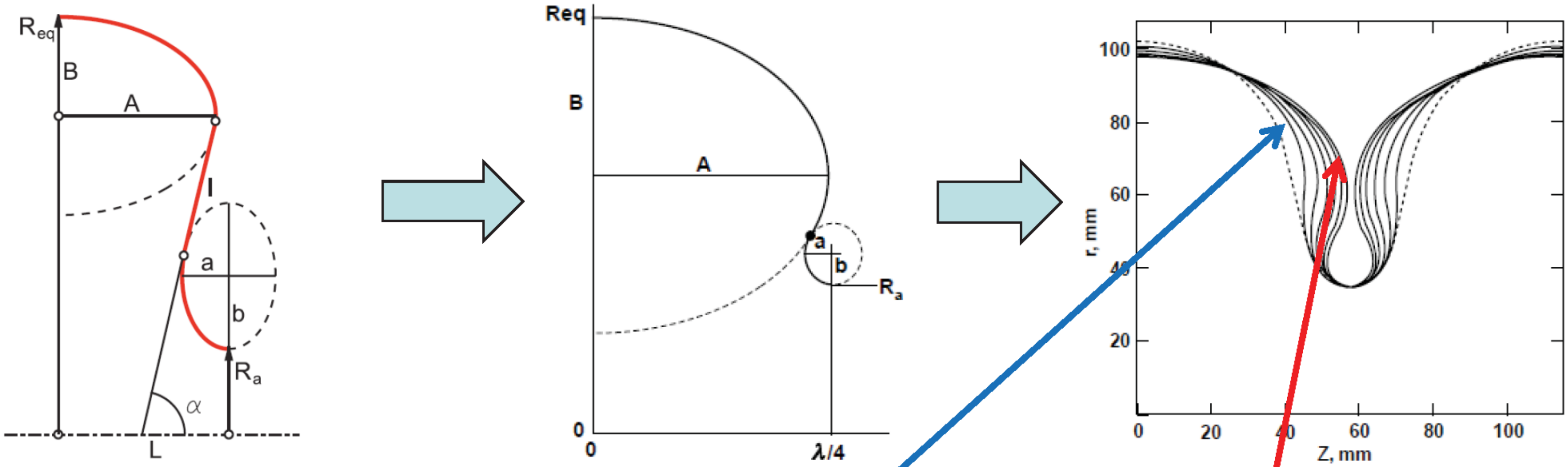
- *HOMs impedances* ($k_{\perp}$, $k_{\parallel}$) *higher*
- *cell-to-cell coupling* (k_{cc}) *weaker*

} (-)

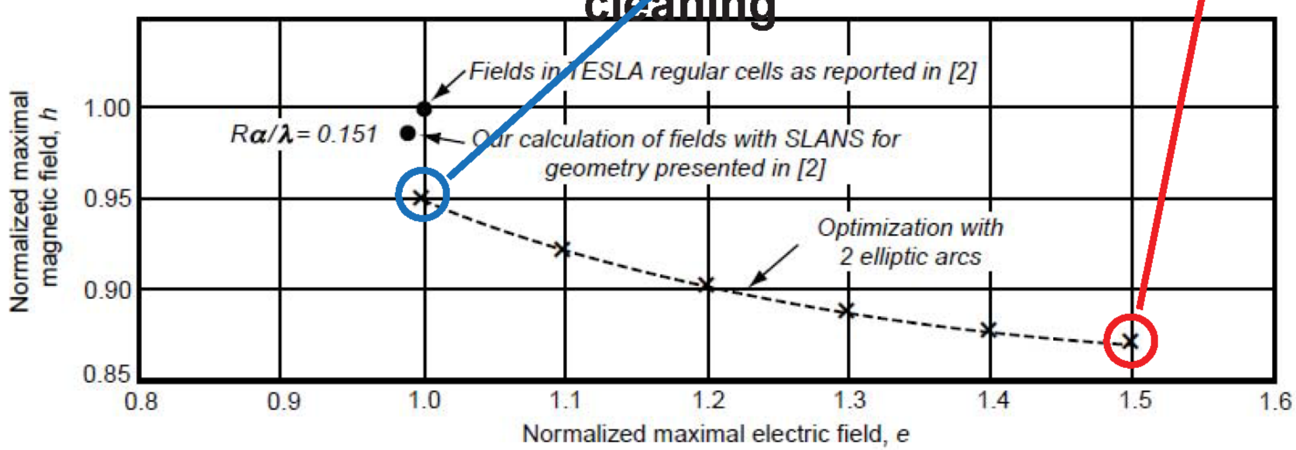
Pre-tuning is difficult for multi-cell cavity

Intuitive understanding for controlling E-peak and B-peak

Re-entrant shape : The world-record holder of highest Eacc

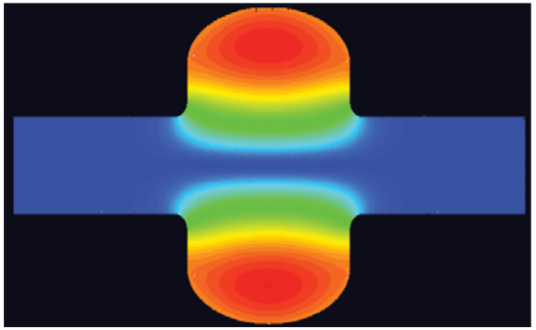


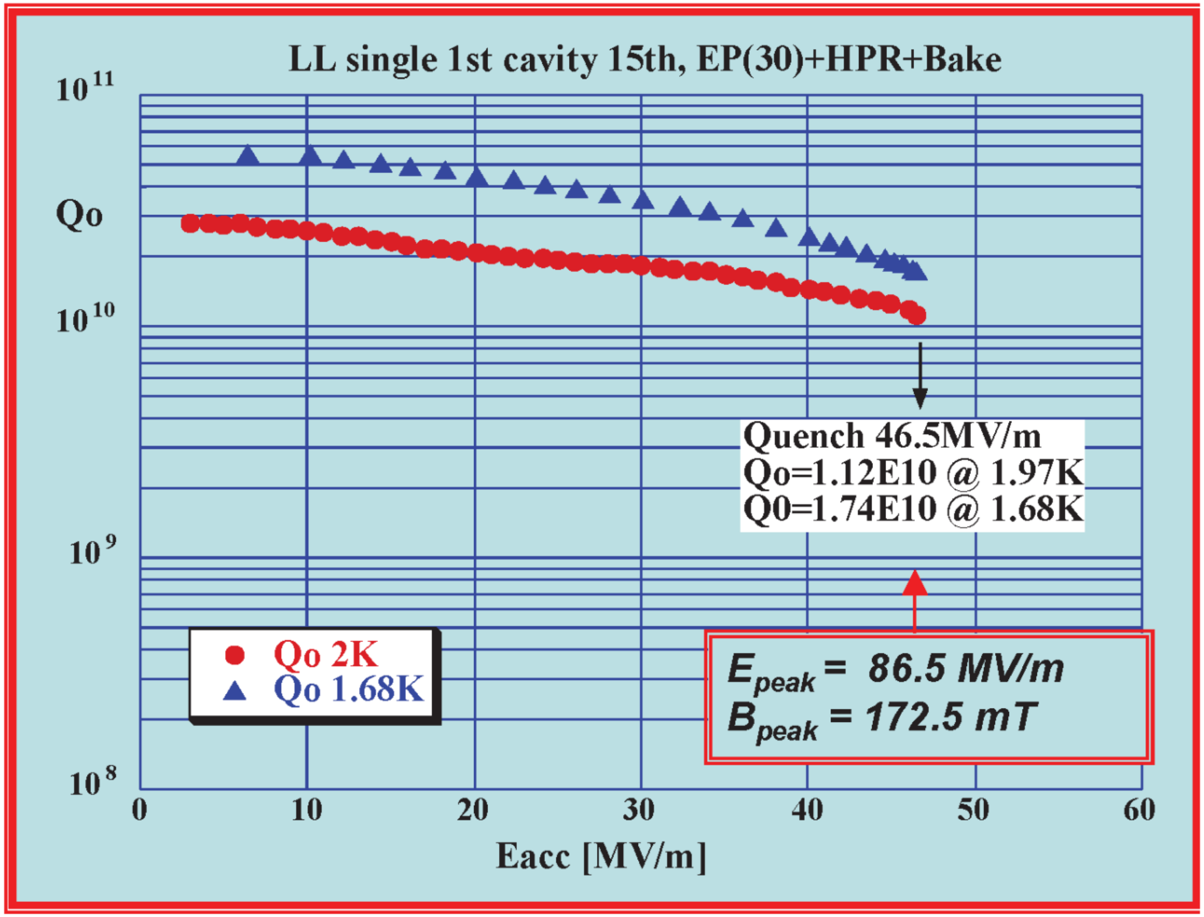
- 3 independent parameters: A, B, a
- potential issue with cavity forming and cleaning



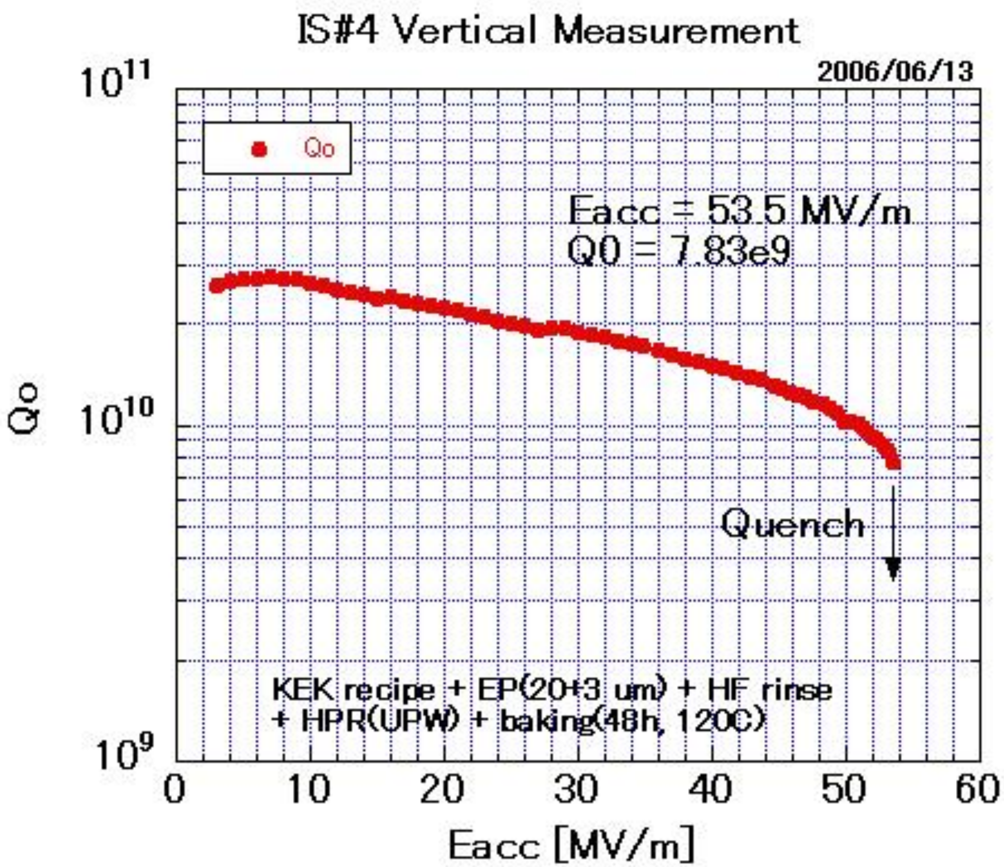
RF test of LL-shape single-cell cavity

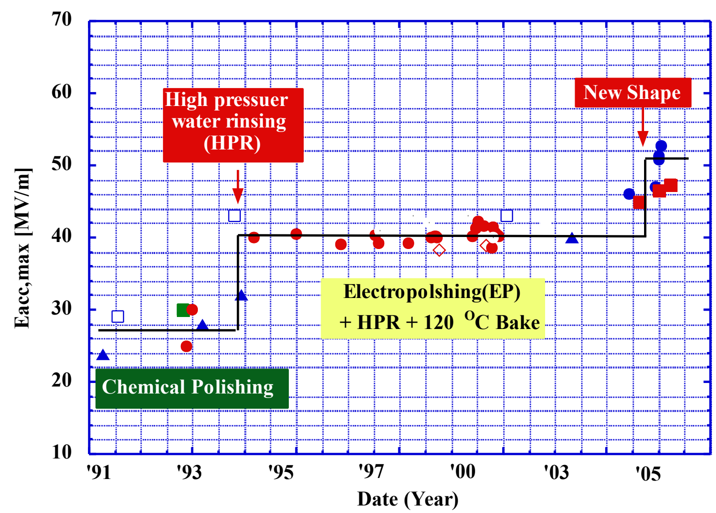
		LL
f_π	[MHz]	1286.6
E_{peak}/E_{acc}	-	1.86
B_{peak}/E_{acc}	[mT/(MV/m)]	3.71
R/Q	[Ω]	130.0
G	[Ω]	279
$Ø_{iris}$	[mm]	61





RF Test of LL single-cell cavity / Eacc = 53.5 MV/m



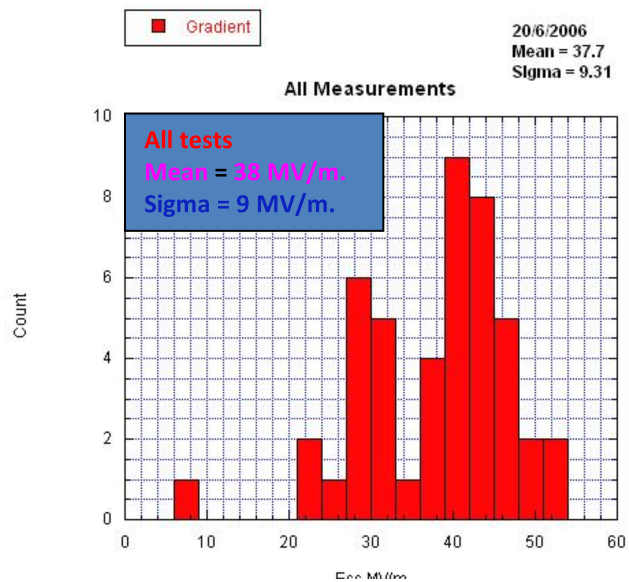


Eacc = 53.5 MV/m was achieved.
This had been the world record until RE single-cell cavity reached beyond.

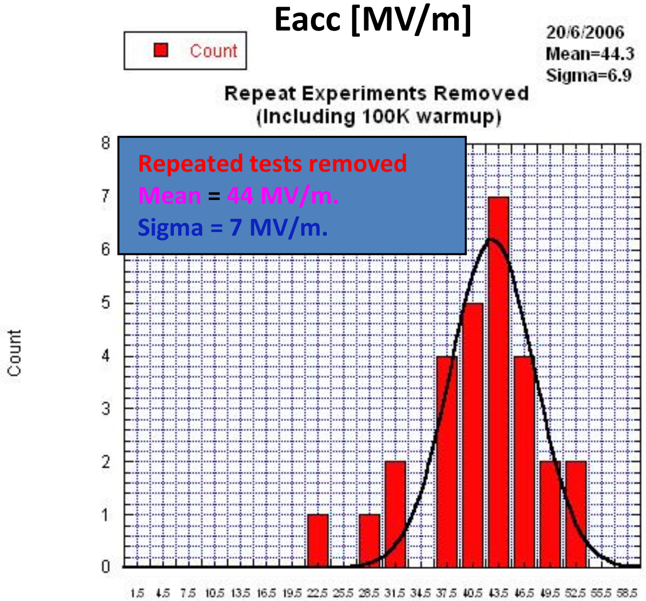
Press release

’05 28th Sept. NHK news, “Good morning Japan”
’05 12th Oct. Nikkan Kogyo News
’05 21st Oct. Energy News Weekly
’05 1st Nov. Daily Yomiuri
’06 24th Jan. Nihon Keizai News

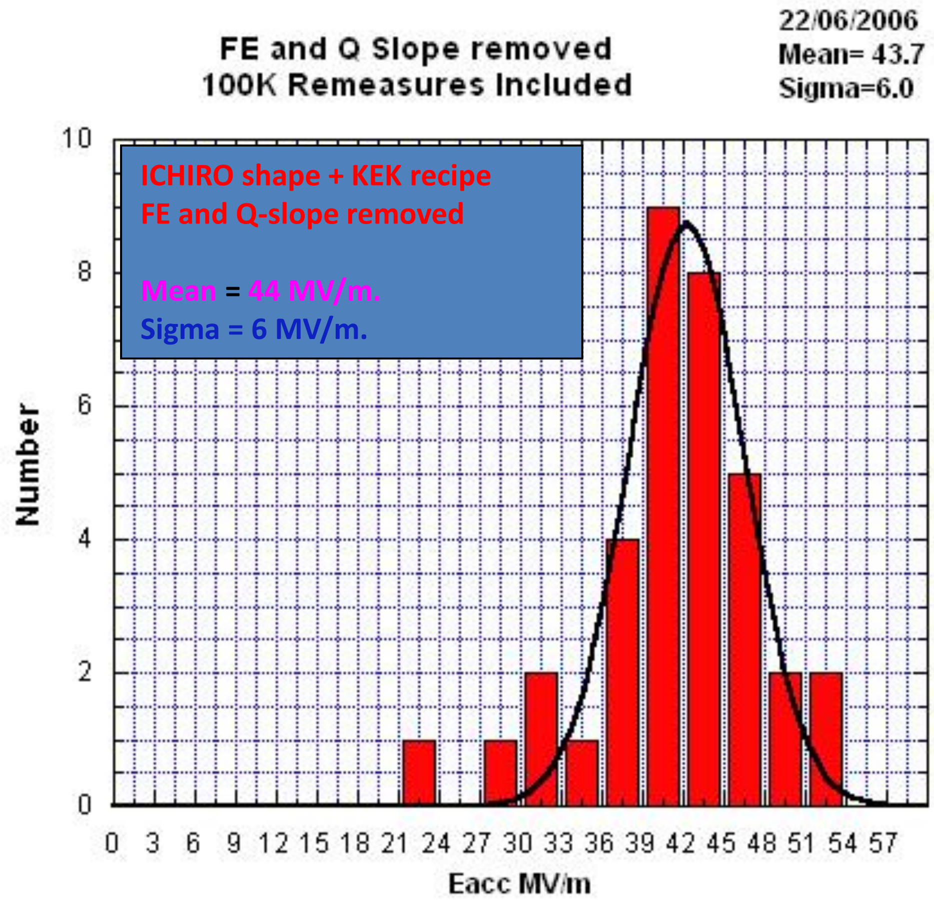
Series RF tests of LL single-cell cavities



Eacc [MV/m]



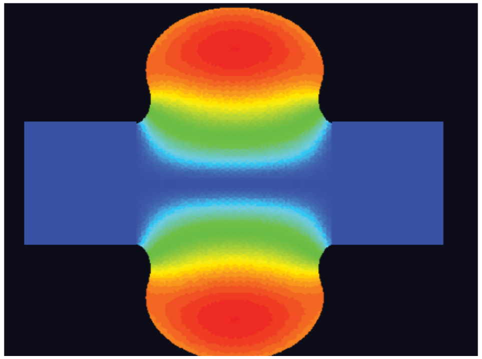
Eacc [MV/m]

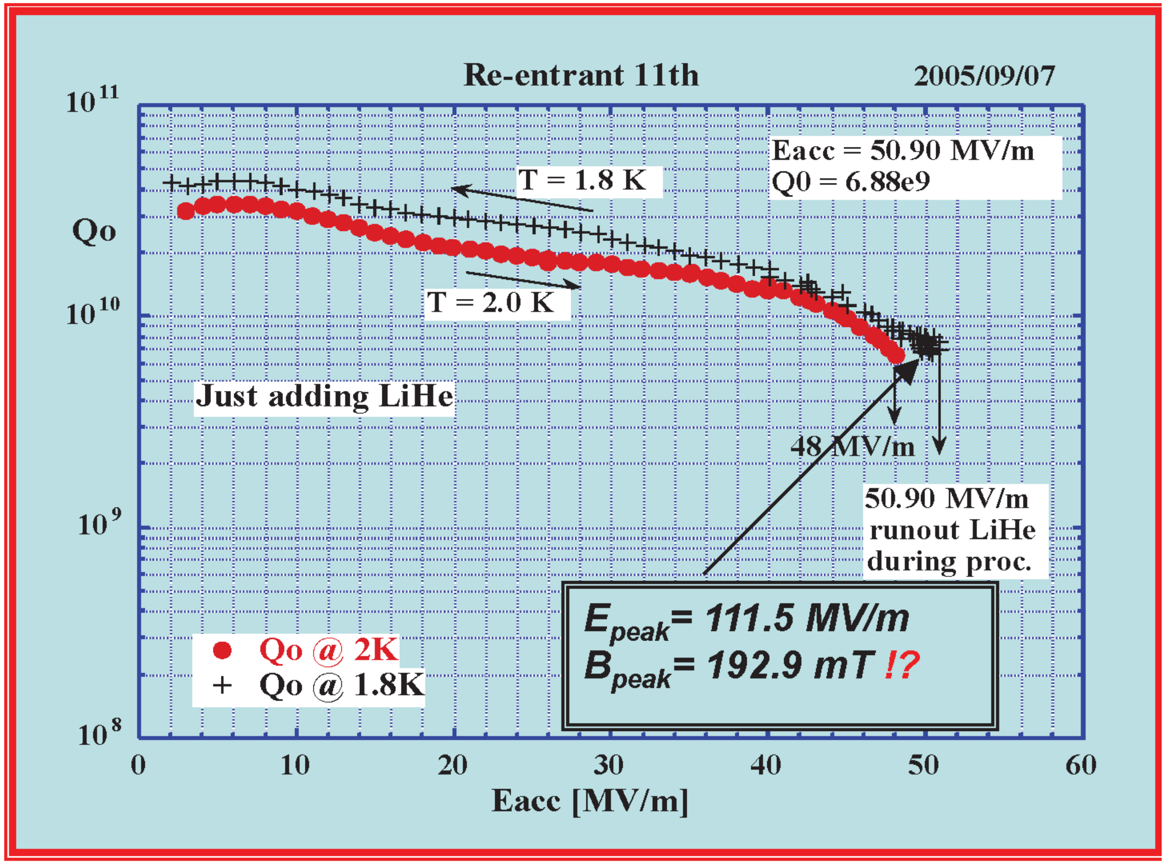


Eacc [MV/m]

RF test of RE-shape single-cell cavity fabricatedy by Cornell Univ.

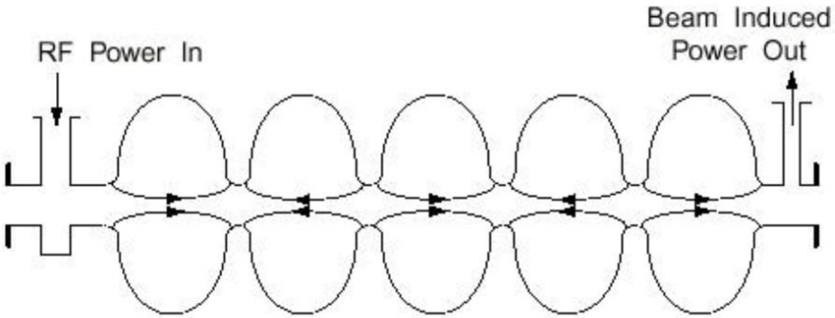
		RE
f_{π}	[MHz]	1278.6
E_{peak}/E_{acc}	-	2.19
B_{peak}/E_{acc}	[mT/(MV/m)]	3.79
R/Q	[Ω]	126.0
G	[Ω]	278
$Ø_{iris}$	[mm]	68



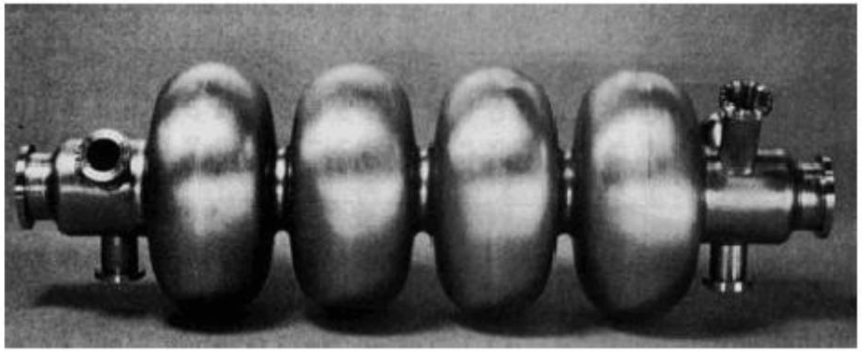


This cavity reached Eacc > 60 MV/m. I believe this cavity might be the world-record holder of highest Eacc. Sorry, I could not find the plot, that I, F. Furuta, and K. Saito measured at KEK…

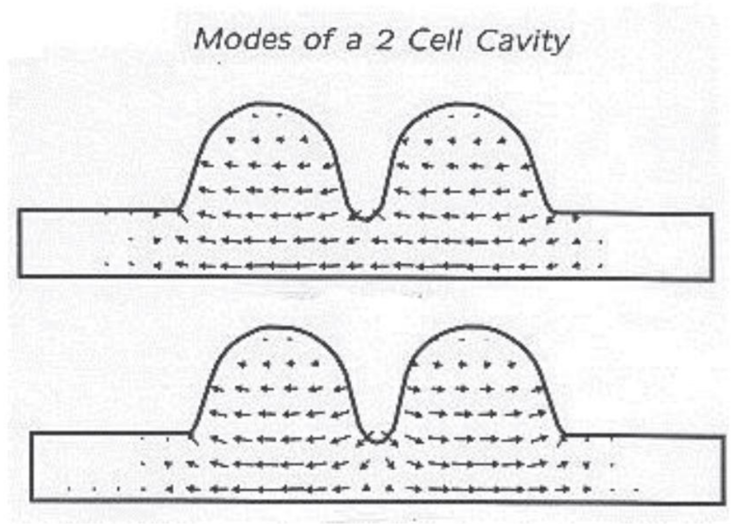
Multi-cell cavities

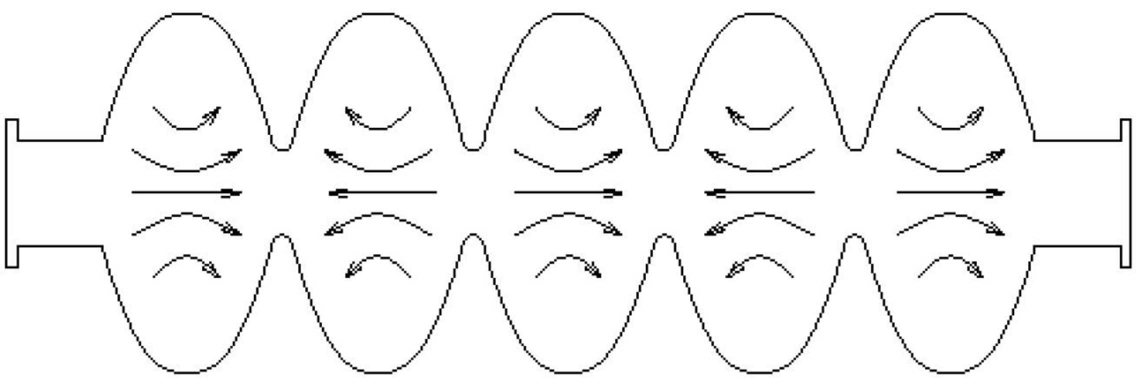


(c)



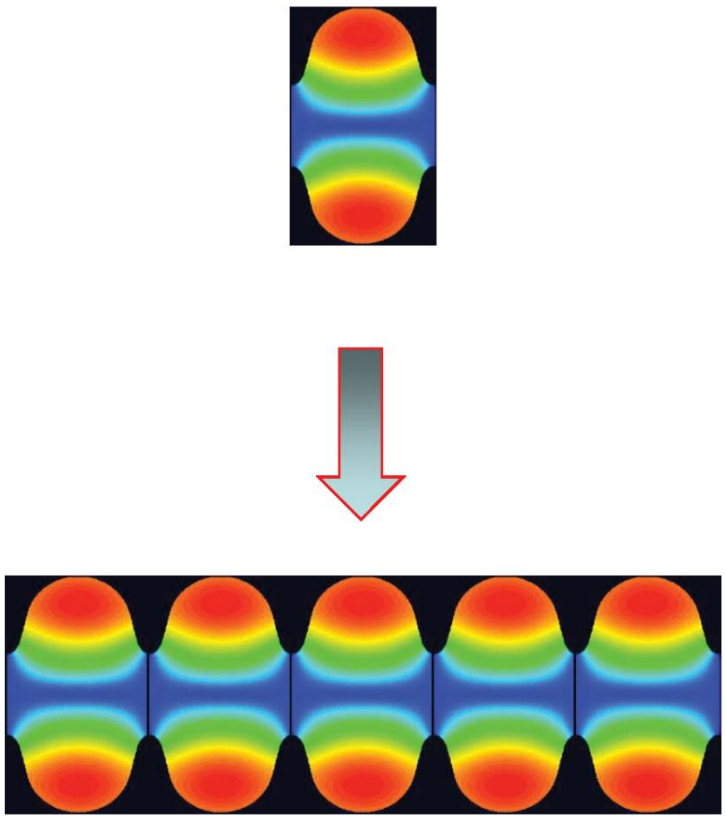
Multi-cell cavities





: Sketch of the electric field lines of the π-mode of a 5-cell

Multi-cell cavities



Single-cell is attractive from the RF-point of view:

- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation

- *But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.*

A multi-cell structure is less expensive and offers higher real-estate gradient but:

Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells

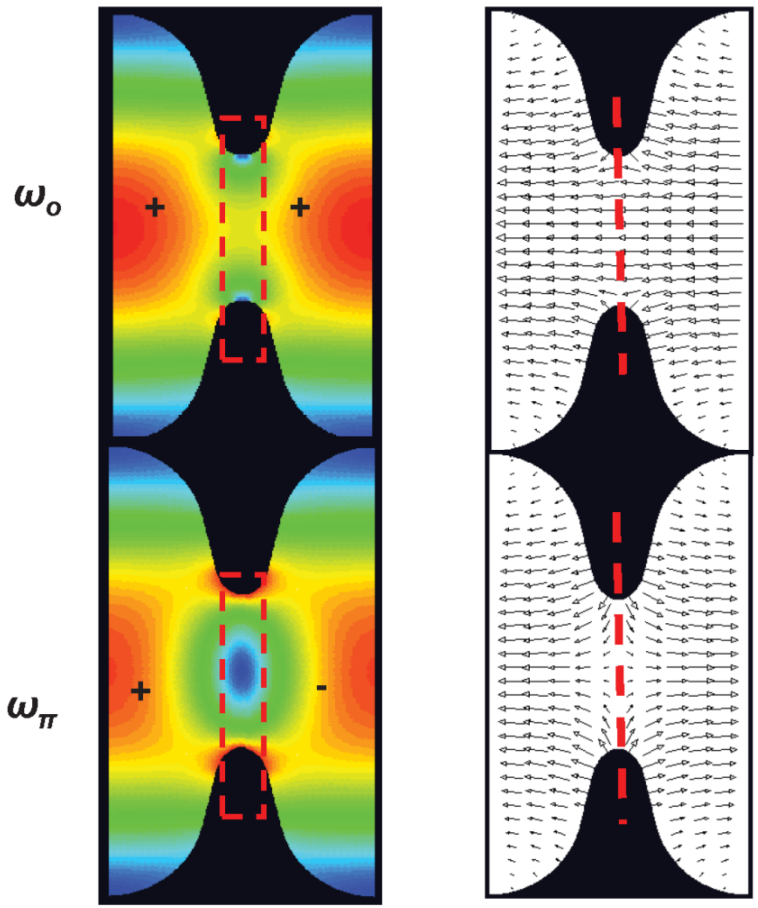
- *Other problems arise: HOM trapping…*

Pros and cons of Multi-cell cavities

- Cost of accelerators are lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics)
- Higher real-estate gradient (better fill factor)

- Field flatness vs. N
- HOM trapping vs. N
- Power capability of fundamental power couplers vs. N
- Chemical treatment and final preparation become more complicated
- The worst performing cell limits whole multi-cell structure

Coupling between cells



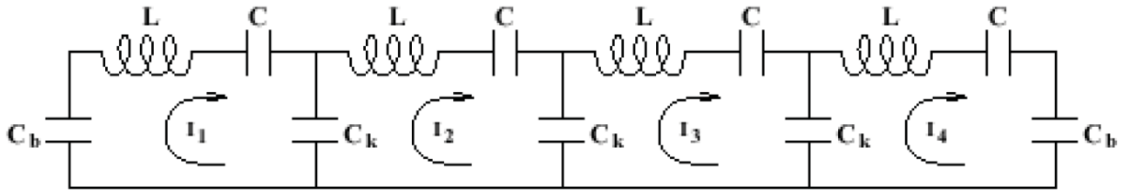
Symmetry plane for the H field

Symmetry plane for the E field which is an additional solution

The normalized difference between these frequencies is a measure of the energy flow via the coupling region

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\frac{\omega_\pi + \omega_0}{2}}$$

Coupling between cells



$$k = \frac{C}{C_k} \qquad C_b = C_k / 2$$

Mode frequencies:

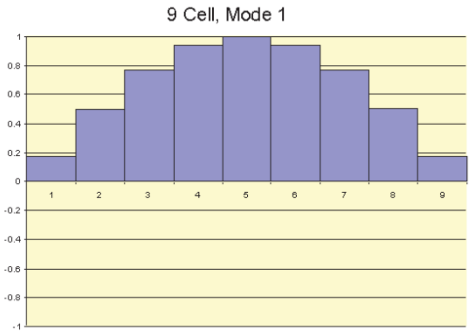
$$\frac{\omega_m^2}{\omega_0^2} = 1 + 2k\left(1 - \cos\frac{\pi m}{n}\right)$$

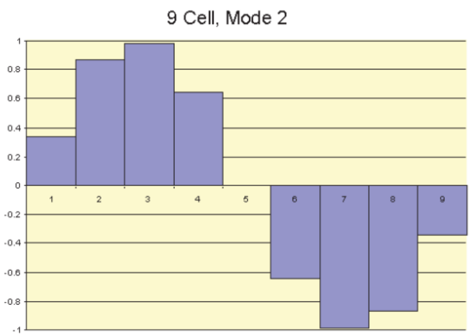
$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

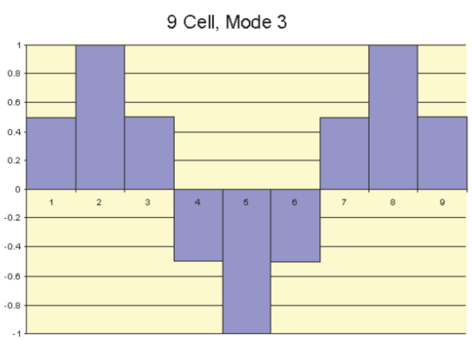
Voltages in cells:

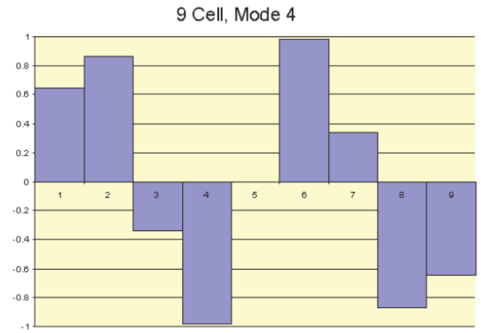
$$V_j^m = \sin\left(\pi m \frac{2j-1}{2n}\right)$$

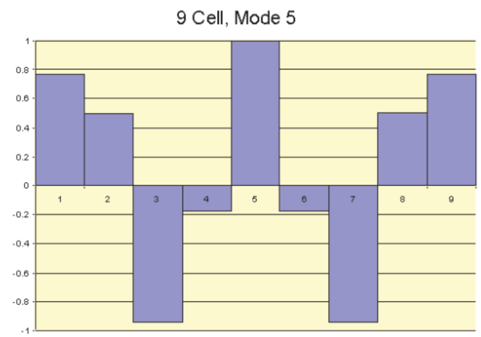
Pass-band mode analysis

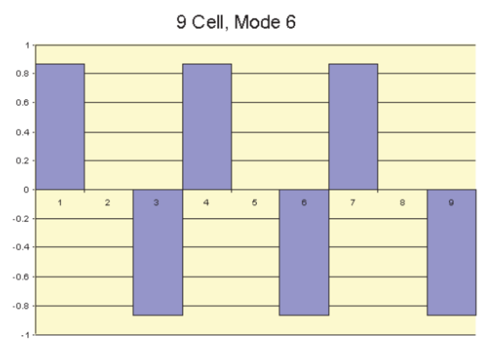


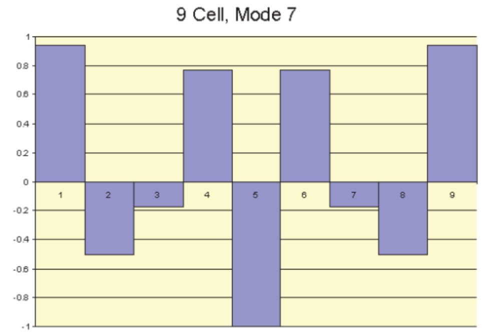


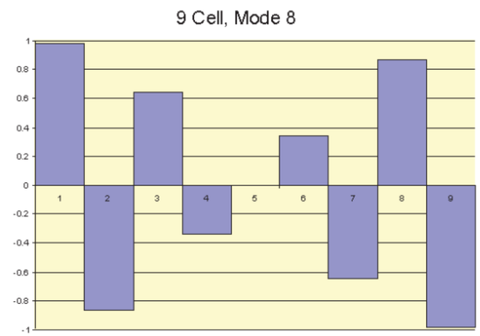


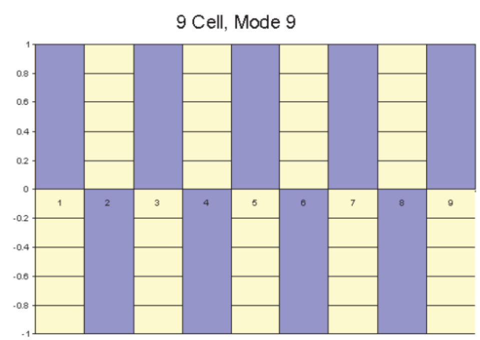




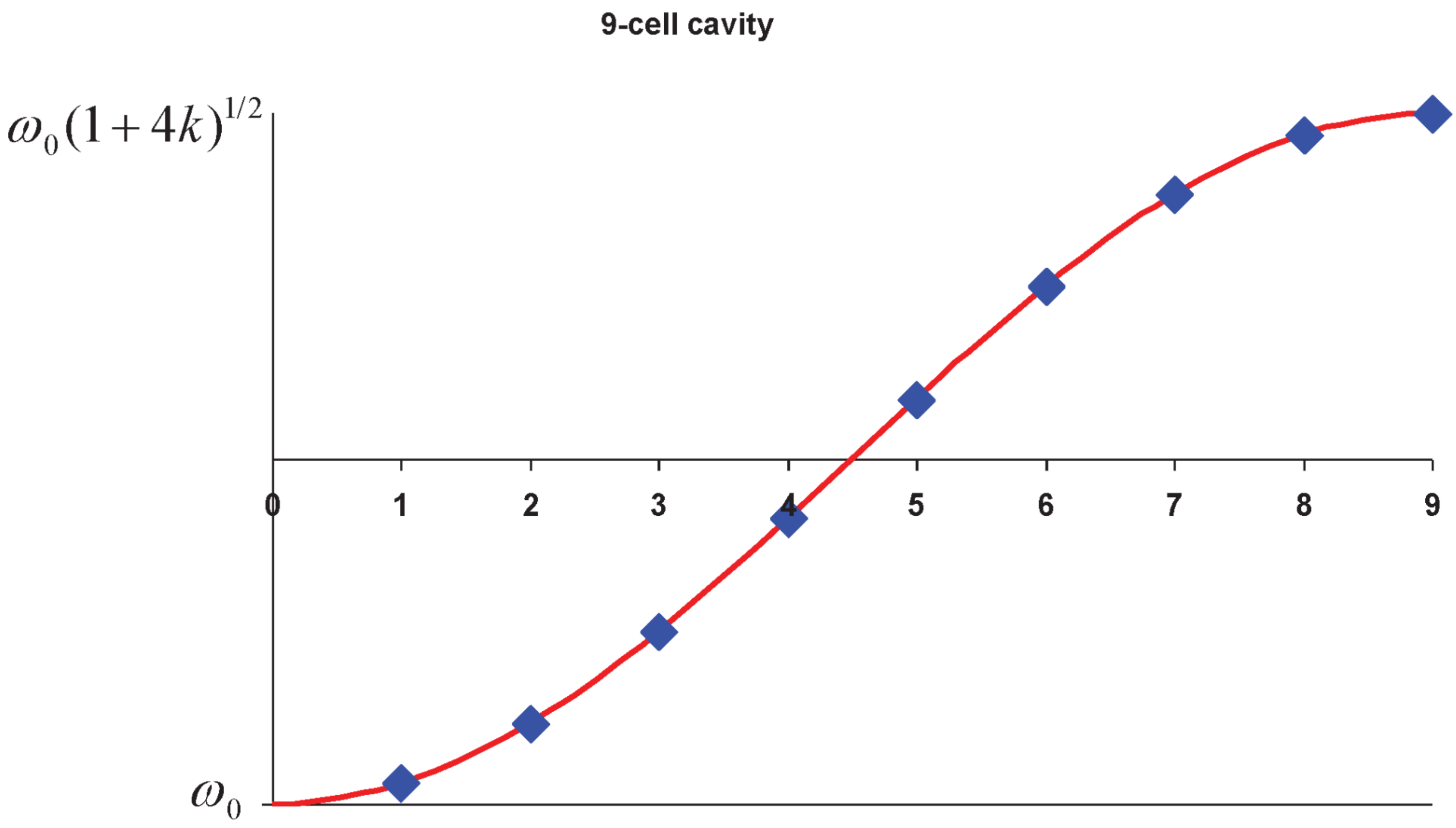








Pass-band mode : Frequency

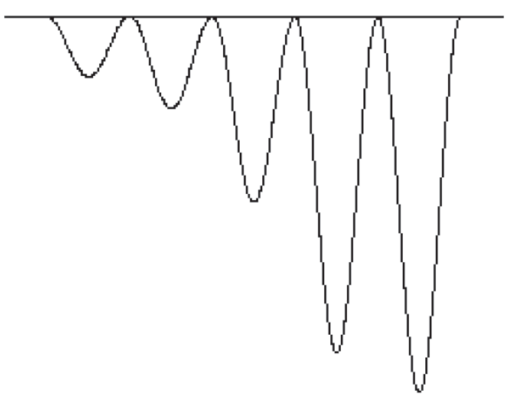


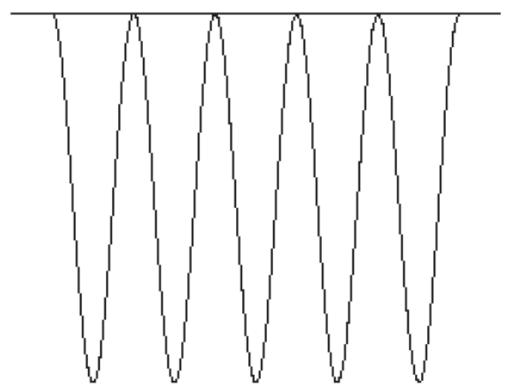
Field flatness

Geometrical differences between cells causes a mixing of the eigenmodes

Sensitivity to mechanical deformation depends on mode spacing

$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

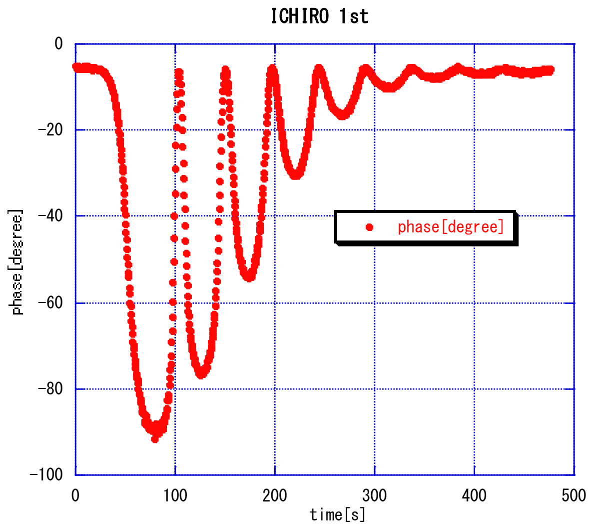




If cell-to-cell coupling is weak, field-flatness is easily broken by mechanical deformation of cells.

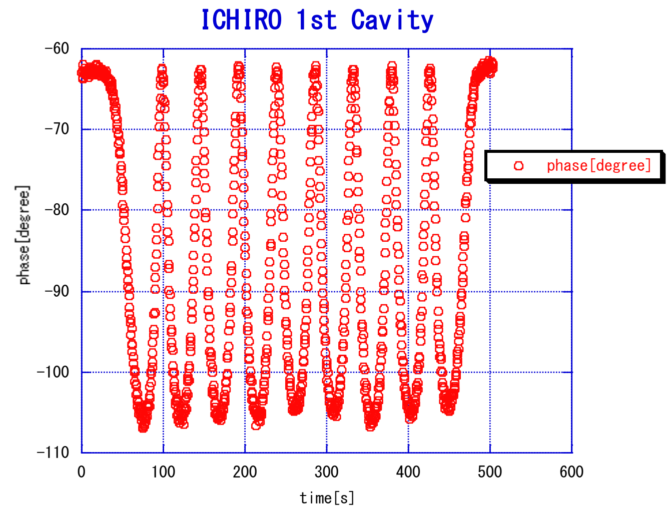
Field flatness after pre-tuning of LL 9-cell cavity

π mode frequency 1298.774 MHz



Field flatness = 0.1 %
(as delivered to KEK)

π mode frequency 1298.547 MHz

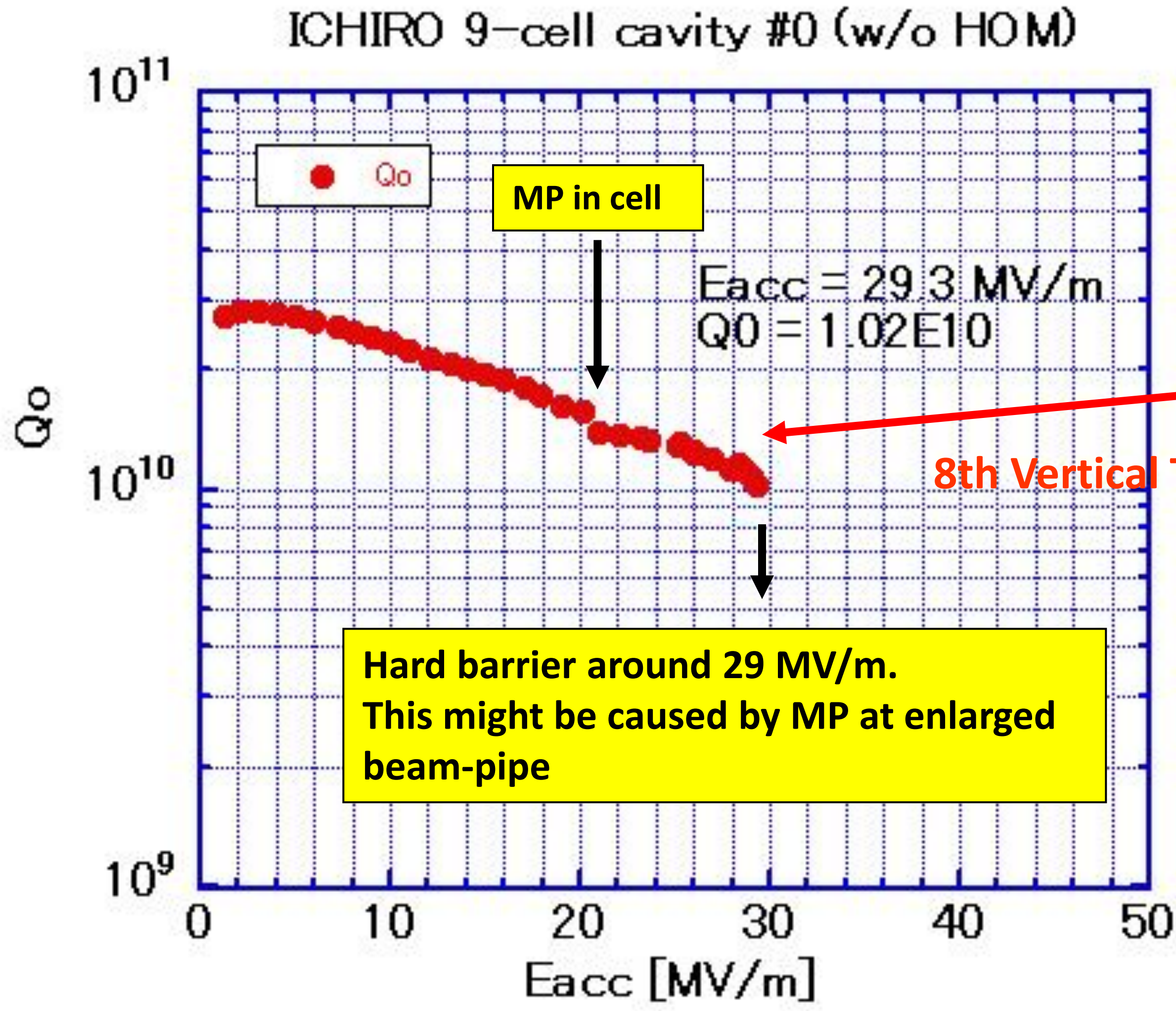


Field flatness = 98 %
(after pre-tuning)

Cavity	Field flatness (min/max) as delivered / after pre-tuning	Freq. target 1298.141 (MHz) @R.T. as delivered / after pre-tuning
1st	0.1% / 98%	1298.774 / 1298.547

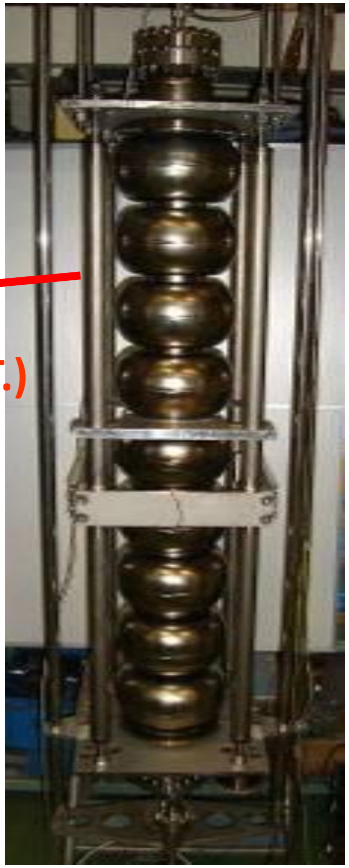
Cell-to-cell coupling is as small as 1.6%, but no problem in pre-tuning.

RF Test of LL 9-cell cavity



8th Vertical Test (V.T.)

LL 9-cell 1st Cavity #0 w/o HOM



No Q-disease was found.

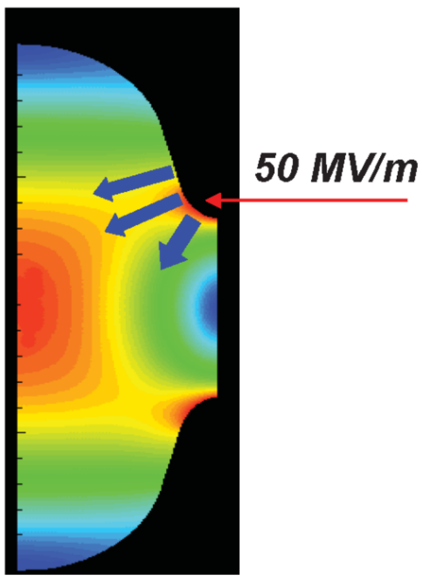
Mechanical design

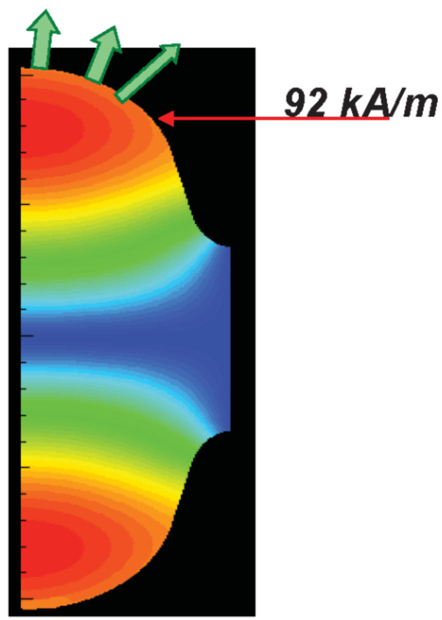
The mechanical design of a cavity follows its RF design:

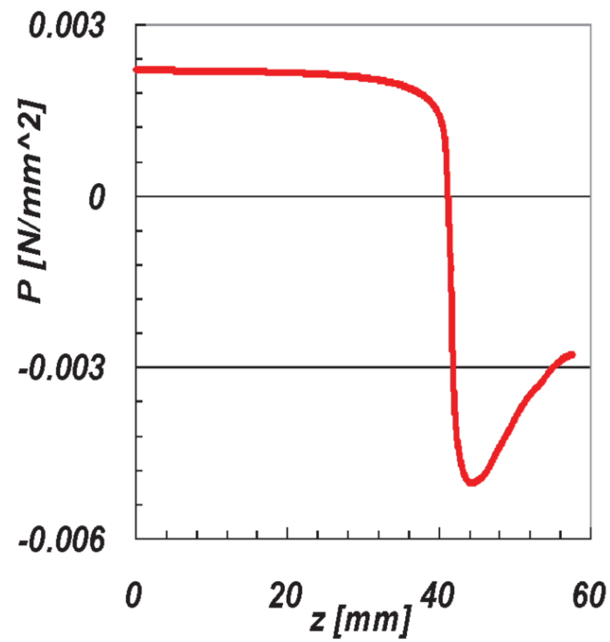
- **Lorentz Force Detuning**
- **Mechanical Resonances**

$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$

Lorentz Force Detuning

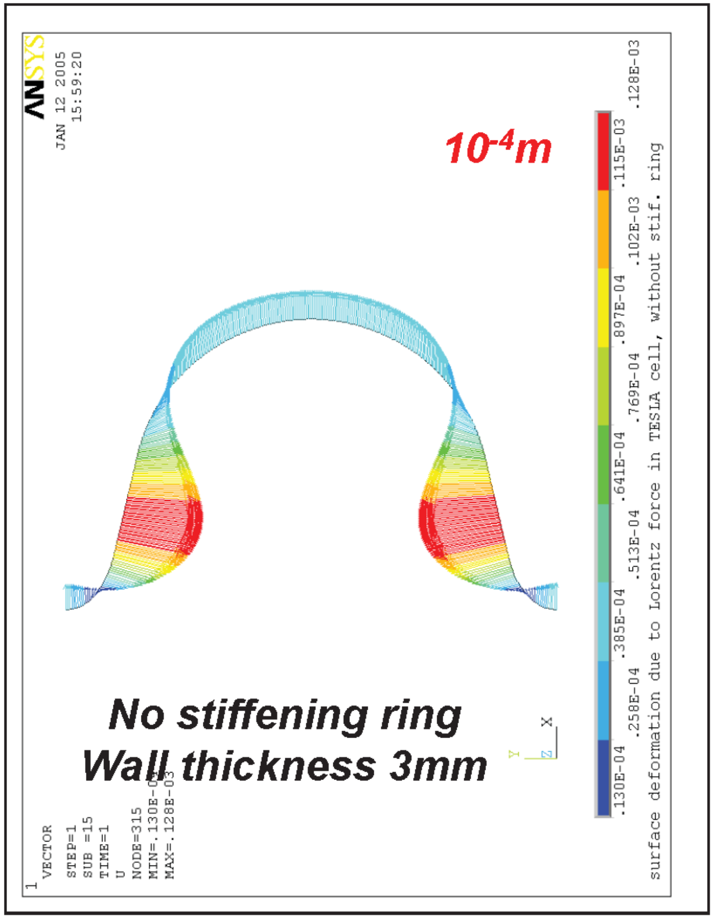


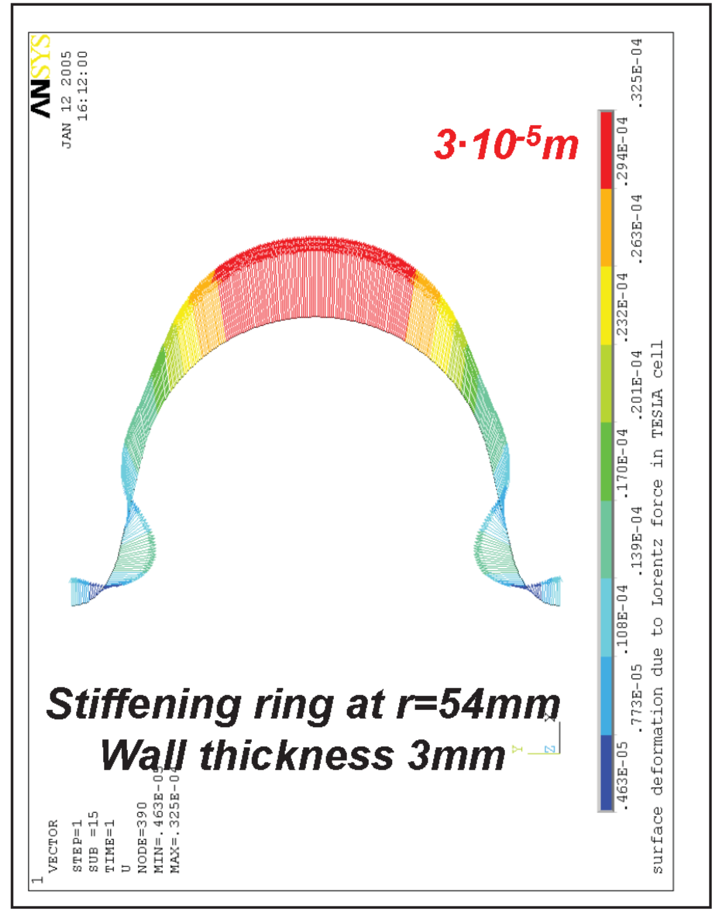




E and H at E_{acc} = 25 MV/m in TESLA inner-cup

Mechanical design





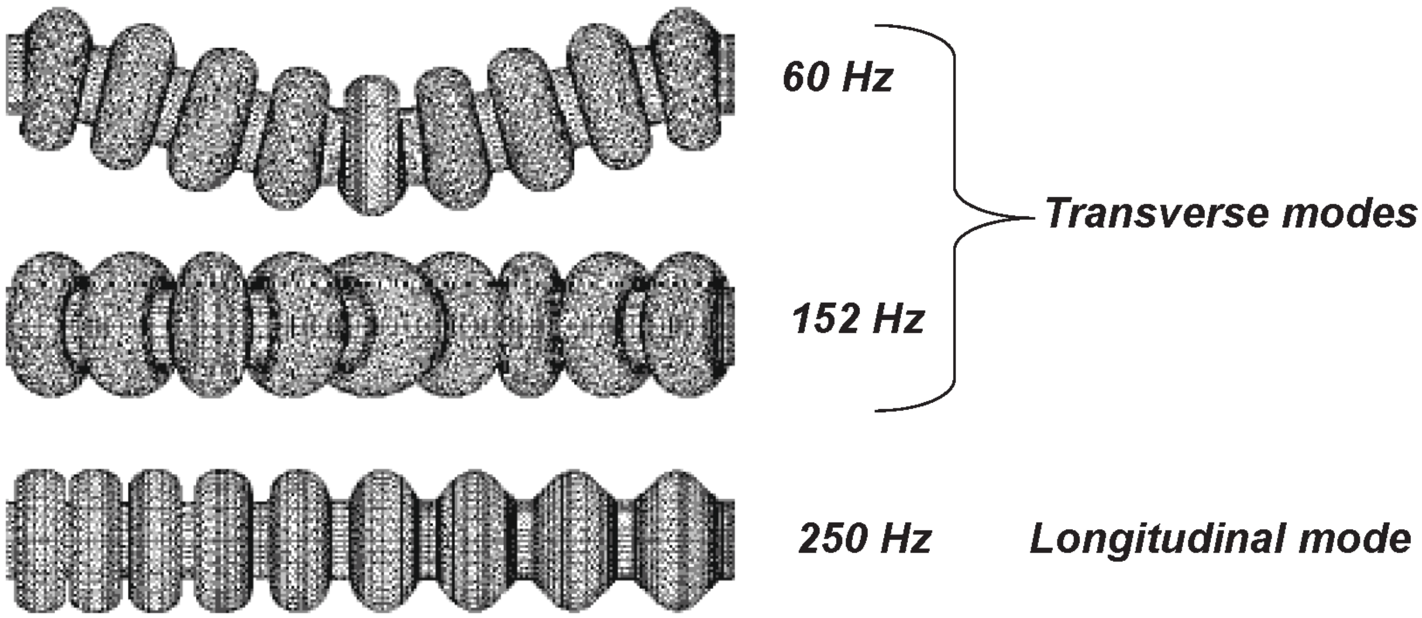
k_L = -1 Hz/(MV/m)^2

Essential for the operation of a pulsed accelerator

$$\Delta f = k_L (E_{acc})^2$$

Mechanical design

Mechanical Resonances of a multi-cell cavity

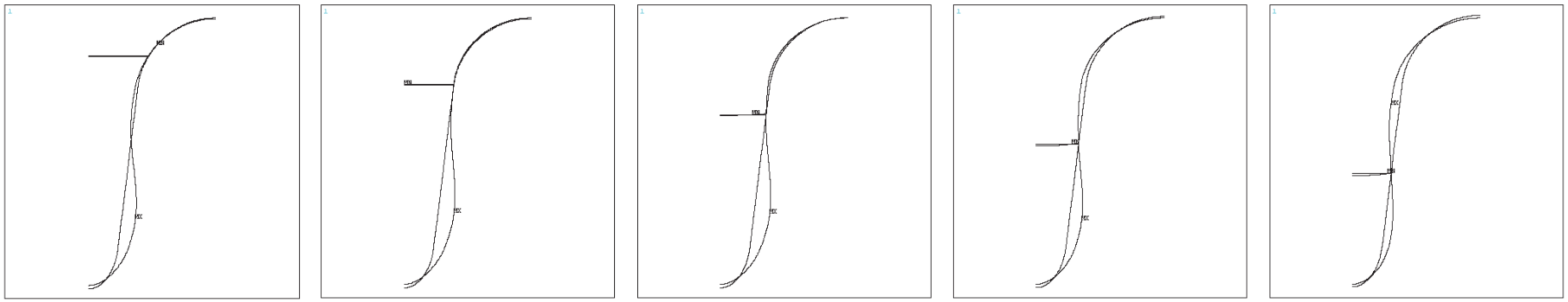


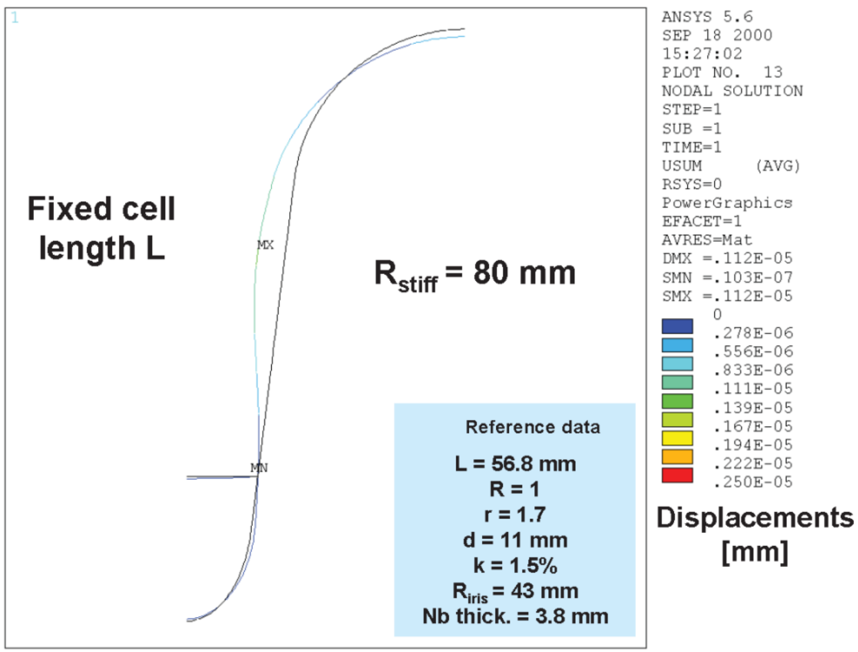
TESLA structure

The mechanical resonances modulate frequency of the accelerating mode.
Sources of their excitation: vacuum pumps, ground vibrations…

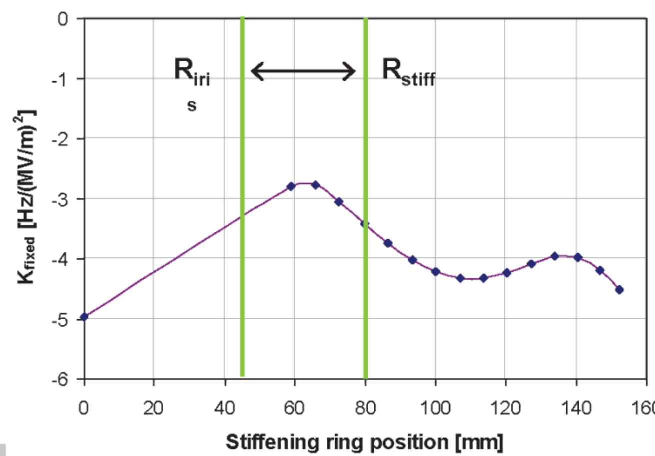
These mechanical resonant modes are also closely related to the microphonics.

Optimum stiffener-ring positioning

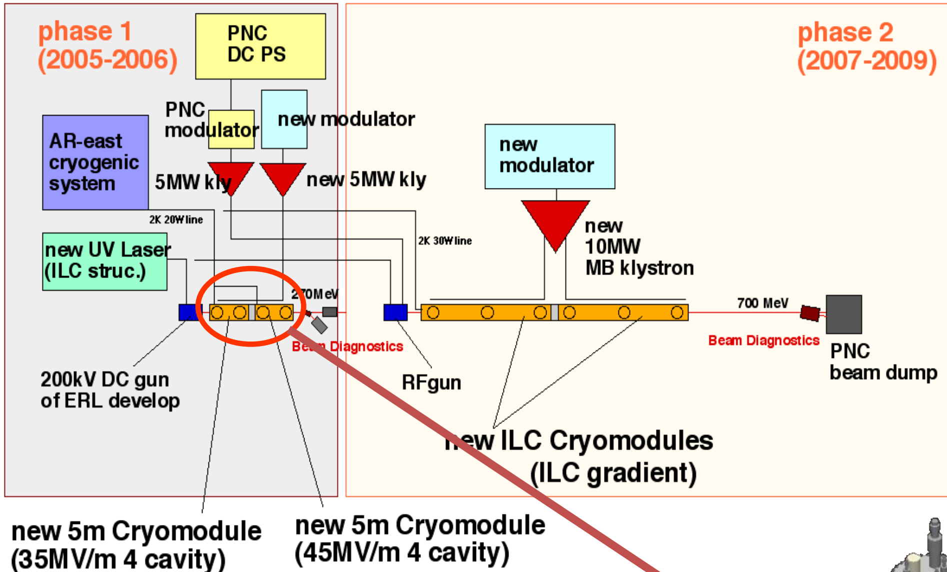




The Lorentz forces coefficients for 15 different stiffening ring positions are evaluated automatically with ANSYS, preparing the geometry and reading the fields from the SFO output from SUPERFISH



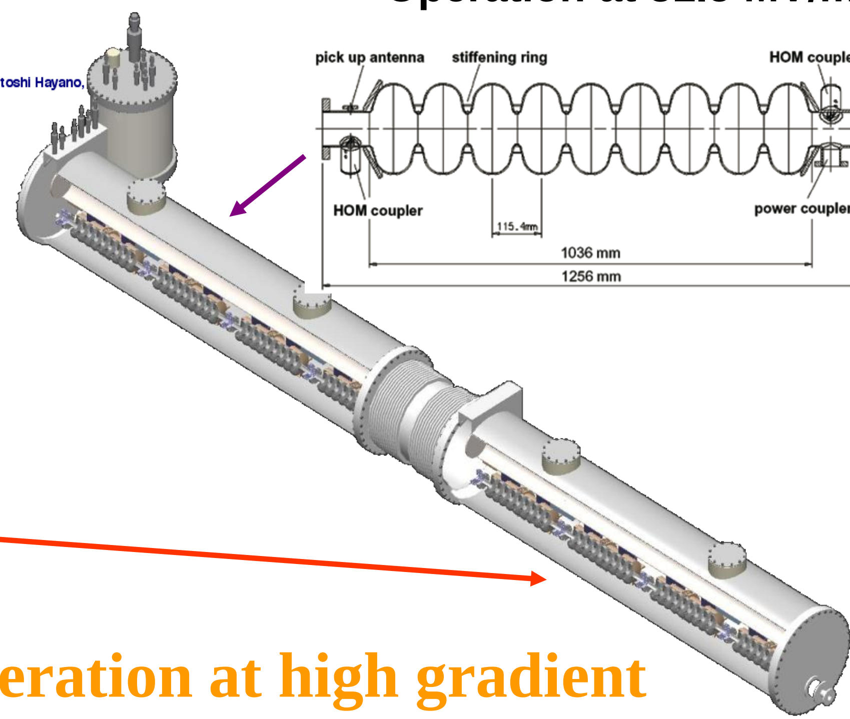
RF Test of LL 9-cell cavity in Cryomodule



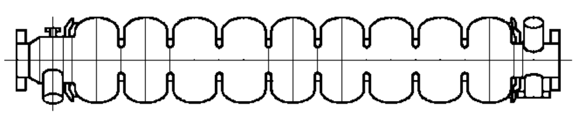
STF @ KEK

STF Phase 1

TESLA design cavities
Operation at 31.5 MV/m



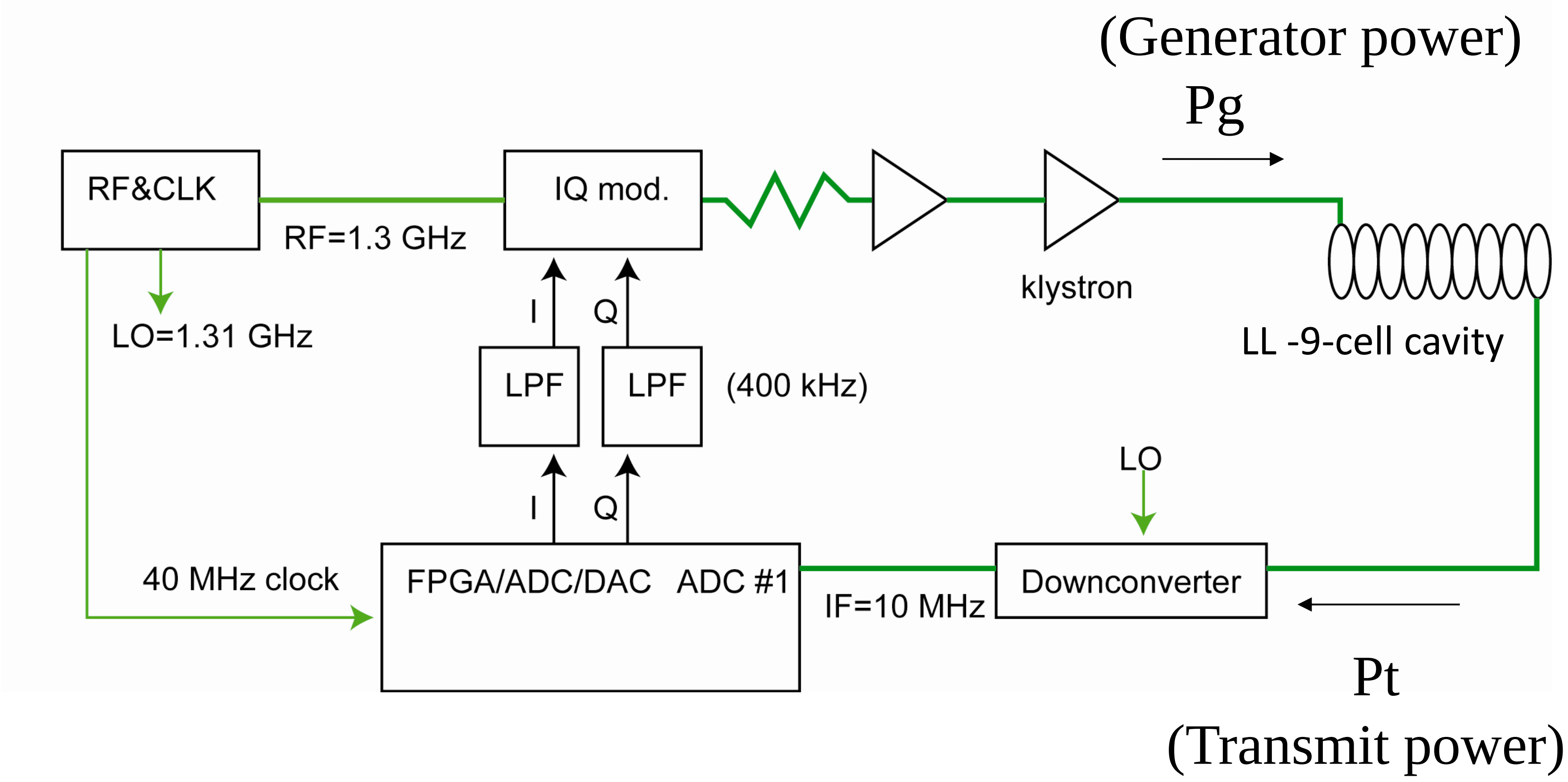
LL 9-cell cavity



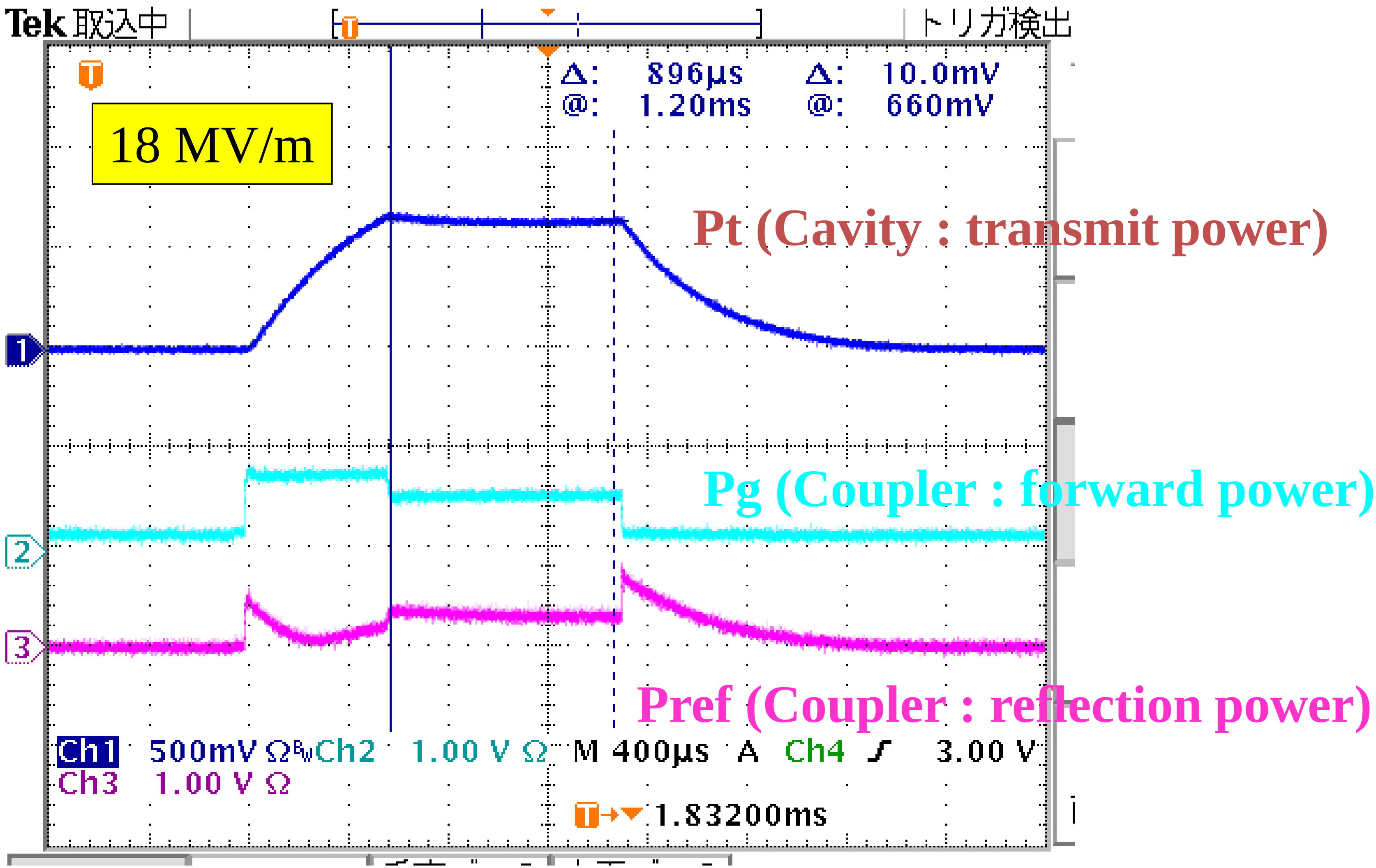
Already fabricated.

Operation at high gradient

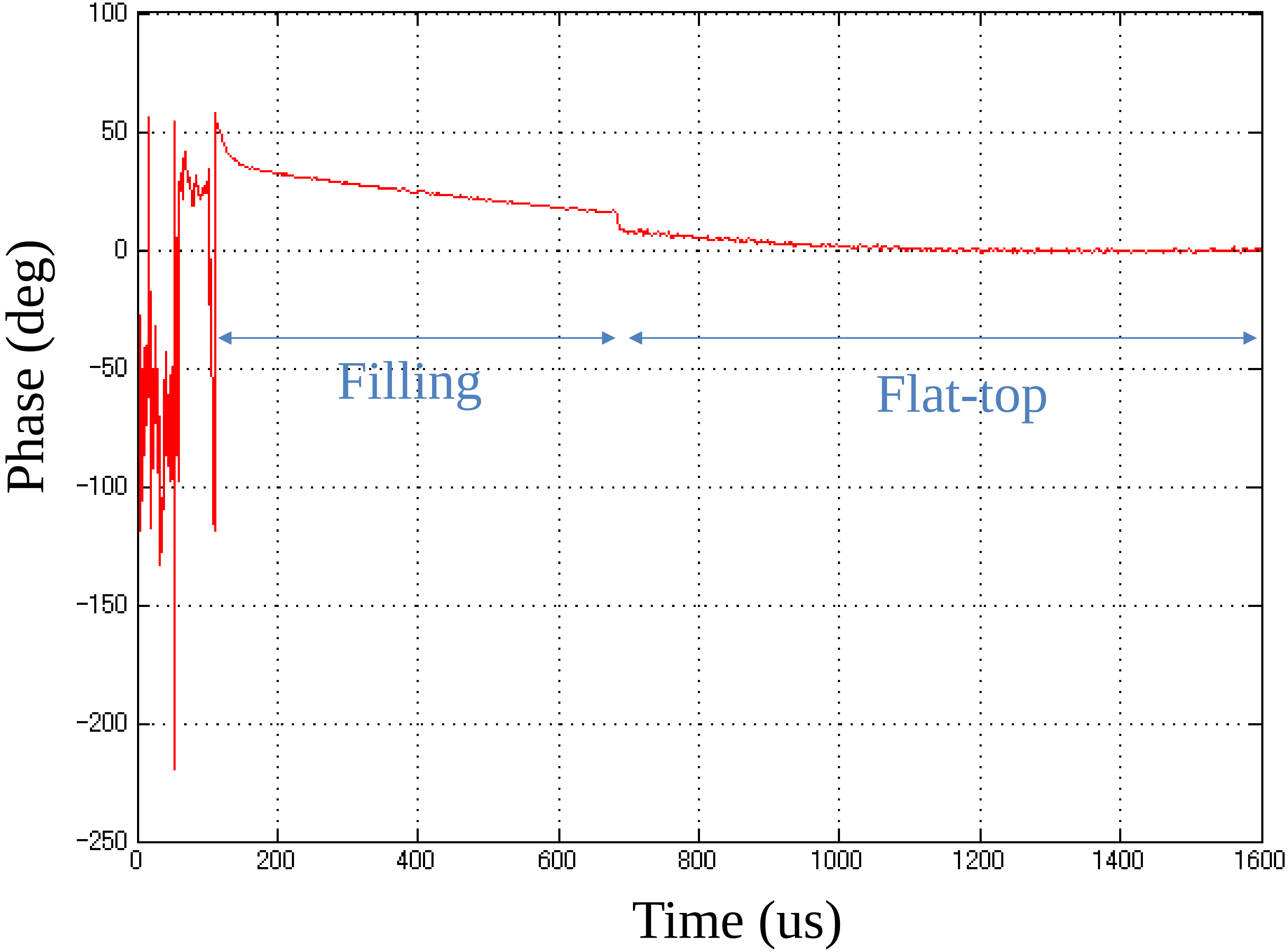
I Q measurement by LLRF (LL 9cell cavity in cryomodule)



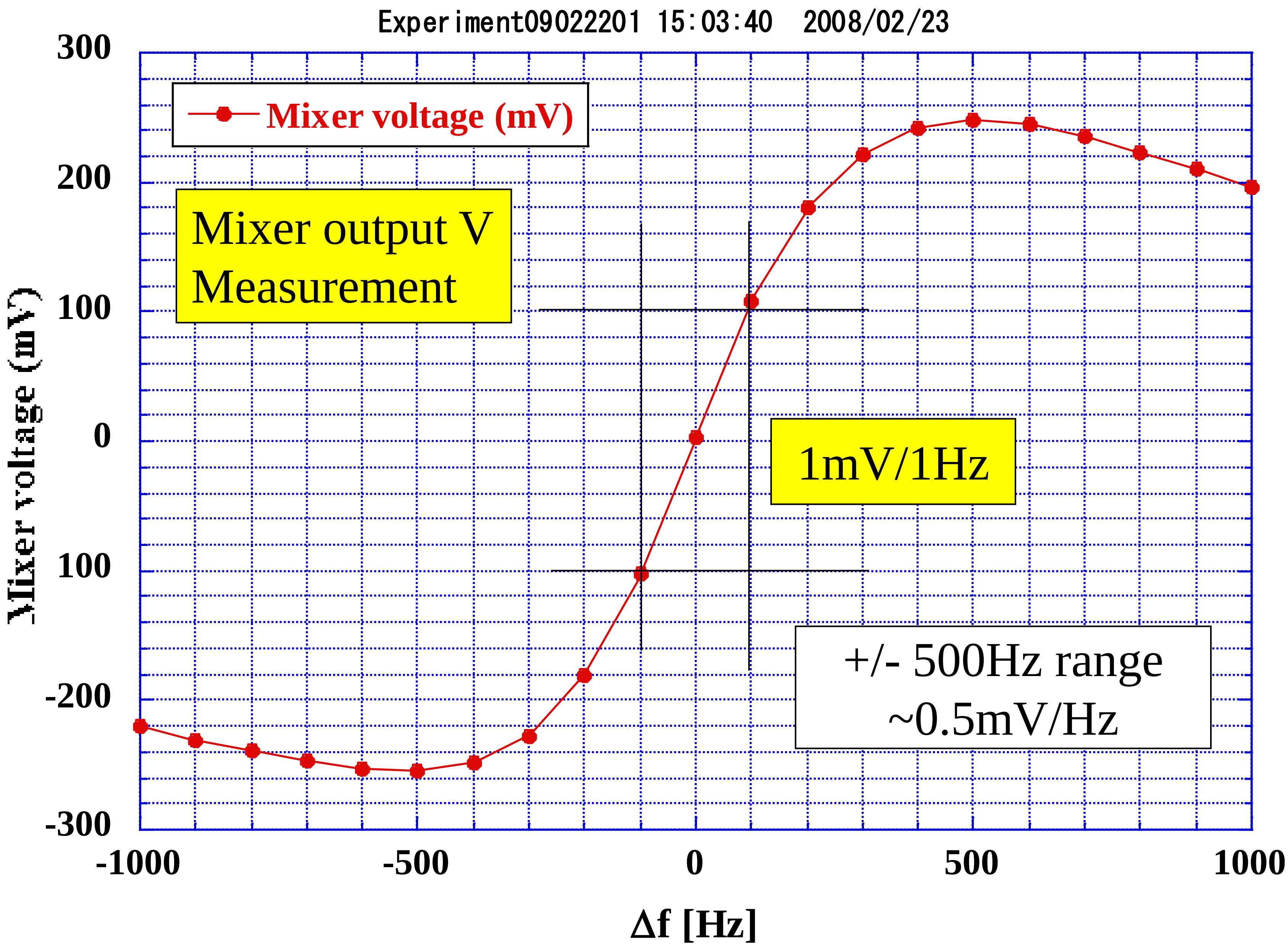
High-power RF Test of LL 9-cell cavity in cryomodule



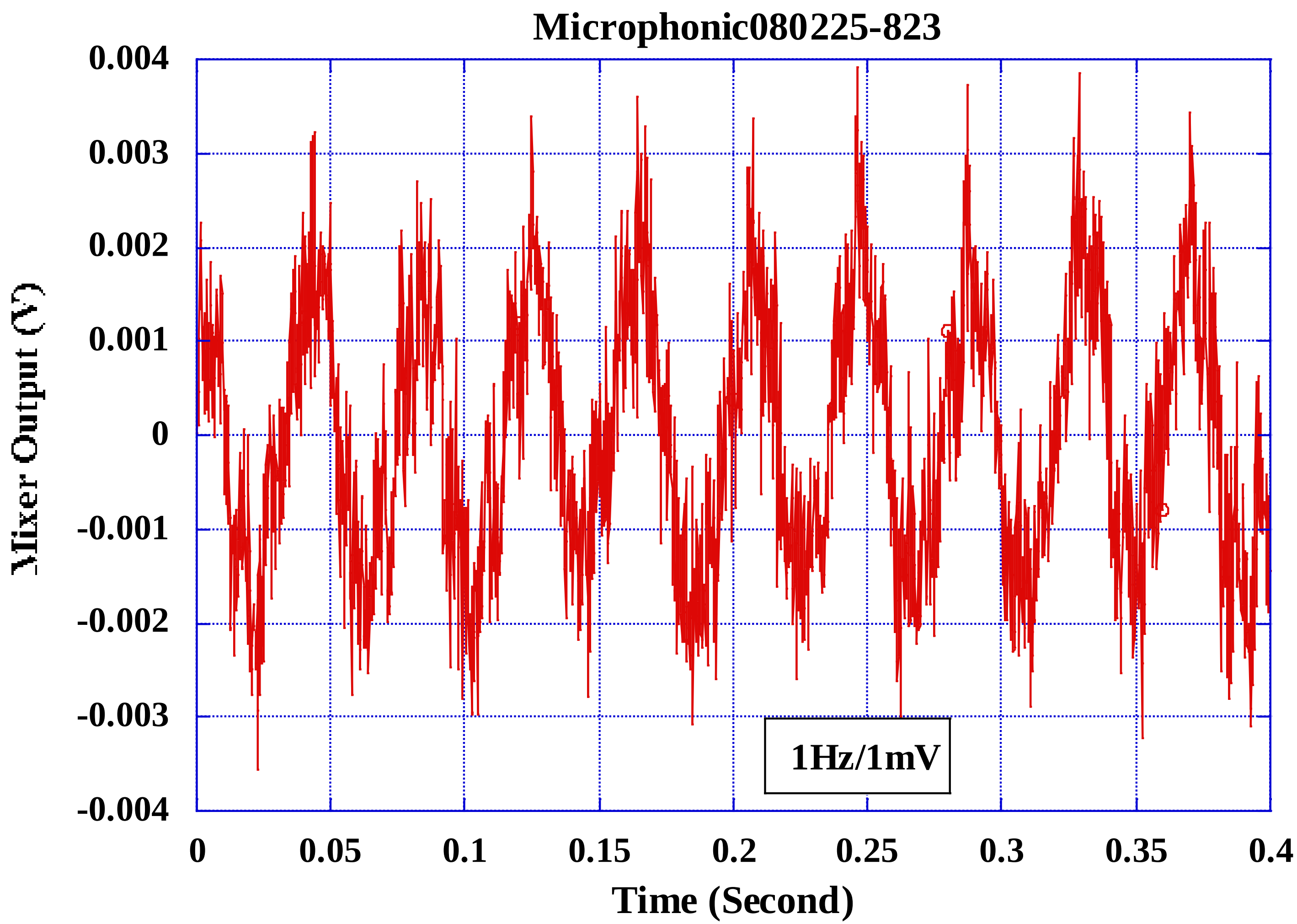
High-power RF Test of LL 9-cell cavity in cryomodule
Phase measurements (LLRF)



Mixer Output Signal @ 25dBm (Pg)



Evaluation of Microphonics (LL 9-cell cavity in CM)



Microphonics is ± 3mV,

which corresponds to ± 3Hz in frequency and ±0.5^{O} in phase variation.