Damping rings' lattice design

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Outline

- Equilibrium emittances and optics conditions for different cells
  - FODO
  - Double Bend Achromat (DBA)
  - Theoretical Minimum Emittance (TME)
  - Multi-Bend Achromat (MBA)
  - Examples from low emittance rings
- Wiggler effect in DR parameters
  - Radiation integrals, energy loss/turn, damping times, energy spread, bunch length, transverse emittance
- The ILC and CLIC DR optics

Equilibrium emittance reminder

$$\epsilon_x = \frac{C_q \gamma^2}{\mathcal{J}_x} \frac{\oint \frac{\mathcal{H}_x(s)}{|\rho_x|^3} ds}{\oint \frac{1}{\rho_x^2} ds} \quad C_q = \frac{55}{32\sqrt{3}} \frac{h}{m_0 c} = 3.83 \times 10^{-13} \text{ m}$$

with the dispersion emittance defined as

$$\mathcal{H}(s) = \beta(s)\eta(s)'^2 + 2\alpha(s)\eta(s)\eta'(s) + \gamma(s)\eta(s)^2$$

- For isomagnetic ring with separated function magnets the equilibrium emittance is written

$$\epsilon_x = 1470 \frac{E^2}{\rho} \frac{1}{l_{\text{bend}}} \int_0^{l_{\text{bend}}} \mathcal{H}_x(s) ds$$

- Smaller bending angle and lower energy reduces emittance
- For evaluating the dispersion emittance, the knowledge of the lattice function is necessary

Optics functions in a FODO cell

- Some basic assumptions
  - Quadrupoles are represented by thin lenses
  - The absolute value of their focal length is the same f
  - The space between quadrupoles is filled completely by dipoles with bending radius ρ and length L and bending angle $\theta = \frac{L}{\rho}$
- With these approximations, the horizontal beta function at the focusing quadrupole is

$$\beta_x = \frac{4f\rho\sin\theta(2f\cos\theta+\rho\sin\theta)}{\sqrt{16f^4-[\rho^2-(4f^2+\rho^2)\cos 2\theta]^2}}$$

- The dispersion at the same location is

$$\eta_x = \frac{2f\rho(2f+\rho\tan\frac{\theta}{2})}{4f^2+\rho^2}$$

- From symmetry, $\alpha_x = \eta_{px} = 0$

- Consider the transfer matrix M and the "optics" matrix

$$A = \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{pmatrix}$$

- The evolution of the optics function between to points $\mathbf{s_0}$ and $\mathbf{s_1}$ is $A(s_1) = M \cdot A(s_0) \cdot M^{\mathsf{T}}$
- The dispersion propagation over a distance Δs with constant bending radius is given by

$$\begin{pmatrix} \eta_x \\ \eta_{px} \end{pmatrix}_{s_1} = M \cdot \begin{pmatrix} \eta_x \\ \eta_{px} \end{pmatrix}_{s_0} + \begin{pmatrix} \rho(1 - \cos\frac{\Delta s}{\rho}) \\ \sin\frac{\Delta s}{\rho} \end{pmatrix}$$

- The transfer matrices to be used for the quadrupole and dipole are

$$M = \begin{pmatrix} 1 & 0 \\ -1/f & 0 \end{pmatrix} \text{ and } M = \begin{pmatrix} \cos\frac{s}{\rho} & \rho\sin\frac{s}{\rho} \\ -\frac{1}{\rho}\sin\frac{s}{\rho} & \cos\frac{s}{\rho} \end{pmatrix}$$

Radiation integrals for a FODO

- The 5th radiation integral can be determined by computing the "curly-H" function evolution along the dipole and integrating to get the mean value
- The algebra is quite involved and it is useful to take the expansion in power series over the bending angle which gives

$$\frac{I_5}{I_2} = \left(4 + \frac{\rho^2}{f^2}\right)^{-\frac{3}{2}} \left[8 - \frac{\rho^2}{2f^2}\theta^2 + O(\theta^4)\right]$$

Radiation integrals for a FODO II

- Some approximations
  - For small bending angle

$$\frac{I_5}{I_2} \approx \left(1 - \frac{\rho^2}{16f^2}\theta^2\right)\left(1 + \frac{\rho^2}{4f^2}\right)^{-\frac{3}{2}} = \left(1 - \frac{L^2}{16f^2}\right)\left(1 + \frac{\rho^2}{4f^2}\right)^{-\frac{3}{2}}$$

  -
  - Considering that $\rho \gg 2f$ (true in most cases)

$$\frac{I_5}{I_2} \approx \left(1 - \frac{L^2}{16f^2}\right)\frac{8f^3}{\rho^3}$$

  - Imposing also $4f \gg L$ (somehow more restrictive)

$$\frac{I_5}{I_2} \approx \frac{8f^3}{\rho^3}$$

Emittance for a FODO cell

- With no quadrupole component in the dipole $\mathcal{J}_x \approx 1$
- The FODO natural emittance can be approximated as

$$\varepsilon_0 \approx C_q \gamma^2 \left(\frac{2f}{L}\right)^3 \theta^3$$

- As already seen, the emittance is scaled to the **square of energy**, but also proportional to the **cube of bending angle** (more cells and a lot of dipoles)
- The emittance is proportional to the **cube of focal length**, i.e. stronger quadrupoles reduce emittance
- The emittance is inversely proportional to the **cube of dipole (or cell) length**, the shortest the cell the better

Minimum Emittance for FODO cell

- The phase advance in a FODO is written as $\cos\mu_x = 1 - \frac{L^2}{2f^2}$ imposing the stability condition $\frac{f}{L} \geq \frac{1}{2}$
- The minimum focal length is obtained for $\mu_x = 180°$ and it is $f = L/2$
- An approximated value of the minimum emittance for a FODO is then

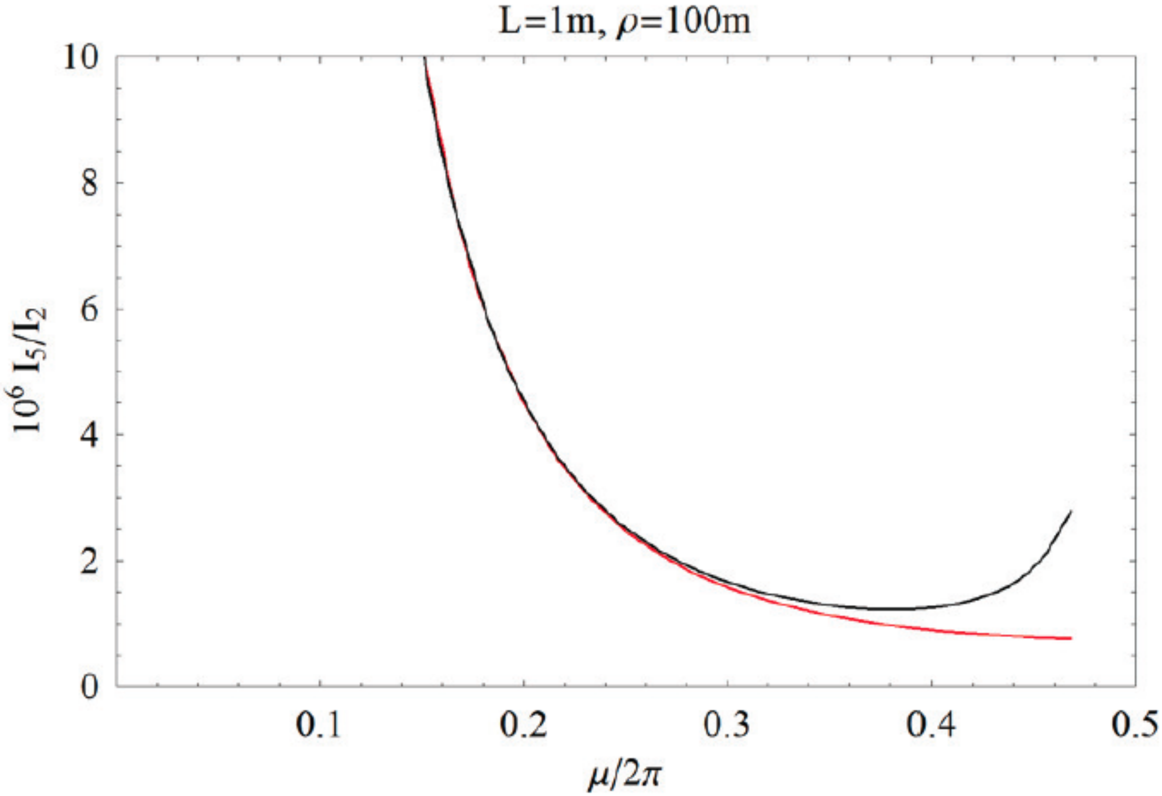
$$\varepsilon_0 \approx C_q \gamma^2 \theta^3$$

- This approximation is indeed breaking down for very high quadrupole strengths (small focal lengths)

Minimum Emittance for FODO cell II

- It can be shown that the actual minimum for a FODO cell is for a phase advance of $\mu_x \approx 137°$
- This sets the minimum emittance of a FODO cell to

$$\varepsilon_{0,\mathrm{FODO,min}} \approx 1.2 C_q \gamma^2 \theta^3$$



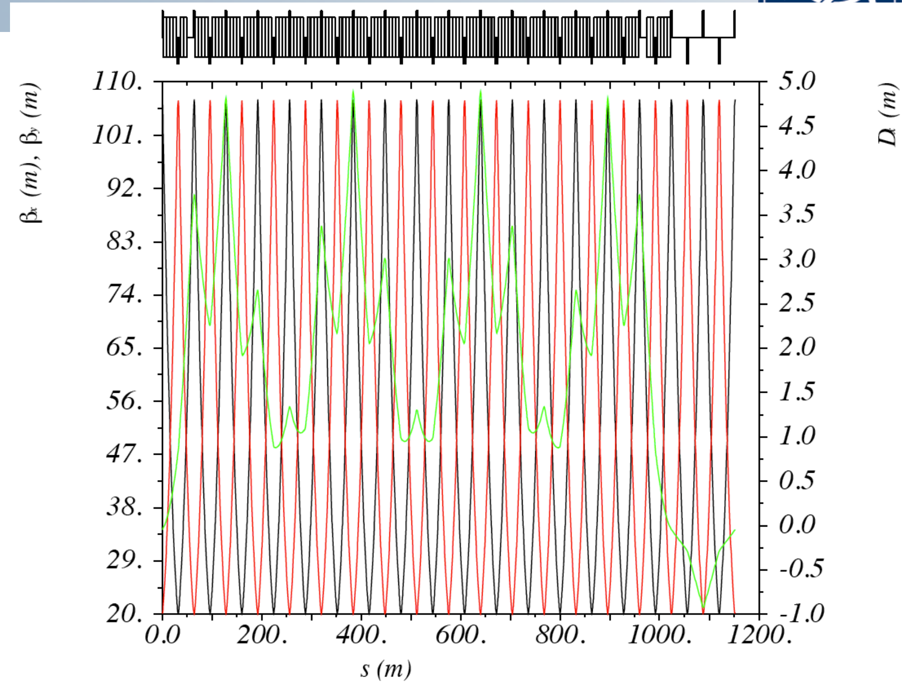
Black line: exact formula.

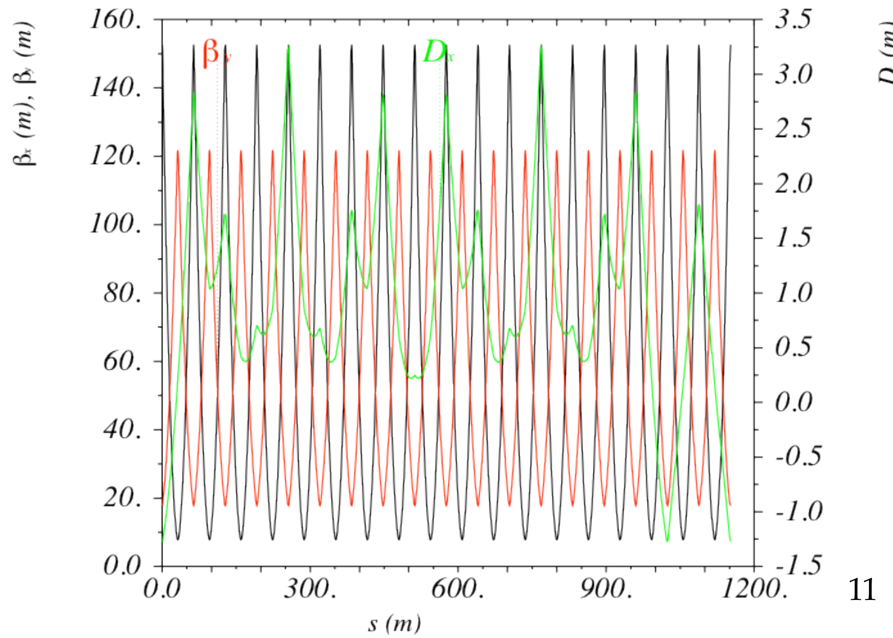
Red line: approximation,

$$\frac{I_5}{I_2} \approx \left(1 - \frac{L^2}{16 f^2}\right) \frac{8 f^3}{\rho^3}$$

An exotic example: SPS low emittance optics

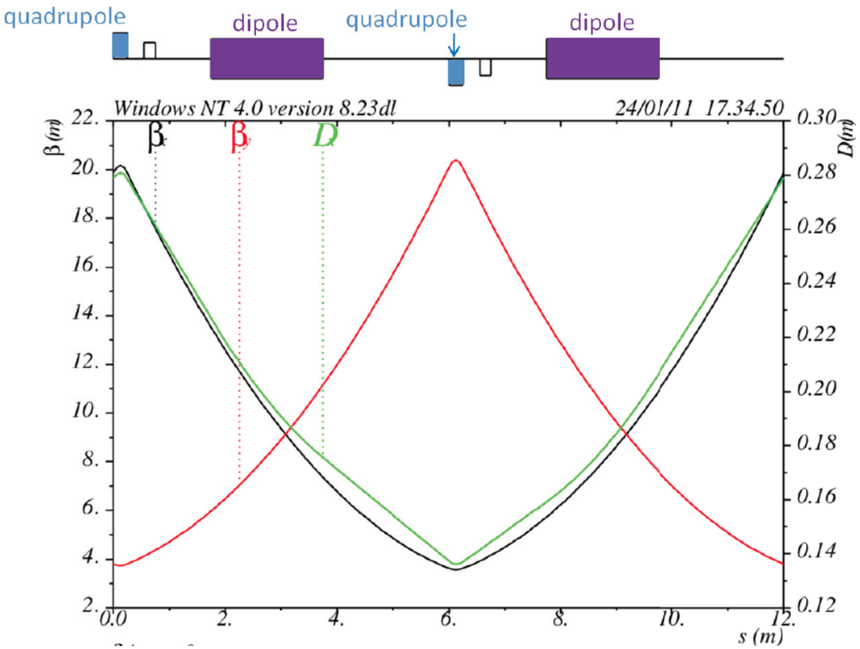
- SPS is a 7km ring with an all FODO cell lattice (6 sextants), with missing dipole
- There are 696 dipoles
- Usually tuned to 90 deg. phase advance for fixed target beams (**Q26**) and since 2012 to 67.5 deg (**Q20**) for LHC beams
- Move horizontal phase advance to 135(3π/4) deg. (**Q40**)
- Normalized emittance with nominal optics @ 3.5GeV of 23.5μm drops to 9μm (1.3nm geometrical)
  - Mainly due to dispersion decrease
  - Almost the normalized emittance of ILC damping rings but still twice the geometrical.
- Damping times of 9s
- Natural chromaticities of -71,-39 (from -20,-27)

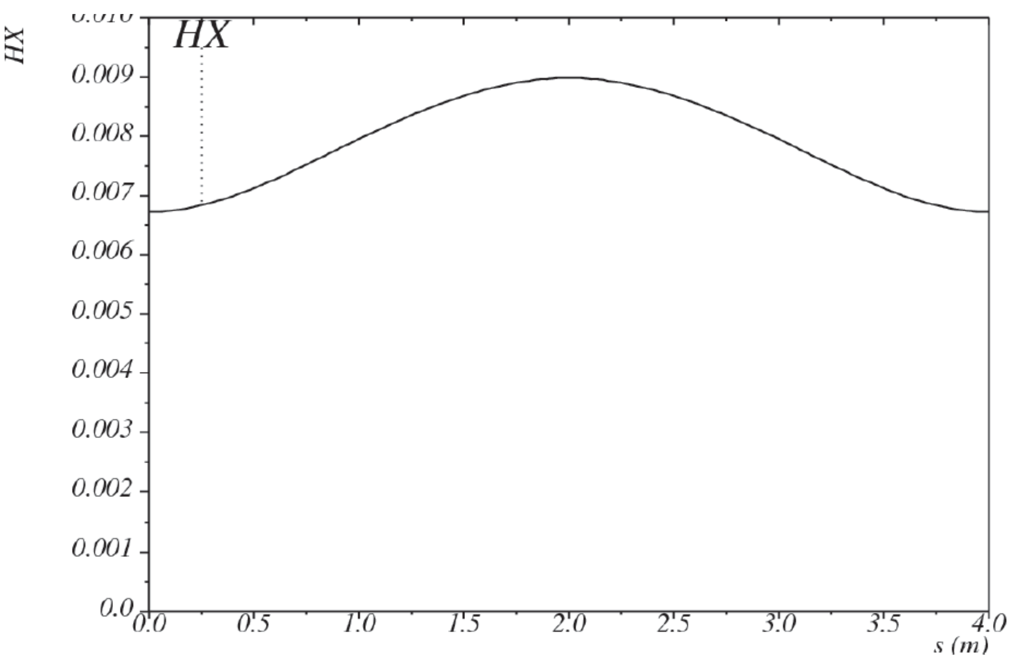




Minimum Emittance for FODO cell II

- The previous examples is actually demonstrating that getting a really small emittance with a FODO cell, a very long ring is needed
- There was no special effort though for minimizing the dispersion emittance, which is almost flat along a dipole





Optics functions through a bend

- Consider the transport matrix of a bending magnet (ignoring edge focusing)

$$\mathcal{M}_{\text{sector}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

- Consider at its entrance the initial optics functions $\beta 0, \alpha 0, \gamma 0, \eta 0, \eta 0'$
- The evolution of the twiss functions, dispersion and dispersion derivative are given by

$$\begin{pmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{pmatrix} = \begin{pmatrix} \text{Cos}\left[\frac{s}{\rho}\right]^2 & -\rho\,\text{Sin}\left[\frac{2s}{\rho}\right] & \rho^2\,\text{Sin}\left[\frac{s}{\rho}\right]^2 \\ \frac{\text{Sin}\left[\frac{2s}{\rho}\right]}{2\rho} & \text{Cos}\left[\frac{2s}{\rho}\right] & -\frac{1}{2}\rho\,\text{Sin}\left[\frac{2s}{\rho}\right] \\ \frac{\text{Sin}\left[\frac{s}{\rho}\right]^2}{\rho^2} & \frac{\text{Sin}\left[\frac{2s}{\rho}\right]}{\rho} & \text{Cos}\left[\frac{s}{\rho}\right]^2 \end{pmatrix} \begin{pmatrix} \beta(0) \\ \alpha(0) \\ \gamma(0) \end{pmatrix}$$

$$\eta(s) = \eta_0 \cos(\frac{s}{\rho}) + \eta_0' \rho \sin(\frac{s}{\rho}) + \rho(1-\cos(\frac{s}{\rho}))$$

$$\eta'(s) = -\frac{\eta_0}{\rho}\sin(\frac{s}{\rho}) + \eta_0' \cos(\frac{s}{\rho}) + \sin(\frac{s}{\rho})$$

Average dispersion emittance

- The dispersion emittance through the dipole is written as

$$\mathcal{H}(s) =$$

$$\gamma 0 \left(\eta 0^2 + \frac{1}{2}\,\eta 0 \left(-4\,\rho + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\right) + \frac{1}{2}\left(3\,\rho^2 - 4\,\rho^2\,\mathrm{Cos}\left[\frac{s}{\rho}\right] + \rho^2\,\mathrm{Cos}\left[\frac{2\,s}{\rho}\right]\right)\right) +$$

$$\beta 0 \left(\frac{1}{2}\left(1 - \mathrm{Cos}\left[\frac{2\,s}{\rho}\right]\right) + 2\,\mathrm{Sin}\left[\frac{s}{\rho}\right]\eta 0' + (\eta 0')^2\right) +$$

$$\alpha 0 \left(\frac{1}{2}\left(-4\,\rho\,\mathrm{Sin}\left[\frac{s}{\rho}\right] + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\mathrm{Sin}\left[\frac{s}{\rho}\right]\right) + \frac{1}{2}\left(-4\,\rho + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\right)\eta 0' + \eta 0\left(2\,\mathrm{Sin}\left[\frac{s}{\rho}\right] + 2\,\eta 0'\right)\right)$$

and its average along the dipole of length *l*

$$\langle\mathcal{H}(s)\rangle =$$

$$\gamma 0 \left(\eta 0^2 - \frac{2\,\eta 0\,\rho\left(l - \rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right]\right)}{l} + \frac{\rho^2\left(6\,l - 8\,\rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right] + \rho\,\mathrm{Sin}\left[\frac{2\,l}{\rho}\right]\right)}{4\,l}\right) +$$

$$\beta 0 \left(\frac{1}{2} - \frac{\rho\,\mathrm{Sin}\left[\frac{2\,l}{\rho}\right]}{4\,l} - \frac{2\,\rho\left(-1 + \mathrm{Cos}\left[\frac{l}{\rho}\right]\right)\eta 0'}{l} + (\eta 0')^2\right) +$$

$$\alpha 0 \left(-\frac{4\,\rho^2\,\mathrm{Sin}\left[\frac{l}{2\,\rho}\right]^4}{l} - \frac{2\,\rho\left(l - \rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right]\right)\eta 0'}{l} + \frac{2\,\eta 0\left(\rho - \rho\,\mathrm{Cos}\left[\frac{l}{\rho}\right] + l\,\eta 0'\right)}{l}\right)$$

Optics functions for minimum emittance

- Take the derivative of the dispersion emittance with respect to the initial optics functions and equate it to zero to find the minimum conditions
- Non-zero dispersion (general case)

$$\beta_0 = \frac{\rho^3\ (2\ (-1+\theta^2+\mathrm{Cos}[2\,\theta])+\theta\,\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{\theta\,\rho^4\ (-9\,\theta+2\,\theta^3+8\,\theta\,\mathrm{Cos}[\theta]+\theta\,\mathrm{Cos}[2\,\theta]+8\,\mathrm{Sin}[\theta]-4\,\mathrm{Sin}[2\,\theta])}}$$

$$\alpha_0 = \frac{\rho^2\ (-\theta+\theta\,\mathrm{Cos}[2\,\theta]+4\,\mathrm{Sin}[\theta]-2\,\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{\theta\,\rho^4\ (-9\,\theta+2\,\theta^3+8\,\theta\,\mathrm{Cos}[\theta]+\theta\,\mathrm{Cos}[2\,\theta]+8\,\mathrm{Sin}[\theta]-4\,\mathrm{Sin}[2\,\theta])}}$$

$$\eta_0 = \rho - \frac{\rho\,\mathrm{Sin}[\theta]}{\theta} \quad \text{and} \quad \eta_0' = \frac{-1+\mathrm{Cos}[\theta]}{\theta}$$

- Zero dispersion (and its derivative)

$$\beta_0 = \frac{\rho^2\ (6\,\theta-8\,\mathrm{Sin}[\theta]+\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{-\rho^2\ (9-6\,\theta^2-16\,\mathrm{Cos}[\theta]+7\,\mathrm{Cos}[2\,\theta]+8\,\theta\,\mathrm{Sin}[\theta]+2\,\theta\,\mathrm{Sin}[2\,\theta])}}$$

$$\alpha_0 = \frac{4\,\rho\,\mathrm{Sin}\left[\frac{\theta}{2}\right]^4}{\sqrt{-\frac{9\,\rho^2}{2}+3\,\theta^2\,\rho^2-\frac{1}{2}\,\rho\ (-16\,\rho\,\mathrm{Cos}[\theta]+7\,\rho\,\mathrm{Cos}[2\,\theta]+2\,\theta\,\rho\ (4\,\mathrm{Sin}[\theta]+\mathrm{Sin}[2\,\theta]))}}$$

Minimum emittance conditions

- In the general case, the equilibrium emittance takes the form

$$\epsilon_x = \frac{1}{\mathrm{Jx}\,\theta^2}\left(735\sqrt{2}\,\mathrm{En}^2\sqrt{\theta\,(-9\,\theta + 2\,\theta^3 + 8\,\theta\,\mathrm{Cos}[\theta] + \theta\,\mathrm{Cos}[2\,\theta] + 8\,\mathrm{Sin}[\theta] - 4\,\mathrm{Sin}[2\,\theta])}\right)$$

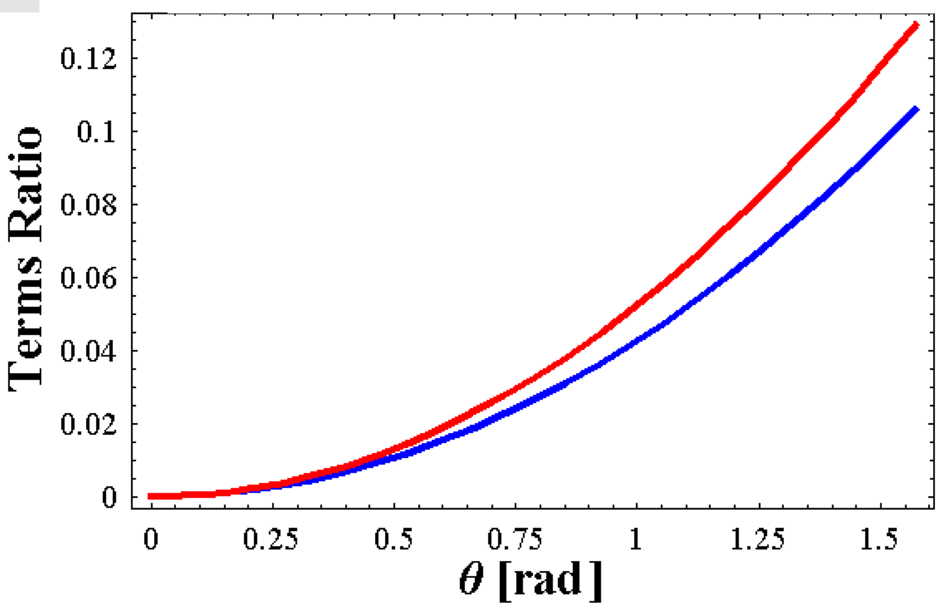
and expanding on θ we have

$$\epsilon_x = \frac{49\sqrt{\frac{5}{3}}\,\mathrm{En}^2\,\theta^3}{2\,\mathrm{Jx}} - \frac{7\left(\sqrt{\frac{3}{5}}\,\mathrm{En}^2\right)\theta^5}{4\,\mathrm{Jx}} + O[\theta]^6$$

- In the 0-dispersion case,

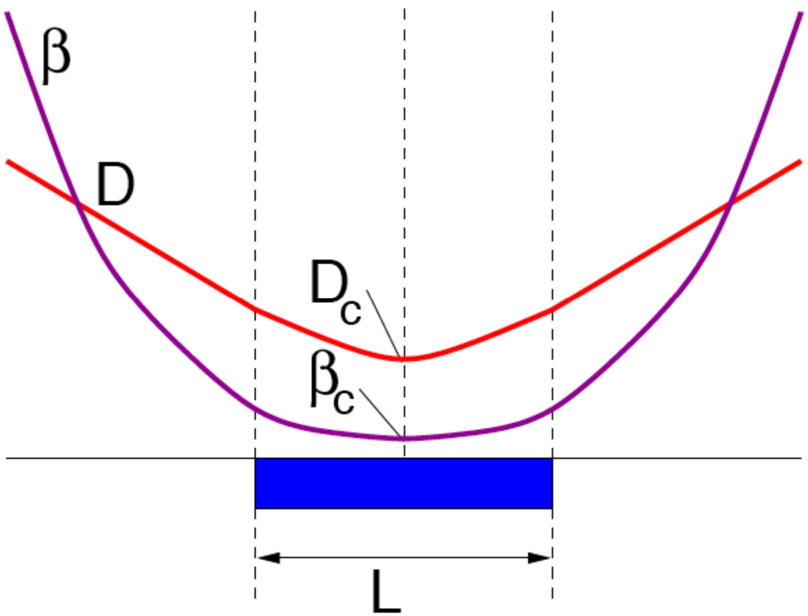
$$\epsilon_x = \frac{735\sqrt{2}\,\mathrm{En}^2\sqrt{-9 + 6\,\theta^2 + 16\,\mathrm{Cos}[\theta] - 7\,\mathrm{Cos}[2\,\theta] - 4\,\theta\,(2 + \mathrm{Cos}[\theta])\,\mathrm{Sin}[\theta]}}{\mathrm{Jx}\,\theta}$$

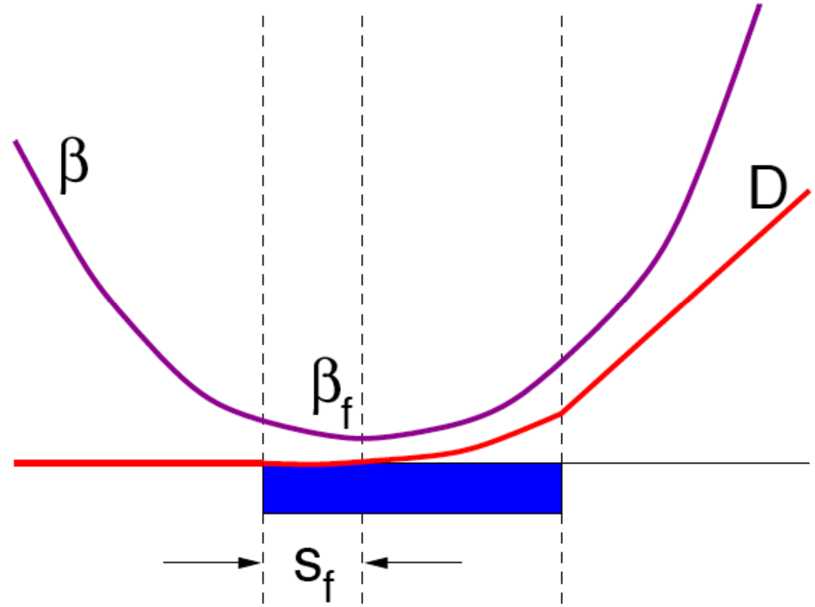
$$\epsilon_x = \frac{49\sqrt{15}\,\mathrm{En}^2\,\theta^3}{2\,\mathrm{Jx}} - \frac{77\,\mathrm{En}^2\,\theta^5}{4\left(\sqrt{15}\,\mathrm{Jx}\right)} + O[\theta]^6$$



- The second order term is negligible (less the 1% for θ < 20 deg.)
- Note that in both cases the emittance depends on the 3rd power of the bending angle
- The emittance for non-zero dispersion is 3 times smaller

Optics functions for minimum emittance





$$\beta_0 = \frac{8\,\rho\,\theta}{\sqrt{15}} - \frac{4\,\rho\,\theta^3}{5\sqrt{15}} + \frac{257\,\rho\,\theta^5}{7350\sqrt{15}} + O[\theta]^6$$

$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{5}{3}}\,\theta^2 + \frac{269\,\theta^4}{1470\sqrt{15}} + O[\theta]^6$$

$$\eta_0 = \frac{\rho\,\theta^2}{6} - \frac{\rho\,\theta^4}{120} + O[\theta]^6$$

$$\eta_0' = -\frac{\theta}{2} + \frac{\theta^3}{24} - \frac{\theta^5}{720} + O[\theta]^6$$

$$\beta_0 = 2\sqrt{\frac{3}{5}}\,\rho\,\theta - \frac{2\,\rho\,\theta^3}{5\sqrt{15}} + \frac{11\sqrt{\frac{3}{5}}\,\rho\,\theta^5}{2450} + O[\theta]^6$$

$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{3}{5}}\,\theta^2 + \frac{13}{490}\sqrt{\frac{3}{5}}\,\theta^4 + O[\theta]^6$$

$$\eta_0 = \eta_0' = 0$$

- Introduce the dimensionless quantities $\bar{\beta_0} = \frac{\beta_0}{\beta_{0;\min}}$

$$\bar{\eta_0} = \frac{\eta_0}{\eta_{0;\min}} \quad \text{and} \quad F = \frac{\epsilon_x}{\epsilon_{x;\min}}$$

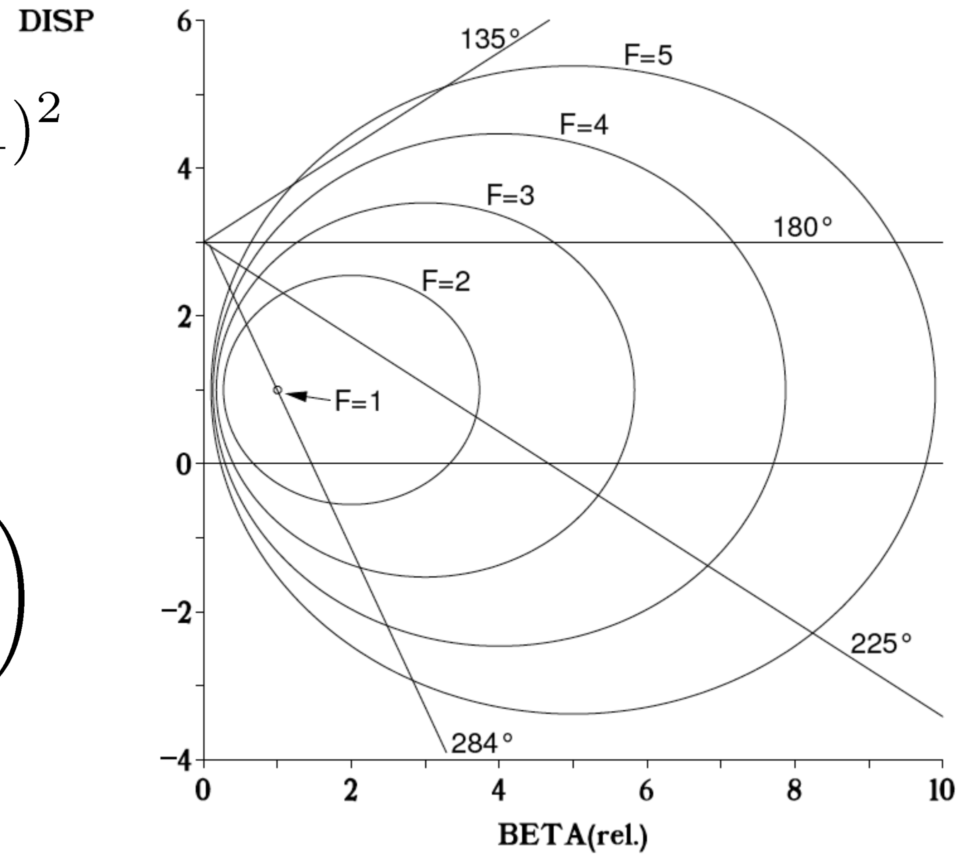
- Introduce them into the expression of the mean dispersion emittance to get

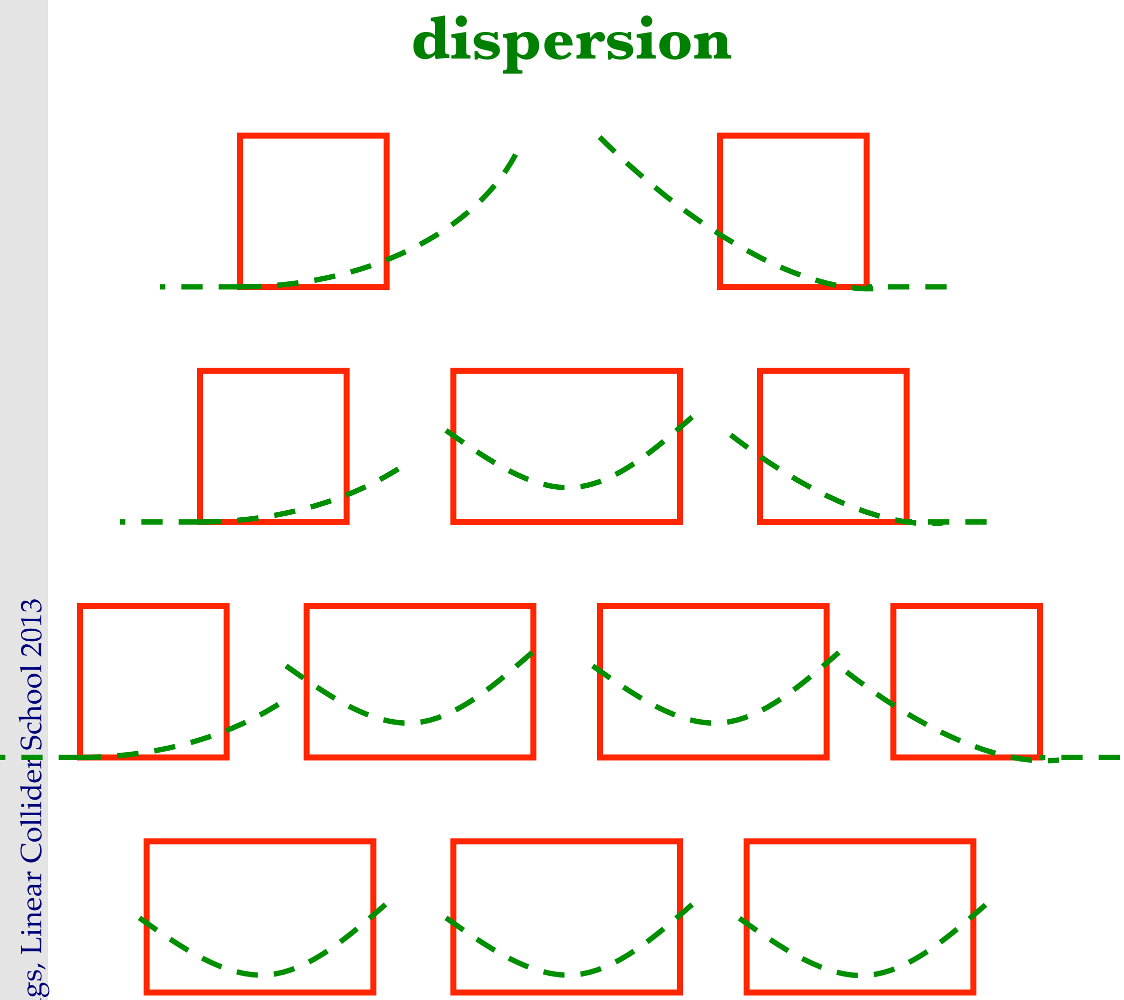
$$F^2 = (\bar{\beta_0} - F)^2 + 5/4(\bar{\eta_0} - 1)^2$$

- The curves of equal relative emittance are ellipses
- The phase advance for a mirror symmetric cell is

$$\mu = \arctan\left(\frac{6}{\sqrt{15}}\frac{\bar{\beta_0}}{\bar{\eta_0} - 3}\right)$$

- The optimum phase advance for reaching the minimum emittance (F=1) is unique (284.5°)!





- Double Bend Achromat (DBA)
- Triple Bend Achromat (TBA)
- Quadruple Bend Achromat (QBA)
- Minimum Emittance Lattice (MEL)

Chasmann-Green cell

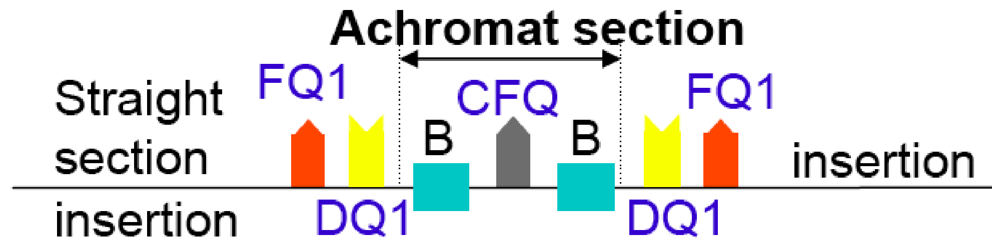
- Double bend achromat with unique central quadrupole
- Achromatic condition is assured by tuning the central quadrupole
- Minimum emittance with a quadrupole doublet in either side of the bends
- The required focal length of the quad is given by

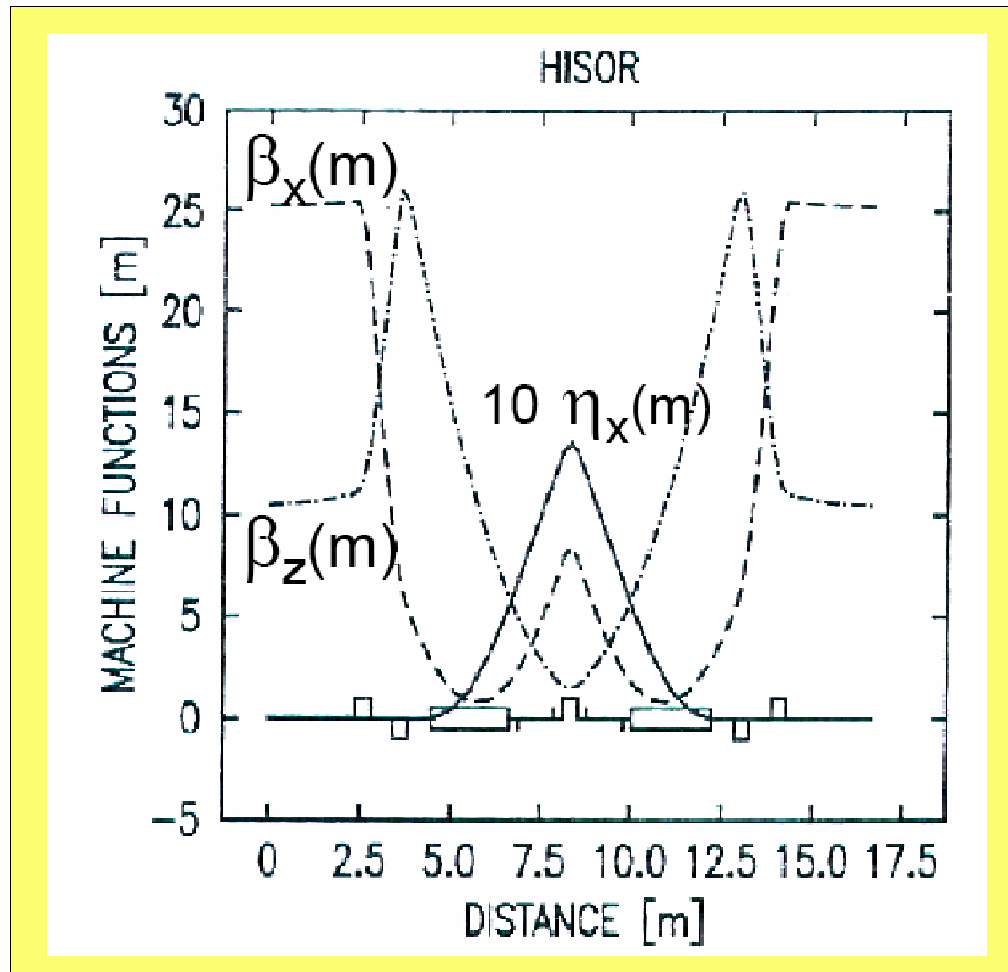
$$f = \frac{1}{2}\left(L_{\rm drift} + \frac{1}{2}L_{\rm bend}\right)$$

and the dispersion

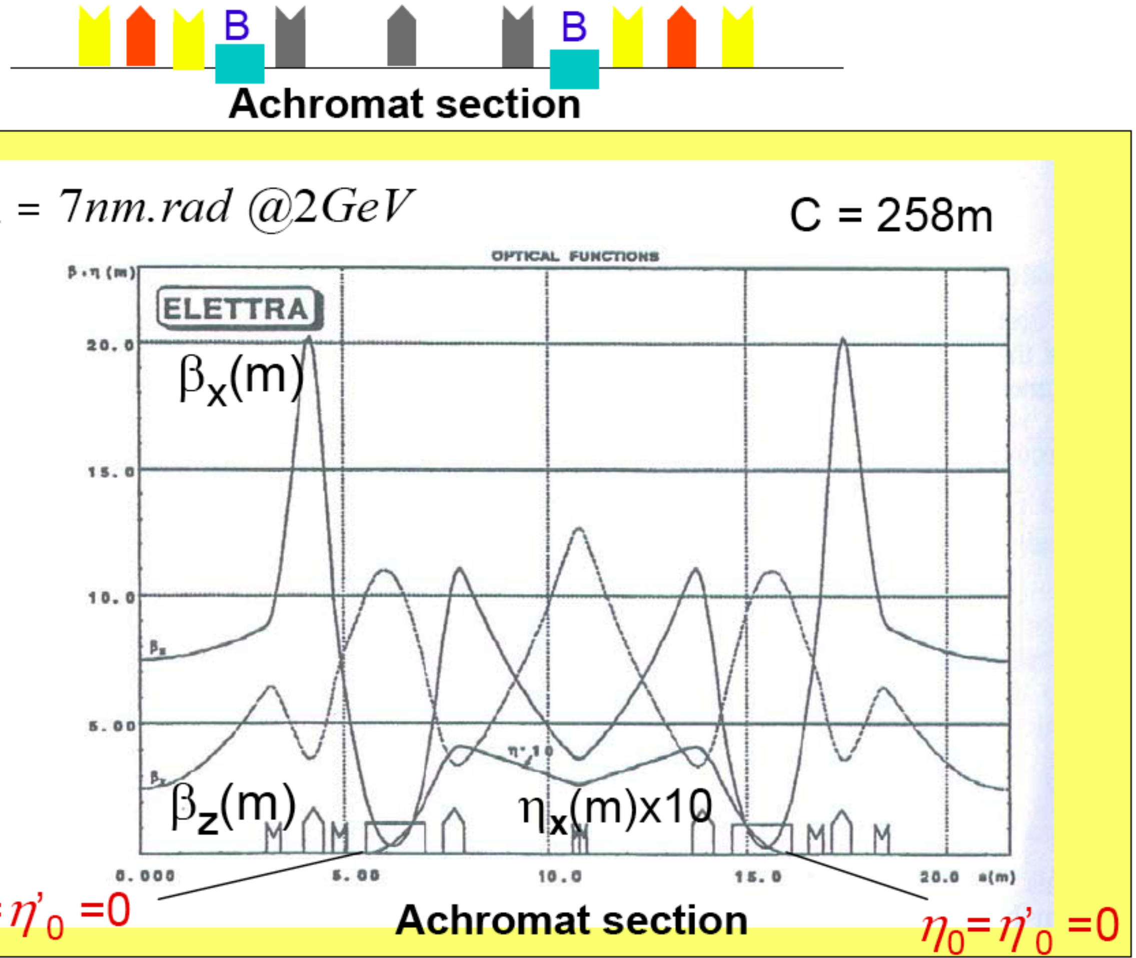
$$D_c = \left(L_{\rm drift} + \frac{1}{2}L_{\rm bend}\right)\theta$$

- Disadvantage the limited tunability and reduced space



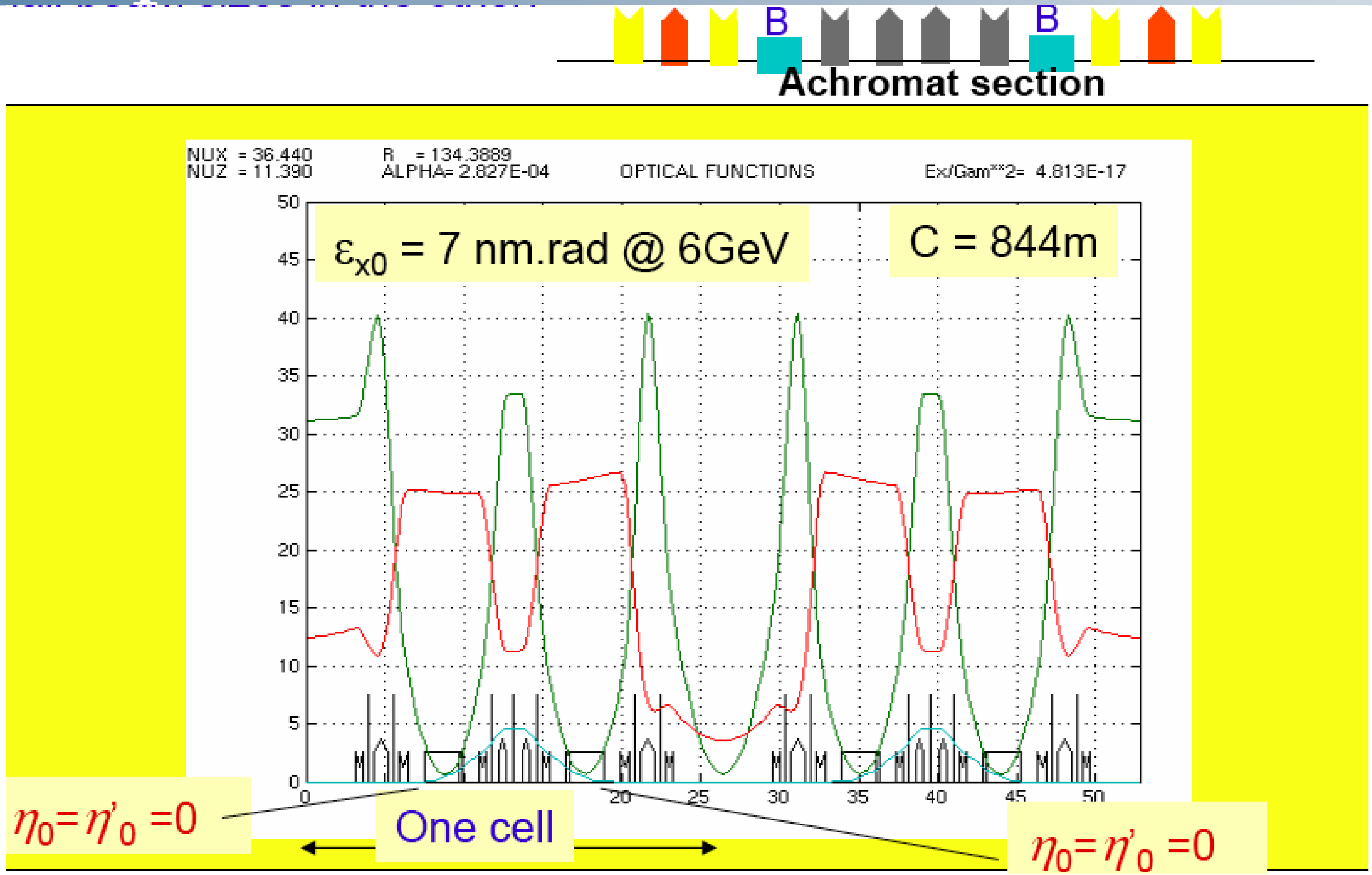


DBA with triplet



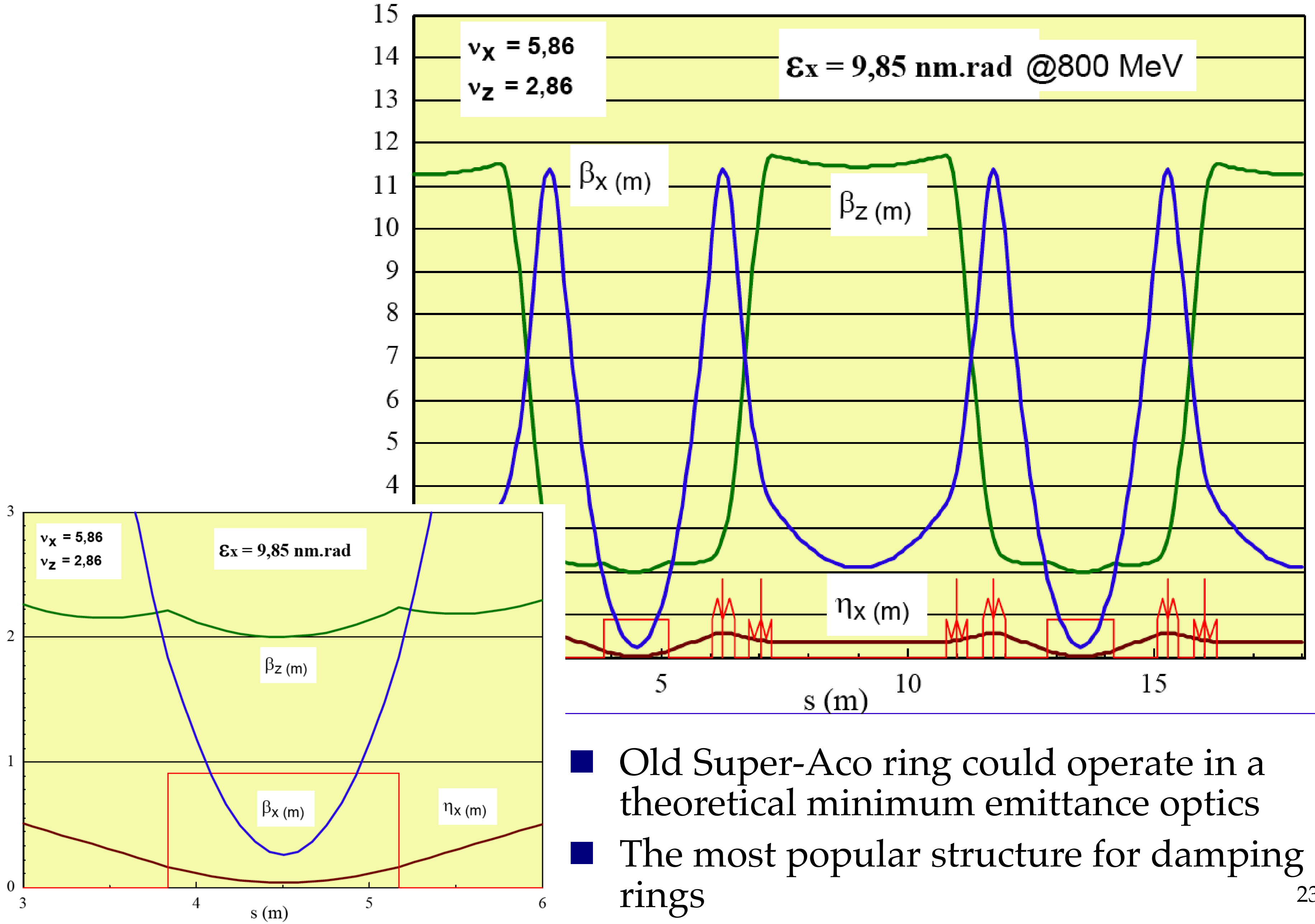
- Central triplet between the two bends and two triplets in the straight section to achieve the minimum emittance and achromatic condition
- Elettra (Trieste) uses this lattice achieving almost the absolute minimum emittance for an achromat
- Disadvantage the increased space in between the bends

Expanded DBA



- Original lattice of ESRF storage ring, with 4 quadrupoles in between the bends
- Alternating moderate and low beta in intertions

Theoretical minimum emittance optics



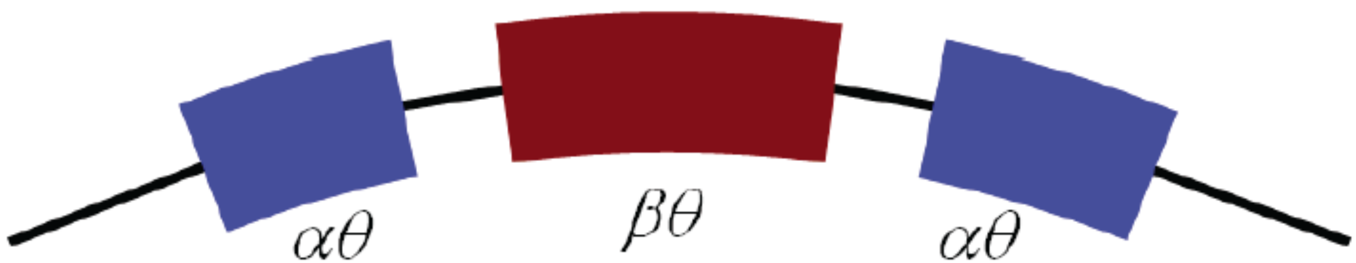
- Old Super-Aco ring could operate in a theoretical minimum emittance optics
- The most popular structure for damping rings

Natural emittance for different lattice cells

Lattice style	Minimum emittance	Conditions/comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2} C_q \gamma^2 \theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
137° FODO	$\varepsilon_0 \approx 1.2 C_q \gamma^2 \theta^3$	minimum emittance FODO
DBA	$\varepsilon_0 \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5}L \quad \alpha_{x,0} \approx \sqrt{15}$
TME	$\varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,\min} \approx \frac{L\theta}{24} \quad \beta_{x,\min} \approx \frac{L}{2\sqrt{15}}$

Combining DBA and TME

- Most of the rings would benefit from areas with zero dispersion for placing special equipment including beam transfer, RF and damping wigglers
- A combination of TME in between dipoles were dispersion vanishes would be beneficial
- These **Multi-bend achromats** (MBA) are the latest trend of X-ray storage rings upgrade projects
- Consider the simple case with variable lengths but same bending radius, and *M* dipoles



- The bending angles then satisfies the condition

$$2\alpha + (M-2)\beta = M$$

Multi-bend achromat emittance

- The radiation integrals are additive quantities so they are

$$I_{5,\text{cell}} \approx \frac{2}{4\sqrt{15}} \frac{(\alpha\theta)^4}{\rho} + \frac{(M-2)}{12\sqrt{15}} \frac{(\beta\theta)^4}{\rho} = \frac{6\alpha^4 + (M-2)\beta^4}{12\sqrt{15}} \frac{\theta^4}{\rho}$$

and $$I_{2,\text{cell}} \approx 2\frac{\alpha\theta}{\rho} + (M-2)\frac{\beta\theta}{\rho} = [2\alpha + (M-2)\beta]\frac{\theta}{\rho}$$

- Their ratio is given by $$\frac{I_{5,\text{cell}}}{I_{2,\text{cell}}} \approx \frac{1}{12\sqrt{15}} \left[\frac{6\alpha^4 + (M-2)\beta^4}{2\alpha + (M-2)\beta}\right] \theta^3$$
- It is minimized for the following conditions

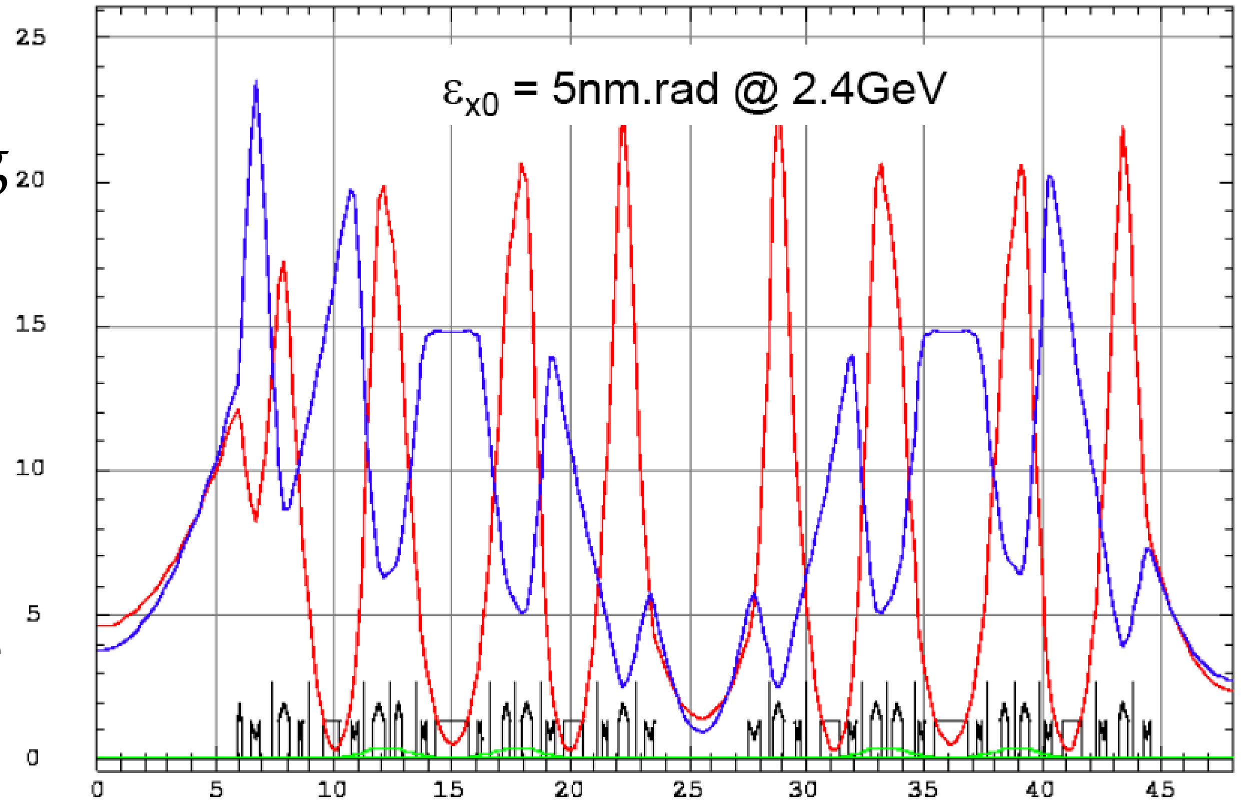
$$\frac{\alpha}{\beta} = \frac{1}{\sqrt[3]{3}}, \qquad \frac{6\alpha^4 + (M-2)\beta^4}{2\alpha + (M-2)\beta} \approx \frac{M+1}{M-1}$$

- This implies that the central bends are longer by a factor of $\sqrt[3]{3}$
- The minimum natural emittance is given by

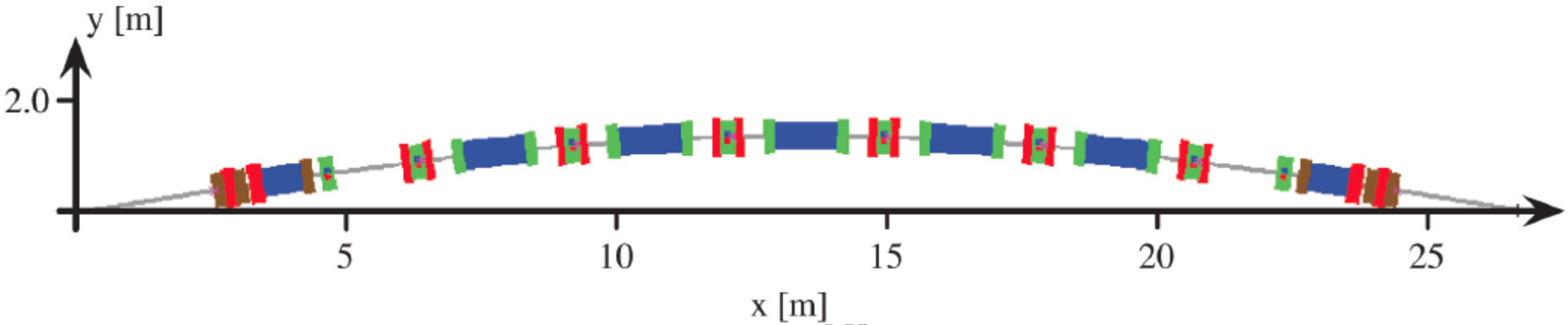
$$\varepsilon_0 \approx C_q \gamma^2 \frac{1}{12\sqrt{15}} \left(\frac{M+1}{M-1}\right) \theta^3, \qquad 2 < M$$

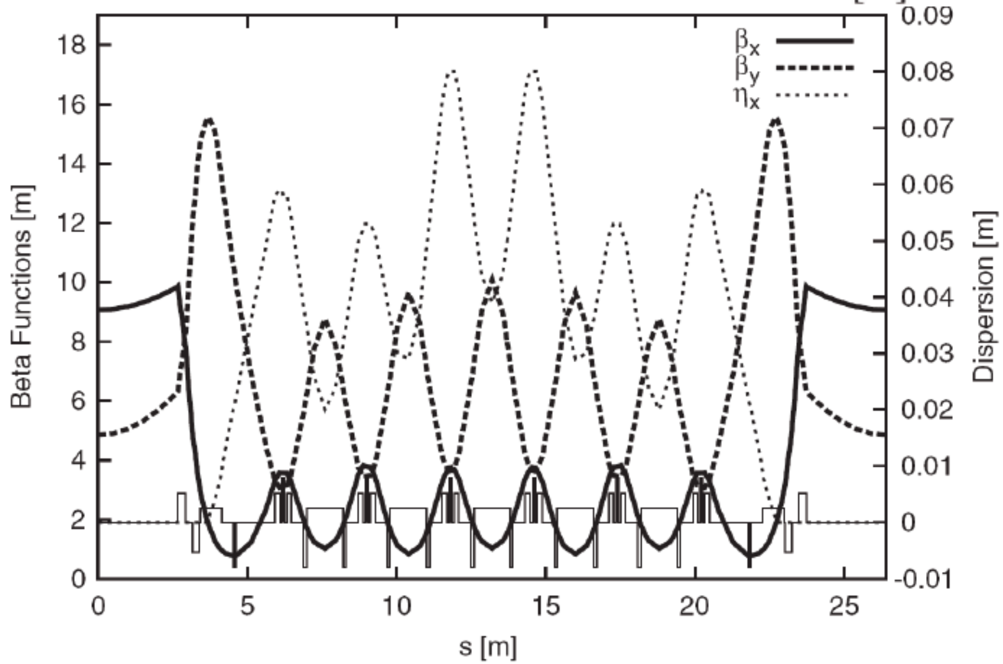
Triple Bend Achromat

- Three bends with the central one with theoretical minimum emittance conditions
- Strict relationship between the bending angles and lengths of dipoles in order to achieve dispersion matching $\frac{L_2^3}{\rho_2^2} = 3\frac{L_1^3}{\rho_1^2}$
- A unique phase advance of 255º is needed for reaching the minimum emittance
- This minimum is half the DBA one
- Example, the **Swiss Light Source**



Seven-Bend achromat: MAXIV





Beam energy	3 GeV
Circumference	528 m
Number of cells	20
Horizontal emittance (no IDs)	0.326 nm
Horizontal emittance (with IDs)	0.263 nm

- Very compact ring design with vertical focusing provided by including gradient in the bending magnets

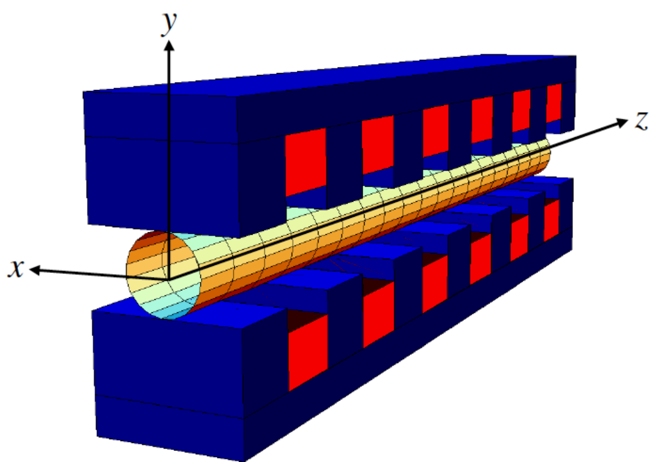
Summary of lattice performance

$$\varepsilon_0 \approx F C_q \gamma^2 \theta^3$$

Lattice style	F
90° FODO	$2\sqrt{2}$
137° FODO	1.2
Double-bend achromat (DBA)	$\frac{1}{4\sqrt{15}}$
Multi-bend achromat	$\frac{1}{12\sqrt{15}}\left(\frac{M+1}{M-1}\right)$
TME	$\frac{1}{12\sqrt{15}}$

Damping wigglers

- A wiggler magnet is a magnetic device producing a vertical field which alternates in polarity along the beam direction.
- In general, wiggler magnets give rise to both radiation damping and quantum excitation
- They result in different equilibrium values of damping times emittance and energy spread which depend both on the wiggler magnet parameters and on the lattice functions through the wiggler
- In the first order approximation, the vertical field component of a wiggler raises along the beam axis is



$$B_y = B_w \sin(k_z z)$$

with

Peak field = B_w

Period = $\lambda_w = \dfrac{2\pi}{k_z}$

Radiation integrals for damping wigglers

- The contribution from a wiggler to the *i*th synchrotron integrals can be written as $I_i = I_{ia} + I_{iw}$

with I_{ia} and I_{iw} the synchrotron integrals produced in the arcs and in the wigglers

- Assuming that wiggler magnets with sinusoidal field variation are installed in the dispersion free region of the ring, the radiation integrals for the wigglers can be written

$$I_{2w} = \frac{L_{ID}}{2\rho_w^2}\ , \qquad I_{3w} = \frac{4}{3\pi}\frac{L_{ID}}{\rho_w^3}\ , \qquad I_{4w} = -\frac{3}{32\pi^2}\frac{\lambda_w^2}{\rho_w^4}L_{ID}\ ,$$

$$I_{5w} = \frac{\lambda_w^4}{4\pi^4\rho_w^5}\left[\frac{3}{5\pi} + \frac{3}{16}\right]\langle\gamma_x\rangle L_{ID} - \frac{9\lambda_w^3}{40\pi^4\rho_w^5}\langle\alpha_x\rangle L_{ID} + \frac{\lambda_w^2}{15\pi^3\rho_w^5}\langle\beta_x\rangle L_{ID}$$

with the wiggler length L_{ID} equal to $\lambda_w \cdot N_p$, i.e the product of wiggler period length and the number of periods

- The 4th and 5th radiation integrals arise from the dispersion generated by the wiggler magnet (self-dispersion), although the 4th is quite small for small wiggler periods

Relative damping factor

- The change of the damping rate due to the wiggler is conventionally defined by the relative damping factor

$$F_w \equiv \frac{I_{2w}}{I_{2a}} = \frac{L_w B_w^2}{4\pi(B\rho)B_a} = \frac{1}{4\pi \cdot 0.0017\,[\mathrm{Tm}]} \frac{L_w B_w^2}{\gamma B_a}$$

with B_a the bending field of the arc dipoles. When $F_w > 1$ the damping is dominantly achieved by the wigglers

- The energy loss per turn is

$$U_0 = U_{0a}(1+F_w) = 3.548 \times 10^{-12}[\mathrm{MeV}]\gamma^3 B_a[\mathrm{T}](1+F_w)$$

- The horizontal damping partition number is $J_x = \frac{J_{xa}+F_w}{1+F_w}$ and still very close to 1 for wiggler dominated rings

Damping times with wigglers

- The damping times are very influenced in a large extend by the damping wigglers

$$\tau_x = \frac{2E_0T_0}{J_xU_0} = \frac{3(B\rho)C}{2\pi\, r_0 c\gamma^3 B_a(J_{xa}+F_w)} = E_2\frac{C}{B_a\gamma^2(J_{xa}+F_w)}$$

$$\tau_y = \frac{2E_0T_0}{J_yU_0} = \frac{3(B\rho)C}{2\pi\, r_0 c\gamma^3 B_a(1+F_w)} = E_2\frac{C}{B_a\gamma^2(1+F_w)}$$

$$\tau_p = \frac{2E_0T_0}{J_\varepsilon U_0} = \frac{3(B\rho)C}{2\pi\, r_0 c\gamma^3 B_a(3-J_{xa}+2F_w)} = E_2\frac{C}{B_a\gamma^2(3-J_{xa}+2F_w)}$$

with

$$E_2 = \frac{3(B\rho)}{2\pi\, r_0 c\gamma} = \frac{3\cdot 0.0017\,[\mathrm{Tm}]}{2\pi\, r_0 c} = 960.13\,\left[\frac{\mathrm{T}\cdot\mathrm{sec}}{\mathrm{m}}\right]$$

Equilibrium emittance with wigglers

- Assuming that $\langle\alpha_x\rangle$ is small, the largest dominant term for the 5th radiation give

$$I_{5w} = \frac{\lambda_w^2}{15\pi^3\rho_w^5}\langle\beta_x\rangle L_{ID}$$

Sinusoidal field model

$$I_{5w} = \frac{\lambda_w^2}{384\,\rho_w^5}\langle\beta_x\rangle L_{ID}$$

hard-edge field model

- Assuming the hard-edge model, the emittance in a wiggler dominated ring with TME arc cells can be written as

$$\gamma\epsilon_{x0} = \frac{C_q\gamma^3}{12\,(J_{xa}+F_w)}\left[\frac{\epsilon_r\,\theta^3}{\sqrt{15}} + \frac{F_w|B_w^3|\lambda_w^2\langle\beta_x\rangle}{16(B\rho)^3}\right]$$

- This approximation ignores the details of the dispersion suppressor optics at the start and end of the arcs, but is still a fairly accurate description, especially when the number of TME cells per arc is large

Energy spread and mom. compaction

- The equilibrium rms energy spread is

$$\sigma_\delta = \gamma\sqrt{\frac{C_q I_3}{2I_2 + I_3}} = \gamma\left[\frac{C_q|B_a|}{(B\rho)}\frac{1+F_w\left|\frac{B_w}{B_a}\right|}{3-J_{xa}+2F_w}\right]^{1/2}$$

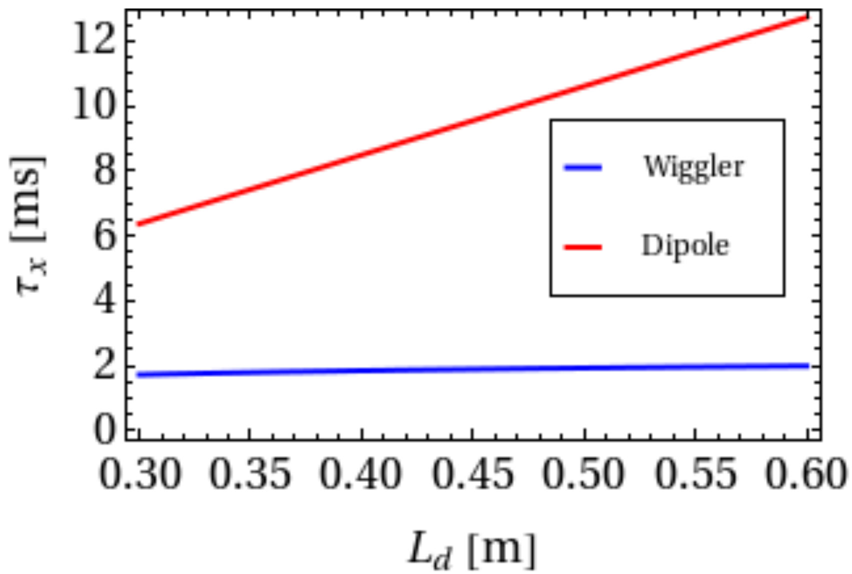
- The equilibrium bunch length depends also on the momentum compaction factor and the parameters of the RF system
- Considering the ratio of the emittance with respect to the absolute emittance minimum, ϵ_r, the momentum compaction factor with wigglers takes a complicated form

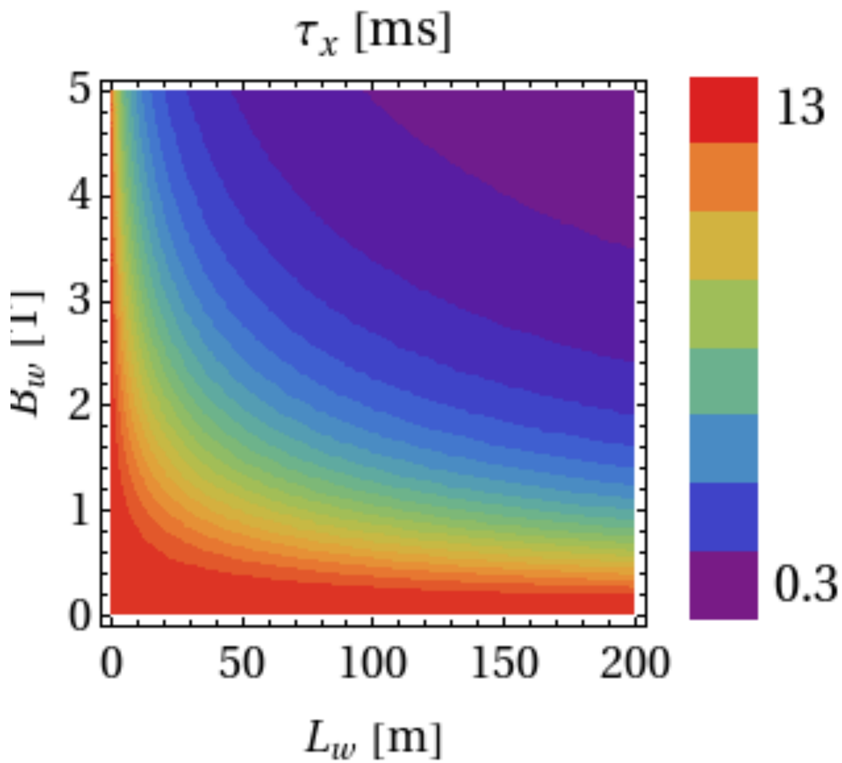
$$\alpha_p = \frac{3\pi}{2}\left(\frac{4\sqrt{15}}{9}\right)^{2/3}\frac{(B\rho)(1+F_w)^{2/3}}{C|B_a|\gamma^2}$$

$$\times \left(\frac{\gamma\epsilon_{x0}}{C_q} - \frac{|B_w^3|\lambda_w^2\langle\beta_x\rangle\gamma^3}{192(B\rho)^3}\frac{F_w}{J_{xa}+F_w}\right)^{2/3} \times \frac{\sqrt{5}+\sqrt{\epsilon_r^2-1}}{\epsilon_r^{2/3}}$$

CLIC DR – damping times

F. Antoniou, PhD thesis, NTUA 2013

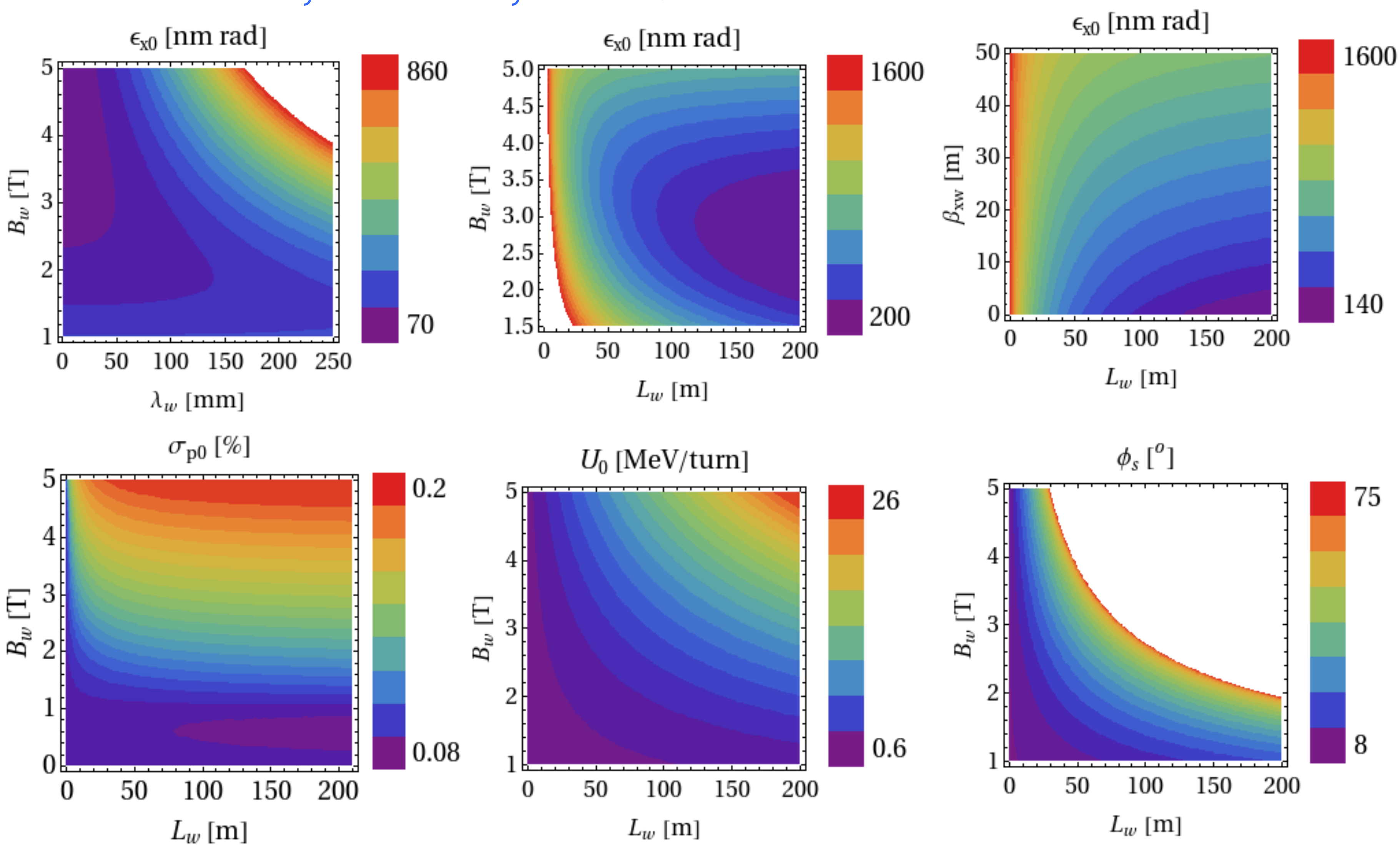




- To damp the beam from 63 μm-rad to 500 nm-rad in less than 20 ms a maximum damping time of 4 ms is required →
- Large dipole fields (or very small dipole length)
  - Cannot be achieved by normal conducting dipoles
- Fast damping times can be achieved for large wiggler fields and/or large wiggler total length

Effect of damping wigglers on CLIC DR

F. Antoniou, PhD thesis, NTUA 2013

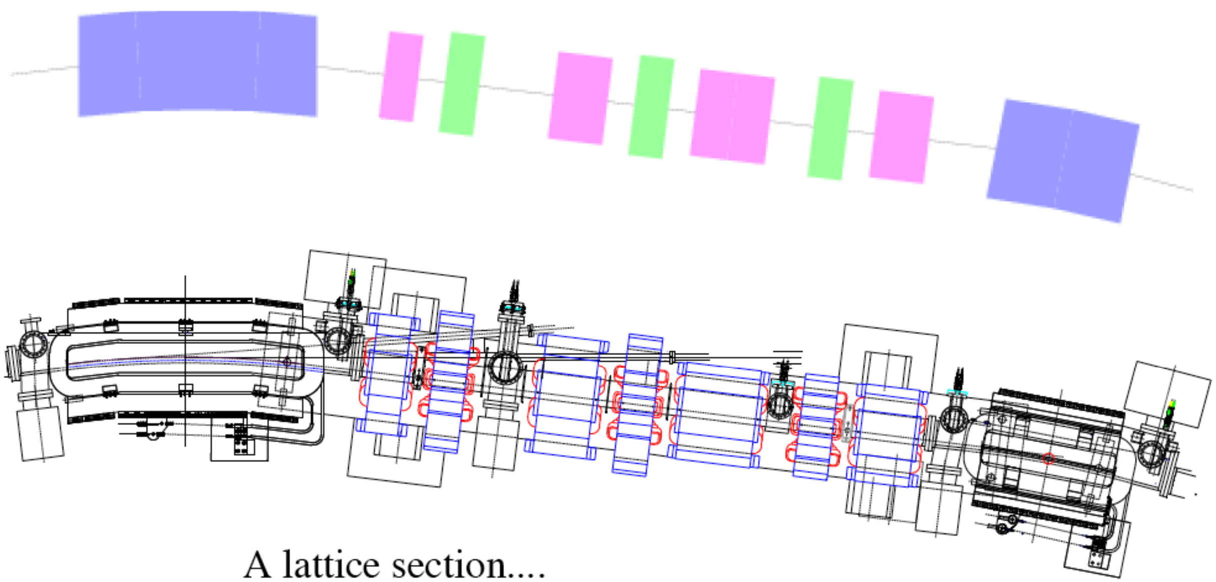


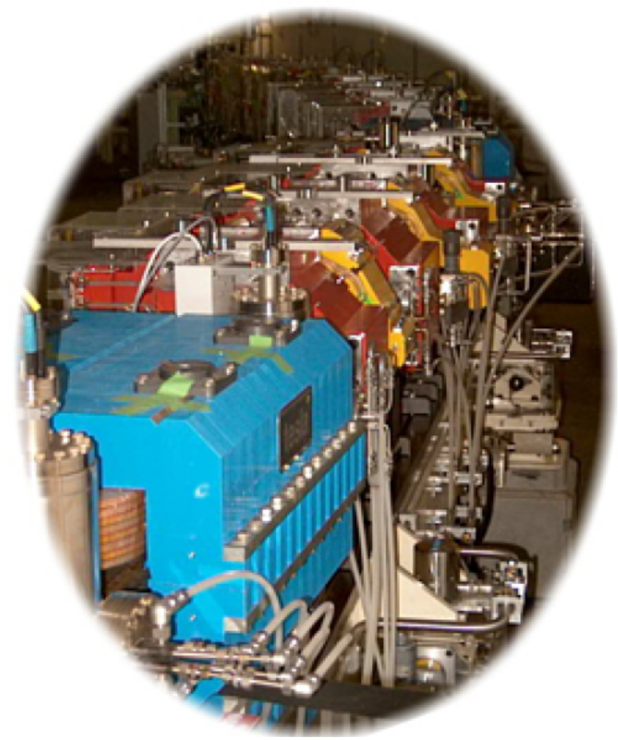
Lattice design phases

- Initial preparation
  - Performance
  - Boundary conditions and constraints
  - Building blocks (magnets)
- Linear lattice design
  - Build modules, and match them together
  - Achieve optics conditions for maximizing performance
  - Global quantities choice working point and chromaticity
- Non-linear lattice design
  - Chromaticity correction (sextupoles)
  - Dynamic aperture
- Real world
  - Include imperfections and foresee corrections

Lattice design inter-phase

Magnet Design: Technological limits, coil space, field quality
Vacuum: Impedance, pressure, physical apertures, space
Radiofrequency: Energy acceptance, bunch length, space
Diagnostics: Beam position monitors, resolution, space
Alignment: Orbit distortion and correction
Mechanical engineering: Girders, vibrations
Design engineering: Assembly, feasibility





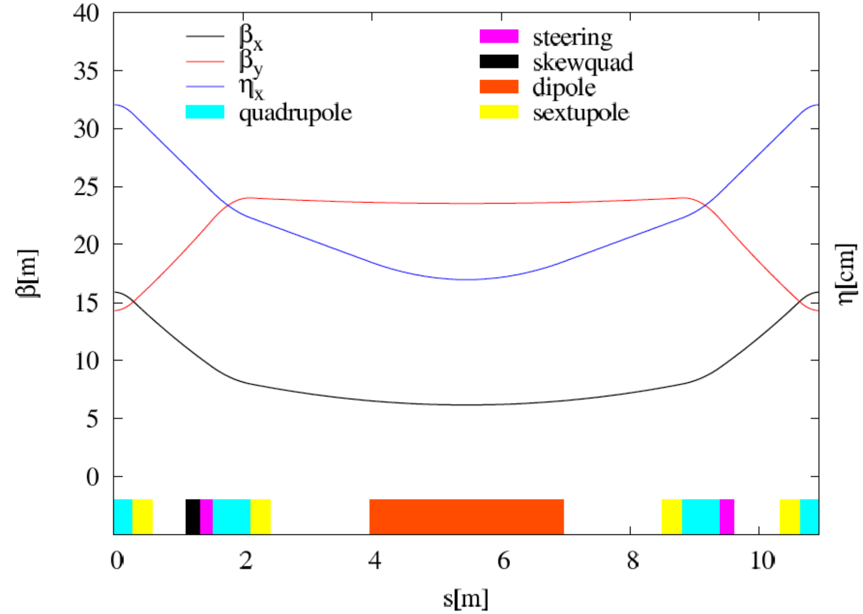
A lattice section....
(top) .....as seen by the lattice designer
(bottom) ..... as seen by the design engineer
(right) ..... and how it looks in reality

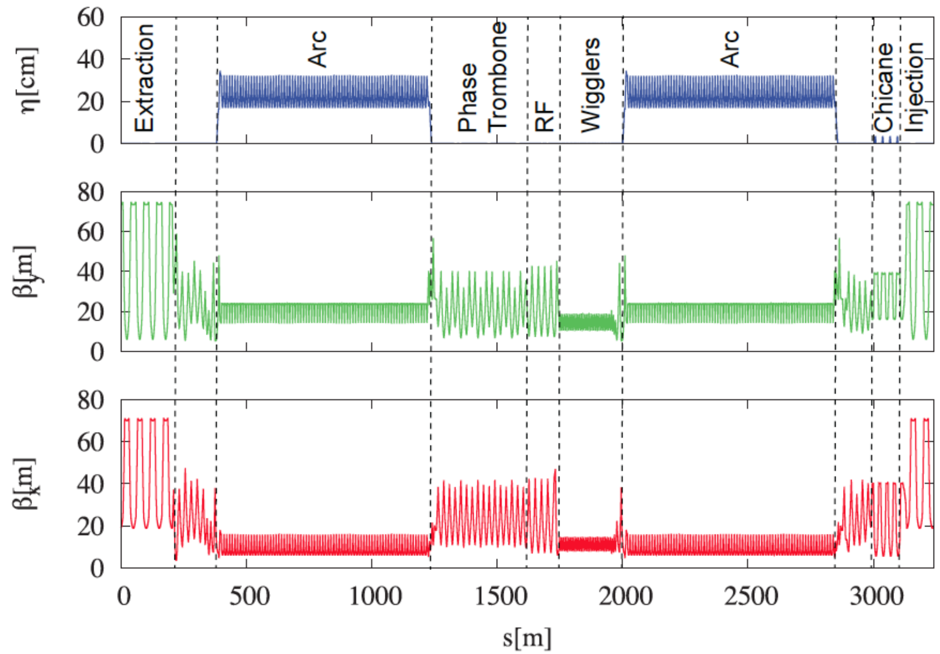
Other devices

Device	Parameter	Purpose
RF cavities	RF phase and Voltage	Acceleration, phase stability
Septum	Position and width	injection
Kicker	Integrated dipole field	injection
Orbit corrector	Integrated dipole field	Orbit correction
Quadrupole corrector	Integrated quad field	Restoring periodicity
Skew quadrupoles	Integrated skew quad field	Coupling correction
Undulators, wigglers	Number of periods, wavelength, field and gap	Synchrotron radiation
Vacuum pumps	Passive	Keep high vacuum
Beam position monitors, other instrumentation	Passive	Position, beam parameters measurement
Absorbers	Passive	Synchrotron radiation absorption

The ILC DR TDR optics

- The ring has a race-track shape with two arcs and two straight sections
- The arc is filled with 75 TME-like cells with one additional defocusing quadrupole for tuning flexibility
- The dispersion is zeroed at the end of the arc by dispersion suppressor cells (two dipoles and 7 quads)
- The straight sections are filled with different type FODO cells, depending on the different function (wigglers, phase trombone, beam transfer, RF)





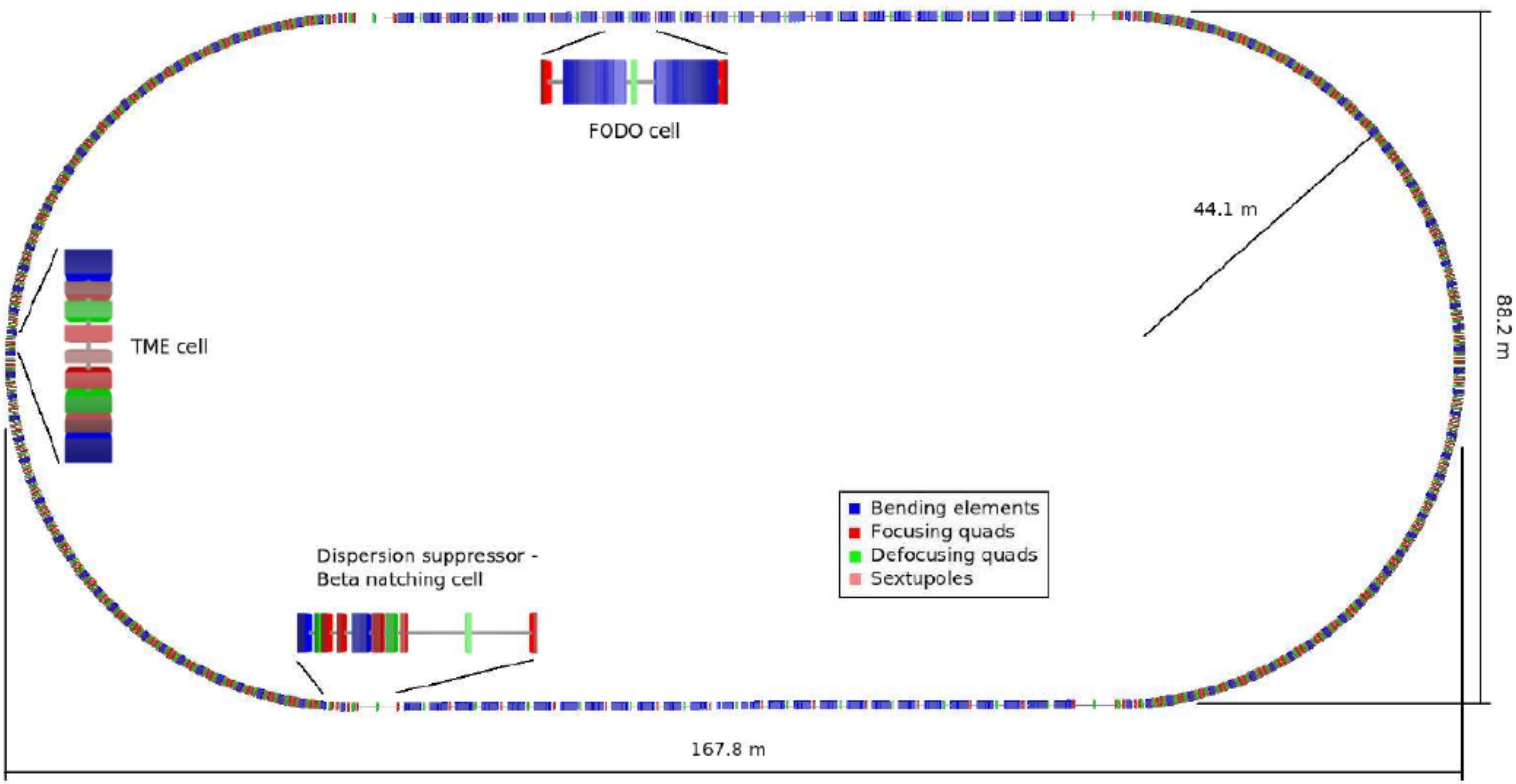
ILC DR magnets

Magnet	Type	Eng. Style	Qty	Power Method
Dipoles:	Corrector	D60L250	304	Individual
	Chicane	D60L940	28	String
	Disp. Supp.	D60L1940	10	String
	Arc	D60L2940	150	String
Quadrupoles:	Arc	Q60L480	482	Individual
	Straight	Q60L700	121	Individual
	Wig/Inj/Ext	Q85L309	50	Individual
	Wiggler	Q85L600	30	Individual
Skew Quads	Corrector	Q60L250	158	Individual
Sextupoles	—	SX60L250	600	Individual
Wigglers	—	WG76L2100	54	Individual
Kickers	Inj/Ext	Striplines	42	Individual
Thin Pulsed Septa	Inj/Ext	—	2	Individual
Thick Pulsed Septa	Inj/Ext	—	2	Individual

Type	Unit	Max Field Max KL	Error
Dipoles	mrad	41	2×10^{-4}
Quadrupoles	m^{-1}	0.35	2×10^{-4}
Sextupoles	m^{-2}	1.23	2×10^{-4}
H correctors	mrad	2	5×10^{-3}
V correctors	mrad	2	5×10^{-3}
Skew quads	m^{-1}	0.03	3×10^{-3}
Wigglers		–	3×10^{-3}

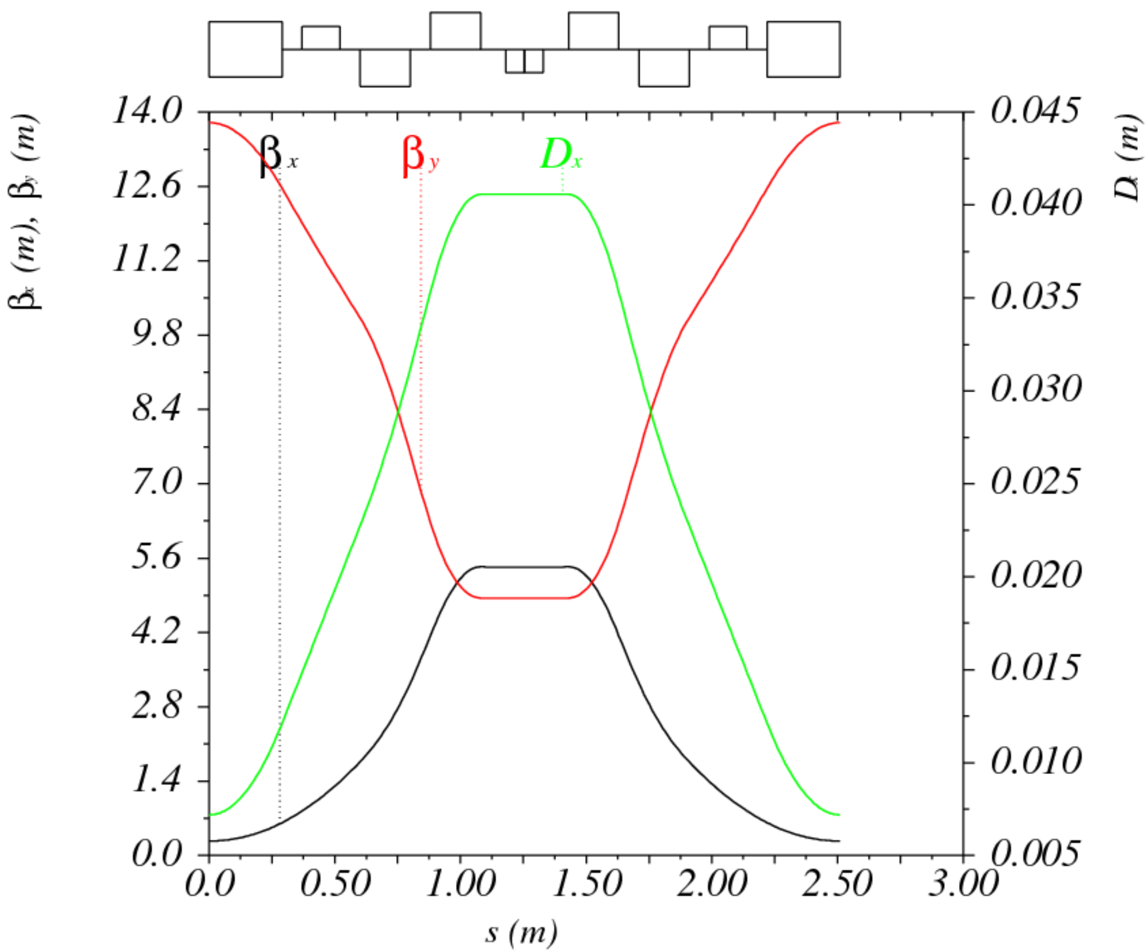
- ILC DR filled with conventional electromagnets for the dipole, quadrupole, sextupole, and corrector magnets.
- This offers flexibility for tuning and optimizing the rings as well as for adjusting the operating beam energy by a few percent around the nominal value of 5 GeV.
- Maximum strength and field tolerances within capabilities of modern magnets

The CLIC DR lattice



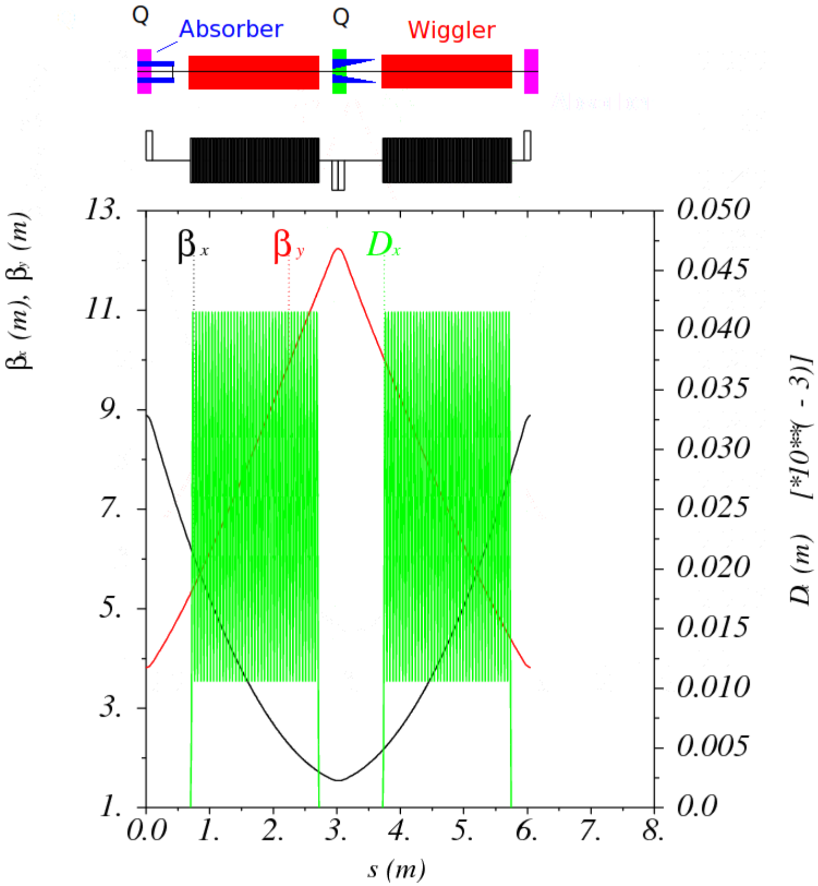
- Racetrack configuration
  - 2 arc sections filled with TME cells
  - 2 long straight sections filled with FODO cells accommodating the damping wigglers

Optimized TME cell

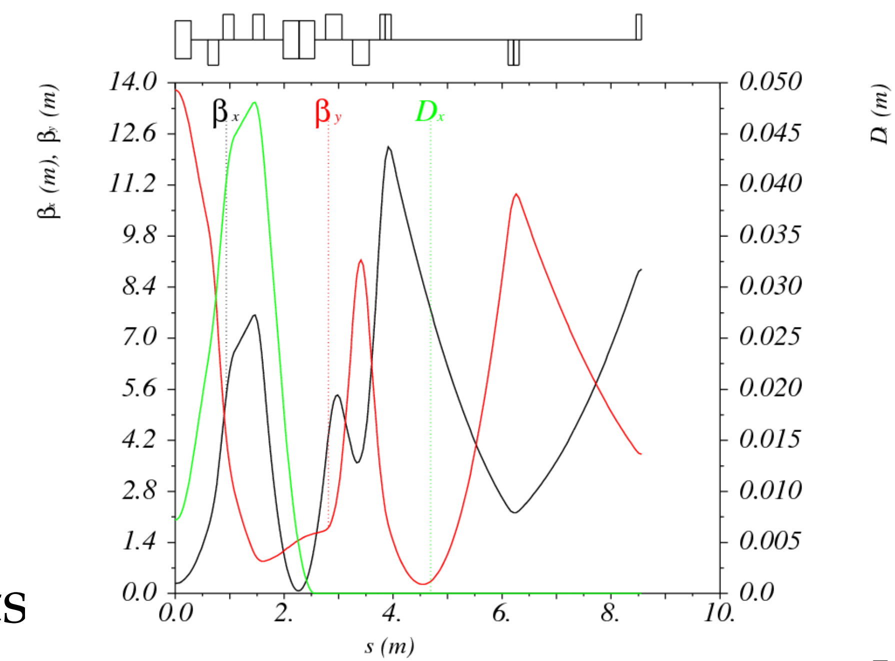


- TME cell with defocusing gradient along the dipole length
  - Reduction of the IBS effect
- Dipole length increased
  - l_d=0.58m (from 0.43m)
- Horizontal phase advance reduced
  - μ_{xTME}=0.408 (from 0.452)
- RF voltage decreased
  - V_{RF}=5.1MV (from 4.5MV)

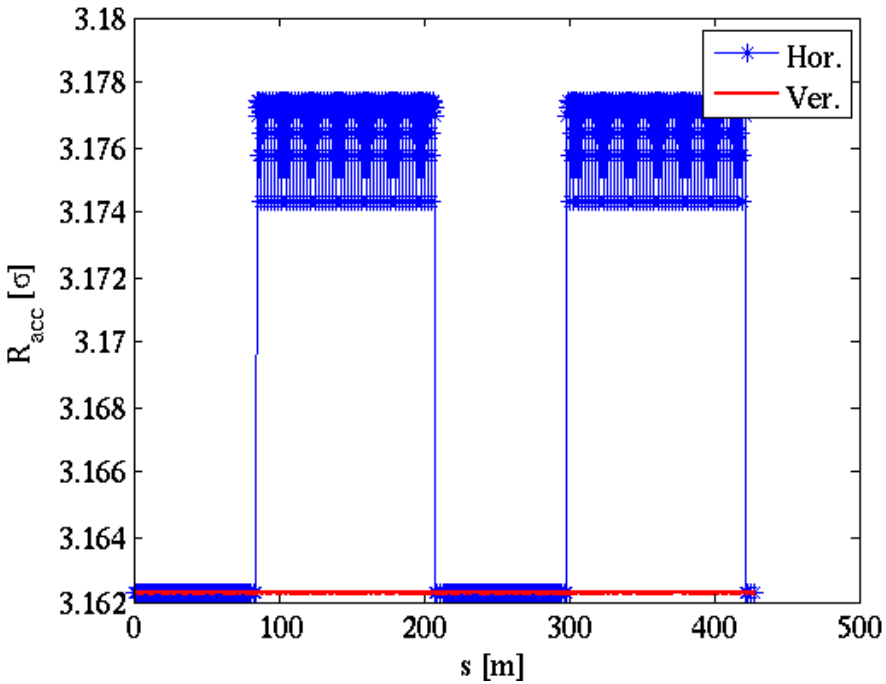
CLIC DR FODO and DS-BM cells

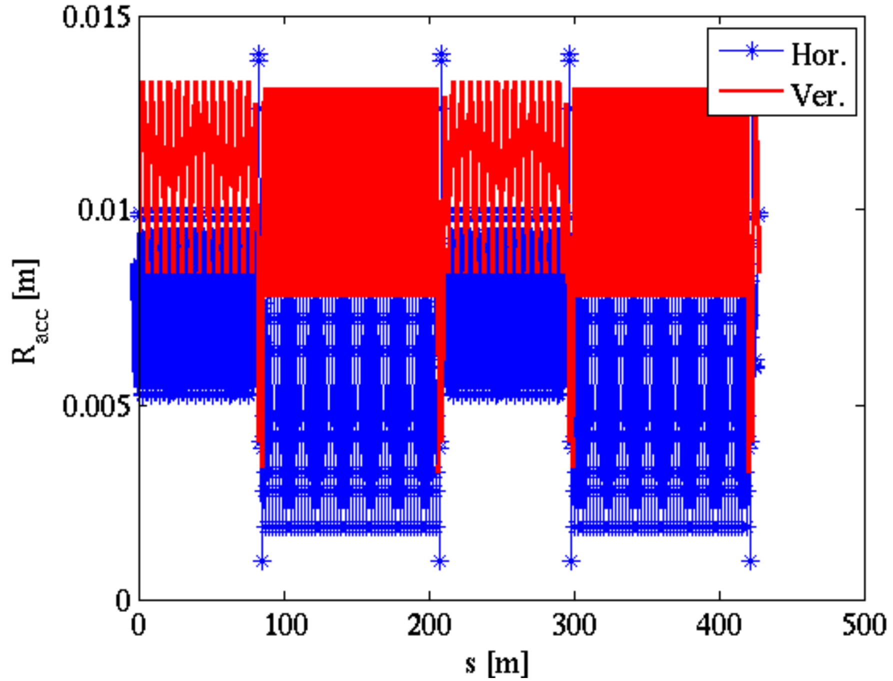


- FODO cells accommodate the damping wigglers (2 wigglers / cell)
- Space is reserved for the **absorption scheme** of synchrotron radiation
- Dispersion suppression – beta matching cell optics
- Space is reserved for **injection/extraction** elements and **RF cavities**



Geometrical acceptance of the DR





- Required geometrical acceptance around the DR to fit the incoming beam
- For Gaussian beams (coming from PDR)

$$R_{\mathrm{min}} = \sqrt{2\beta\varepsilon_{\mathrm{max}} + (D(\delta p/p_0)_{\mathrm{max}})^2}$$

DR- magnetic parameters

Type	Location	Length [m]	Number	Families	Pole tip field [T]	Full aperture H/V [mm]
Dipoles	Arc DS-BM	0.58	96 4	1	0.97	80/20
Quadrupoles	Arc	0.20	376	2	1.0	20/20
	LSS	0.20	28+26	2		
	DS-BM	0.20	24	12		
	DS-BM	0.31	4	2		
Sextupoles	Arc	0.15	188+94	2	0.5	20/20
Wigglers	LSS	2.00	52	1	2.5	80/13

- Summary table of the magnet parameters of the DR
- All maximum fields can be achieved by standard electromagnets magnets (apart wigglers)

Summary

- Low emittance cannot be achieved with FODO cells
- Special lattice cells are needed (TME or MBE)
- Damping wigglers enhance emittance and damping time reduction but slightly increases longitudinal beam characteristics
- Although ILC and CLIC DR parameters differ in some extent, lattice design concept is quite similar
- In order to close the loop, non-linear dynamics, collective effects and technology considerations have to be taken into account